Distributed Coordination

Overview

In network of agents distributed coordination generally means to have the whole group of vehicles working in a cooperative fashion through a distributed protocol.

Roughly speaking coordination algorithms can be divided into three categories:

- Swarming
- Flocking
- Formation Control

Distributed Coordination

Swarming

Swarm Robotics

Swarming aims at achieving an aggregation of the team through local simple interaction.

The key points of swarming robotics are:

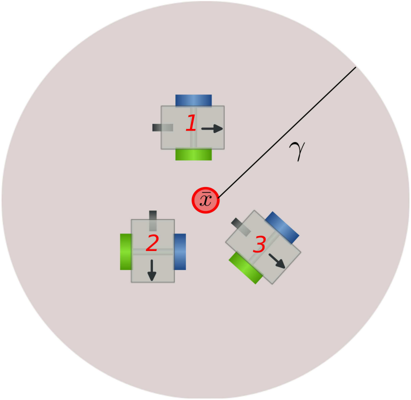
- Focuses on a large number of simple autonomous agents,
- An emergent global behavior arises from local interactions.
- Can provide high robustness and flexibility,

Aggregation (in space)

Consider a team of n agents, the team is said to be showing an aggregative behavior if the following holds:

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| \leq 2\gamma, \quad \forall\, i, j \in 1, \ldots, n \tag{51}$$

with γ the aggregation radius.



State of the Art

Several works can be found in the literature about swarm robotics:

- Veysel Gazi and Kevin M. Passino. "A class of attractions/repulsion functions for stable swarm aggregations". In: *International Journal of Control* 77.18 (2004)
- Wei Li. "Stability Analysis of Swarms With General Topology". In: *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 38.4 (2008), pp. 1084–1097
- D.V. Dimarogonas and K.J. Kyriakopoulos. "Connectedness Preserving Distributed Swarm Aggregation for Multiple Kinematic Robots". In: *IEEE Transactions on Robotics* 24.5 (2008), pp. 1213 –1223. ISSN: 1552-3098
- Andrea Gasparri et al. "A Swarm Aggregation Algorithm Based on Local Interaction with Actuator Saturations and Integrated Obstacle Avoidance". In: *The 2013 IEEE International Conference on Robotics and Automation (ICRA 2013)*. 2013

Distributed Coordination

Flocking

Flocking

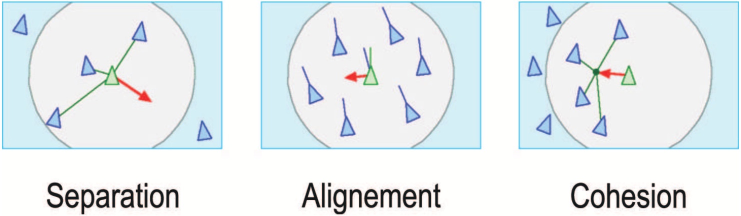
Flocking is a form of collective behavior of large number of agents with an agreement in the direction of motion and velocity.

According to Reynolds' model[1], a flocking behavior is controlled by three simple rules:

- Separation: avoid crowding neighbors (short range repulsion)
- Alignment: steer towards average heading of neighbors
- Cohesion: steer towards average position of neighbors (long range attraction)

[1] Craig W. Reynolds. "Flocks, herds and schools: A distributed behavioral model". In: *SIGGRAPH Comput. Graph.* 21.4 (Aug. 1987), pp. 25–34.

Reynolds Model Simulations



State of the Art

Several works can be found in the literature about flocking:

- A. Jadbabaie, Lin Jie, and A.S. Morse. "Coordination of groups of mobile autonomous agents using nearest neighbor rules". In: *IEEE Transactions on Automatic Control* 48.6 (2003), pp. 988–1001
- R. Olfati-Saber. "Flocking for multi-agent dynamic systems: algorithms and theory". In: *IEEE Transactions on Automatic Control* 51.3 (2006), pp. 401–420
- H.G. Tanner, A. Jadbabaie, and G.J. Pappas. "Flocking in Fixed and Switching Networks". In: *IEEE Transactions on Automatic Control* 52.5 (2007), pp. 863–868
- F. Cucker and S. Smale. "Emergent Behavior in Flocks". In: *IEEE Transactions on Automatic Control* 52.5 (2007), pp. 852–862

The Tanner et al. Framework

Objective: To interpret Reynolds flocking model as a mechanism for achieving velocity synchronization and regulation of relative distances within a group of agents.

Contribution: To derive decentralized controllers which provably lead to flocking, even when information exchange between the agents can change arbitrarily fast.

Limitation: Only applicable to identical vehicles with a double-integrator dynamics

The Tanner et al. Framework

Consider a group of n double integrator vehicles on the plane:

$$\begin{aligned} \dot{r}_i &= v_i \\ \dot{v}_i &= u_i \end{aligned} \tag{52}$$

Assume the control input of each agent i to be:

$$u_i = \alpha_i + a_i \tag{53}$$

where :

- α_i aims at aligning the velocity vectors of all the agents
- a_i is used for collision avoidance and cohesion in the group.

The Tanner et al. Framework

Define the velocity graph as an undirected graph $\mathcal{G}_c = \{\mathcal{V}, \mathcal{E}_c\}$:

- $\mathcal{V}$ the set of vertices (agents)
- $\mathcal{E}_c$ the set of edges (communication links)

Denote with $\mathcal{N}_c(i) = \{j \, : \, (i,j) \in \mathcal{E}_c\}$ the velocity graph neighborhood of agent i

Define the position graph as an undirected graph $\mathcal{G}_s = \{\mathcal{V}, \mathcal{E}_s\}$:

- $\mathcal{V}$ the set of vertices (agents)
- $\mathcal{E}_s = \{(i,j) \, : \, \|r_i - r_j\| \leq R\}$ the set of edges (sensing links)

Denote with $\mathcal{N}_s(i) = \{j \, : \, (i,j) \in \mathcal{E}_s\}$ the position graph neighborhood of agent i

The Tanner et al. Framework

Let a potential function V_{ij} be a differentiable, nonnegative function of the distance $\|r_{ij}\|$ between agents i and j such that:

- $V_{ij}(\|r_{ij}\|) \to \infty$ as $\|r_{ij}\| \to 0$
- $\dfrac{\partial V_{ij}(\|r_{ij}\|)}{\partial \|r_{ij}\|} = 0$ if $\|r_{ij}\| > R$
- $\exists!\, \text{argmin}\, V_{ij}(\|r_{ij}\|) = \delta$ with δ the desired distance

The potential of agent i is:

$$V_i = \sum_{j \in \mathcal{N}_s(i)} V_{ij}(\|r_{ij}\|) \tag{54}$$

The Tanner et al. Framework

The control input of agent i can be defined as:

$$u_i = \underbrace{- \sum_{j \in \mathcal{N}_c(i)} (v_i - v_j)}_{\alpha_i} \underbrace{- \sum_{i=1}^{N} \nabla_{r_i} V_i}_{a_i} \tag{55}$$

Observations:

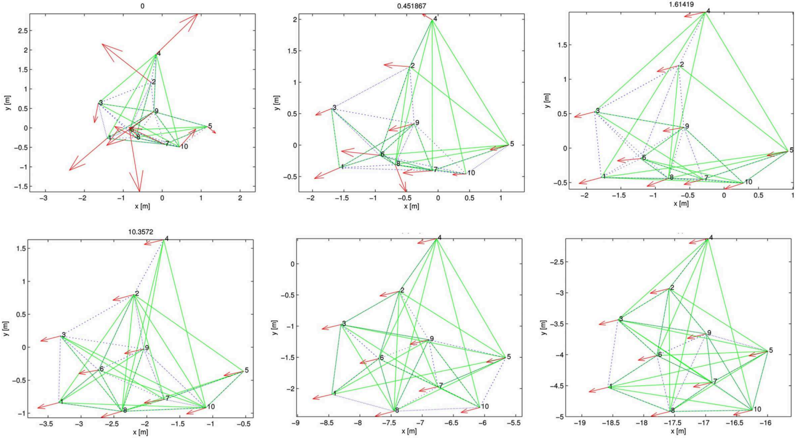
- The control requires only local interaction among the agents
- The agent position must be available
- The agent velocity must be available
- The connectivity must be preserved over time

The Tanner et al. Framework

Theorem: Consider a system of n double integrators vehicles, each steered by control law (55). Assume both the position and velocity graphs be time-varying but always connected. Then:

- all agent velocity vectors become asymptotically the same,
- collisions between interconnected agents are avoided,
- the system approaches a configuration that minimizes all agent potentials.

The Tanner et al. Framework



Distributed Coordination

Formation Control

Control Techniques

The key idea is to design distributed control laws so that the multi-robot system can achieve a desired formation shape.

Different mathematical tools have been applied to reach this objective. Among the others:

- Matrix Theory
- Rigidity Theory

Matrix Theory

Matrix Theory is a fairly simple tool to investigate the stability of a dynamical linear system.

Example of approaches belonging to this category are:

- J.A. Fax and R.M. Murray. "Information flow and cooperative control of vehicle formations". In: *IEEE Transactions on Automatic Control* 49.9 (2004), pp. 1465–1476
- Peng Lin and Yingmin Jia. "Distributed rotating formation control of multi-agent systems". In: *Systems & Control Letters* 59.10 (2010), pp. 587 –595
- Wei Ren and Nathan Sorensen. "Distributed coordination architecture for multi-robot formation control". In: *Robotics and Autonomous Systems* 56.4 (2008), pp. 324 –333

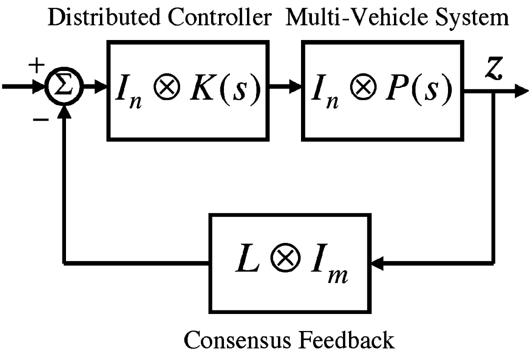
The Fax and Murray Framework

Objective: To provide a system-theoretic framework for addressing the problem of cooperative control of networked multi-vehicle systems using distributed controllers.

Contribution: To prove a Nyquist criterion that uses the eigenvalues of the graph Laplacian matrix to determine the effect of the communication topology on formation stability.

Limitation: Only applicable to identical vehicles with a time-invariant linear dynamics

The Fax and Murray Framework



Observations:

- The coupling occurs through cooperation via the consensus feedback.
- This cooperation requires sharing of information among vehicles, either through sensing or explicit communication.

The Fax and Murray Framework

Consider a group of n vehicles with an identical linear dynamics:

$$\dot{x}_i = A\,x_i + B\,u_i, \quad x_i \in \mathbb{R}^m,\ u_i \in \mathbb{R}^p \tag{56}$$

Each vehicle receives the following measurements:

$$\begin{aligned} y_i &= C_1 x_i, \quad y_i \in \mathbb{R}^k \\ z_{ij} &= C_2(x_i - x_j), \quad j \in \mathcal{N}_i, \quad z_{ij} \in \mathbb{R}^l \end{aligned} \tag{57}$$

A single vehicle cannot drive all the z_{ij} terms to zero simultaneously, thus a single fused signal error measurement is considered:

$$z_i = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} z_{ij} \tag{58}$$

The Fax and Murray Framework

Consider a distributed controller K of the form:

$$\begin{aligned} \dot{v}_i &= F\, v_i + G_1\, y_i + G_2\, z_i \\ u_i &= H\, v_i + D_1\, y_i + D_2\, z_i \end{aligned} \tag{59}$$

The collective dynamics of the system of all n vehicles is:

$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} \tag{60}$$

with:

$$\begin{aligned} M_{11} &= I_N \otimes (A + B\, D_1\, C_1) + (I_N \otimes B\, D_2\, C_2)(L \otimes I_N) \\ M_{12} &= I_N \otimes B\, H \\ M_{21} &= I_N \otimes G_1\, C_1 + (I_N \otimes G_2\, C_2)(L \otimes I_N) \\ M_{22} &= I_N \otimes F \end{aligned} \tag{61}$$

The Fax and Murray Framework

Theorem: A local controller K stabilizes the formation dynamics in (60) if and only if it stabilizes all the n systems:

$$\begin{aligned} \dot{x}_i &= A\,x_i + B\,u_i \\ y_i &= C_1 x_i \\ z_i &= \lambda_i\, C_2 x_i \end{aligned} \tag{62}$$

with $\{\lambda_i\}_{i=1}^n$ the eigenvalues of the normalized graph Laplacian L.

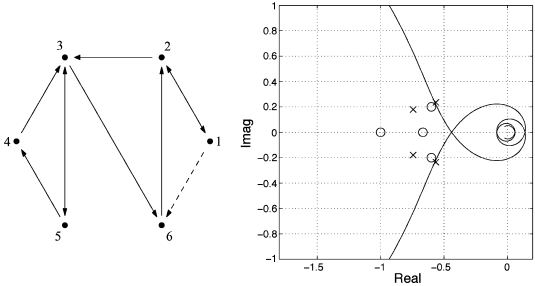
Observations:

- The stability of a formation of n identical vehicles can be verified by stability analysis of a single vehicle with the same dynamics and an output that is scaled by the eigenvalues of the (normalized) Laplacian of the network.

The Fax and Murray Framework

Theorem: Suppose P is a SISO system. Then K stabilizes the relative dynamics of a formation if and only if the net encirclement of $-1/\lambda_i$ by the Nyquist plot of $K(S)\,P(s)$ is zero for all non- zero $\lambda_i > 0$.

Example:



(a) (b)

Nyquist for the $K(s)P(s)$ of a stable vehicle dynamics where the "o" locations correspond to the eigenvalues of the graph defined by the solid arcs "x" locations are for eigenvalues of the graph when the dashed arc is also included.

Rigidity Theory

Rigidity Theory turns out to be useful for the design of control algorithm where range-only measurements are available.

Example of approaches belonging to this category are:

- B. D O Anderson, Yu Changbin, B. Fidan, and J.M. Hendrickx. "Rigid graph control architectures for autonomous formations". In: *IEEE Control Systems* 28.6 (2008), pp. 48–63
- Laura Krick, E. Mireille Broucke, and Bruce A. Francis. "Stabilisation of infinitesimally rigid formations of multi-robot networks". In: *International Journal of Control* 82.3 (2009), pp. 423–439
- Changbin Yu and Brian D. O. Anderson. "Development of redundant rigidity theory for formation control". In: *International Journal of Robust and Nonlinear Control* 19.13 (2009), pp. 1427–1446. ISSN: 1099-1239
- Ming Cao, Changbin Yu, and Brian D.O. Anderson. "Formation control using range-only measurements". In: *Automatica* 47.4 (2011)

Connectivity Maintenance

Motivations

To achieve a collaborative behavior, robots are usually required either to communicate or to sense each other all the time.

$$\Downarrow$$

Decentralized algorithm to preserve the MRS connectivity during the execution of a task must be designed.

Taxonomy

Several approaches have been proposed in the literature to preserve the connectivity maintenance of MRSs.

Roughly speaking those approaches can be divided into two categories:

- Approaches to maintain the local connectivity,
- Approaches to maintain the global connectivity.

Local Connectivity Maintenance I

Definition:
The local connectivity maintenance problem aims at preserving the original set of links defining the connectivity graph over time.

Examples of decentralized algorithms for local connectivity maintenance are:

- Ji Meng and M. Egerstedt. "Distributed Coordination Control of Multiagent Systems While Preserving Connectedness". In: *IEEE Transactions on Robotics* 23.4 (2007), pp. 693–703
- A. Ajorlou, A. Momeni, and A.G. Aghdam. "A Class of Bounded Distributed Control Strategies for Connectivity Preservation in Multi-Agent Systems". In: *IEEE Transactions on Automatic Control* 55.12 (2010), pp. 2828–2833
- D.V. Dimarogonas and K.H. Johansson. "Bounded control of network connectivity in multi-agent systems". In: *IET Control Theory Applications* 4.8 (2010), pp. 1330–1338

The Dimarogonas and Johansson Framework

Objective: A distributed control law that guarantees connectivity maintenance in a network of multiple mobile agents.

Contribution: Design of a bounded control laws that drive the agents to an agreement point while preserving the connectivity properties induced by the inter-agent relative initial positions.

Limitation: Only applicable to vehicles with single integrator dynamics

The Dimarogonas and Johansson Framework

Consider a group of n single integrator vehicles on the plane:

$$\dot{q}_i = u_i \tag{63}$$

Denote as communication graph $\mathcal{G} = \{V, E\}$ an undirected graph
Define the neighborhood $\mathcal{N}_i$ of agent i as:

$$\mathcal{N}_i = \{j \in (1, \dots, n), j \neq i \;::\; \|q_i(0) - q_j(0)\| \leq d\} \tag{64}$$

Define the communication graph $\mathcal{G} = \{V, E\}$ as an undirected graph with $V = \{1, \dots, n\}$ and $E = \{(i, j) \in V \times V \;:\; j \in \mathcal{N}_i\}$.
with:

- $V = \{1, \dots, n\}$
- $E = \{(i, j) \in V \times V \;:\; j \in \mathcal{N}_i\}$

The Dimarogonas and Johansson Framework

Define the bounded potential $\psi : \mathbb{R}^{2n} \rightarrow [0,1]$ for agent i as:

$$\psi_i = \frac{\gamma_i}{(\gamma_i^k + G_i)^{1/k}}, \quad k > 0 \tag{65}$$

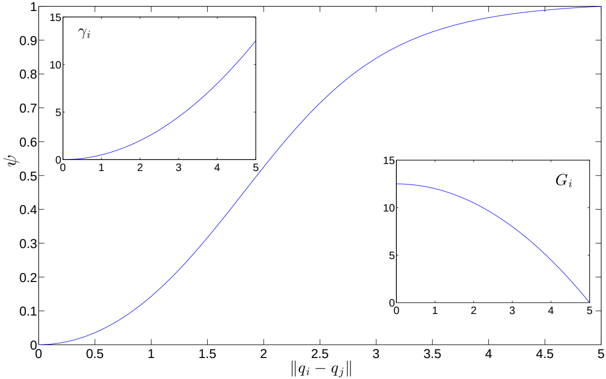
where:

- $\gamma_i(q) = \sum_{j \in \mathcal{N}_i} \frac{1}{2} \|q_i - q_j\|^2$
- $G_i(q) = \prod_{j \in \mathcal{N}_i} \frac{1}{2} \left(d^2 - \|q_i - q_j\|^2 \right)$

Observations:

- γ_i is responsible for the aggregation objective
- G_i is responsible for the maintenance of the initial set of edges

The Dimarogonas and Johansson Framework



Observations:

- γ_i is minimized when the desired objective is fulfilled
- G_i is minimized when a link between two agents tends to be broken

The Dimarogonas and Johansson Framework

Define the gradient control of agent i as:

$$\begin{aligned} u_i &= -K_i \frac{\partial \psi_i}{\partial q_i} \\ &= -\frac{K_i}{(\gamma_i^k + G_i)^{\frac{1}{k}+1}} \left(G_1 \nabla_i \gamma_i - \frac{\gamma_i}{k} \nabla_i G_i \right) \\ &= -\frac{K_i}{(\gamma_i^k + G_i)^{\frac{1}{k}+1}} \sum_{j \in \mathcal{N}_i} \pi_{ij}(q_i - q_j) \end{aligned} \tag{66}$$

Observations:

- γ_i and G_i never go to zero simultaneously,
- The control law is bounded when $\|q_i - q_j\| \rightarrow d$

The Dimarogonas and Johansson Framework

Lemma: The set $\mathcal{Q} : \{q \in \mathbb{R}^{2n} \; : \; G_i > 0, i \in \mathcal{N}\}$ is an invariant set for the trajectory of the closed loop system.

Observations:

- Links available at $t = 0$ are preserved over time
- if $\|q_i(0) - q_j(0)\| \leq d \;\Rightarrow \|q_i(0) - q_j(0)\| \leq d\| \,\forall\, t > 0$
- The neighborhoods $\mathcal{N}_i$, $i \in V$ are static

How flexible are we?

Observations:

- Preserving each link of the communication graph over time is a very restrictive requirement and significantly limits the capability of a MRS.
- As long as the overall graph is still connected: if necessary, redundant links can be removed, and new ones can be added.

$$\Downarrow$$

Imposing the global connectivity maintenance ensures that none of the robots loses connectivity from the rest of the group, while not reducing the ability of the group to move into cluttered environments.

Connectivity Maintenance

Global Connectivity Maintenance

Global Connectivity Maintenance

Definition:
The global connectivity maintenance problem aims at preserving the connectivity of graph over time (links might get broken or created).

An effective framework to achieve it relies on the estimation of the algebraic connectivity to derive a proper control action to avoid disconnections.

Algebraic Connectivity Estimation

Examples of strategies for the estimation of the algebraic connectivity of an undirected graph and its related eigenvector are:

- P. Yang et al. "Decentralized estimation and control of graph connectivity for mobile sensor networks". In: *Automatica* 46.2 (Feb. 2010), pp. 390–396
- Tuhin Sahai, Alberto Speranzon, and Andrzej Banaszuk. "Hearing the clusters of a graph: A distributed algorithm". In: *Automatica* 48.1 (2012), pp. 15 –24
- Mauro Franceschelli, Andrea Gasparri, Alessandro Giua, and Carla Seatzu. "Decentralized estimation of Laplacian eigenvalues in multi-agent systems". In: *Automatica* 49.4 (2013), pp. 1031 –1036

The Franceschelli et al. Framework

Objective: To provide a framework for the estimation of the algebraic connectivity of the Laplacian matrix encoding the network topology of a multi-agent system

Contribution: A decentralized algorithm for the estimation of the whole spectrum of the Laplacian matrix.

Limitation: Only applicable to network topologies described by an undirected graph

The Franceschelli et al. Framework

Idea...

To design a local interaction rule among agents so that their state oscillates only at frequencies corresponding to eigenvalues of the network topology.

$$\Downarrow$$

The problem of decentralized eigenvalues estimation is mapped into a problem of signal processing, solvable by applying any desired frequency estimation algorithm.

The Franceschelli et al. Framework

Each agent performs the following local update rule:

$$\begin{cases} \dot{x}_i(t) = z_i(t) + \sum_{j\in\mathcal{N}_i}(z_i(t) - z_j(t)) \\ \dot{z}_i(t) = -x_i(t) - \sum_{j\in\mathcal{N}_i}(x_i(t) - x_j(t)) \end{cases} \quad (67)$$

The collective dynamics is a time-varying switching autonomous linear system:

$$\begin{bmatrix} \dot{x}(t) \\ \dot{z}(t) \end{bmatrix} = \mathcal{A}(t) \cdot \begin{bmatrix} x(t) \\ z(t) \end{bmatrix} \quad (68)$$

where

$$\mathcal{A} = \begin{bmatrix} \mathbf{0}_{n\times n} & I + \mathcal{L} \\ -I - \mathcal{L} & \mathbf{0}_{n\times n} \end{bmatrix}, \quad (69)$$

I is the $n \times n$ identity matrix and $\mathbf{0}_{n\times n}$ is the null $n \times n$ matrix.

The Franceschelli et al. Framework

Consider the system matrix:

$$\mathcal{A} = \begin{bmatrix} \mathbf{0}_{n\times n} & I+\mathcal{L} \\ -I-\mathcal{L} & \mathbf{0}_{n\times n} \end{bmatrix} \tag{70}$$

Observations:

- Matrix $\mathcal{A}$ is skew symmetric, i.e., $\mathcal{A}^T = -\mathcal{A}$.
- A skew-symmetric matrix has eigenvalues only on the imaginary axis of the Gauss plane.
- The eigenvalues of $\mathcal{A}$ can be analytically derived from the eigenvalues of the Laplacian matrix $\mathcal{L}$

The Franceschelli et al. Framework

Lemma:Let $\mathcal{G}$ be an undirected graph with Laplacian $\mathcal{L}$. Let matrix $\mathcal{A}$ be defined as in (70). To any eigenvalue $\lambda_{\mathcal{L}}$ of $\mathcal{L}$ it corresponds a couple of complex and conjugates eigenvalues $\lambda_{\mathcal{A}}, \bar{\lambda}_{\mathcal{A}}$ of $\mathcal{A}$, that is:

$$\lambda_{\mathcal{A}} = j(1 + \lambda_{\mathcal{L}}), \quad \bar{\lambda}_{\mathcal{A}} = -j(1 + \lambda_{\mathcal{L}}), \tag{71}$$

while the corresponding eigenvectors $v_{\lambda_{\mathcal{A}}}$ are function of the eigenvectors $v_{\lambda_{\mathcal{L}}}$ of $\mathcal{L}$

$$v_{\lambda_{\mathcal{A}}} = \begin{bmatrix} v_{\lambda_{\mathcal{L}}}^T & j v_{\lambda_{\mathcal{L}}}^T \end{bmatrix}^T, \qquad \bar{v}_{\bar{\lambda}_{\mathcal{A}}} = \begin{bmatrix} v_{\lambda_{\mathcal{L}}}^T & -j v_{\lambda_{\mathcal{L}}}^T \end{bmatrix}^T. \tag{72}$$

The Franceschelli et al. Framework

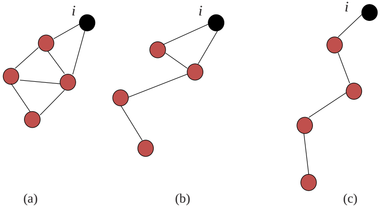
Theorem: Let $\{\lambda_j\}_{j=1}^{m}$ be the set of m distinct eigenvalues of the Laplacian matrix $\mathcal{L}$ and $\delta(\cdot)$ the Dirac's delta function. The module of the Fourier transform of the i-th state components $x_i(t)$ and $z_i(t)$ is:

$$\begin{aligned} |\mathcal{F}[x_i(t)]| = |X_i(f)| = \sum_{j=1}^{m} \frac{a_{j,i}}{2}\,\delta\left(f \pm \frac{1+\lambda_j}{2\pi}\right) \\ |\mathcal{F}[z_i(t)]| = |Z_i(f)| = \sum_{j=1}^{m} \frac{b_{j,i}}{2}\,\delta\left(f \pm \frac{1+\lambda_j}{2\pi}\right) \end{aligned} \tag{73}$$

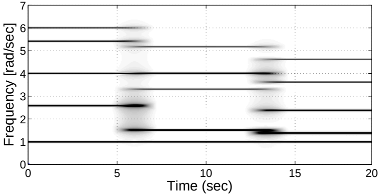
Observations:

- Each agent retrieves the eigenvalues of $\mathcal{L}$ by estimating the frequencies at which its state variable $x_i(t)$ oscillates
- The coefficients $\{a_{j,i},\, b_{j,i}\}$ depend upon the initial conditions $[x(0),\, z(0)]$.

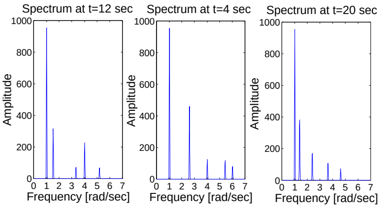
The Franceschelli et al. Framework



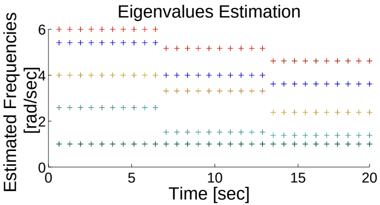
(a) Network Topology



(b) Spectrogram



(c) Topologies Spectrum



(d) Eigenvalues Estimate

Global Connectivity Maintenance III

Examples of decentralized algorithms for global connectivity maintenance are:

- P. Yang et al. "Decentralized estimation and control of graph connectivity for mobile sensor networks". In: *Automatica* 46.2 (Feb. 2010), pp. 390–396
- Paolo Robuffo Giordano, Antonio Franchi, Cristian Secchi, and Heinrich H Blthoff. "A passivity-based decentralized strategy for generalized connectivity maintenance". In: *The International Journal of Robotics Research* 32.3 (2013), pp. 299–323
- Lorenzo Sabattini, Cristian Secchi Nikhil Chopra, and Andrea Gasparri. "Distributed Control of Multi-Robot Systems with Global Connectivity Maintenance". In: *IEEE Transactions on Robotics* (2013). To appear

The Yang et al. Framework

Objective: To provide a complete framework for the estimation and control of the network topology algebraic connectivity for a multi-agent system.

Contribution: A decentralized power iteration algorithm to let agent i estimate the i-th component of the second eigevenctor and choose a motion direction to increase the algebraic connectivity.

Limitation: Only applicable to identical vehicles with a single-integrator dynamics and no integration with external controllers.

The Yang et al. Framework

Consider the following adjacency matrix:

$$A_{ij} = \begin{cases} e^{\dfrac{-\|p^i - p^j\|_2^2}{2\sigma^2}} & \text{if } \|p^i - p^j\|_2 \leq r \\ 0 & \text{otherwise} \end{cases} \tag{74}$$

with r the maximal reliable inter-agent communication distance.

Observations:

- The weight decreases as the inter-agent distance gets larger.
- σ is chosen to satisfy $e^{-r^2/2\sigma^2} = \epsilon$.
- λ_2 increases if the graph adds more links or two agents come closer.

The Yang et al. Framework

The gradient control law for agent k is:

$$
\begin{aligned}
u^k &= \frac{\partial \lambda_2}{\partial p^k} \\
&= \hat{v}_{\mathcal{L}_2}^T \frac{\partial L}{\partial p^k} \hat{v}_{\mathcal{L}_2} \\
&= \sum_{(k,j)\in E} -A_{kj} \left(v_{\mathcal{L}_2}^k - v_{\mathcal{L}_2}^j \right)^2 \frac{p^k - p^j}{\sigma^2}
\end{aligned}
\tag{75}
$$

Each agent must know:

- The location of the neighboring agents $\{p^j\}_{j\in\mathcal{N}_k}$
- The components of the second eigenvector $\{v_{\mathcal{L}_\in}^j\}_{j\in\mathcal{N}_k\cup k}$

The Yang et al. Framework

How to retrieve information about the Fiedler vector?

$$\Downarrow$$

Consider the following estimator:

$$\dot{x} = -k_1 \text{ Ave}\left\{x^i\right\} \mathbf{1} - k_2 \mathcal{L} x - k_3 \left(\text{Ave}\left\{\left(x^i\right)^2\right\} - 1\right) x \qquad (76)$$

Observations:

- The Ave operation can be carried out with a consensus algorithm

The Yang et al. Framework

Theorem: Given any initial condition $x(t_0)$ and positive gains $k_1, k_2, k_3 > 0$, as long as $v_2^T x(t_0) \neq 0$, the gain conditions:

$$\begin{aligned} k_1 &> \lambda_2 k_2 \\ k_3 &> \lambda_2 k_2 \end{aligned} \tag{77}$$

are necessary and sufficient for system (77) to converge to an eigenvector $\|\tilde{v}_2\|$ corresponding to the eigenvalue λ_2 of the Laplacian matrix $\mathcal{L}$ satisfying:

$$\|\tilde{v}_2\| = \sqrt{n\left(\frac{k_3 - k_2\lambda_2}{k_3}\right)} \tag{78}$$

Applications

Collaborative Transportation

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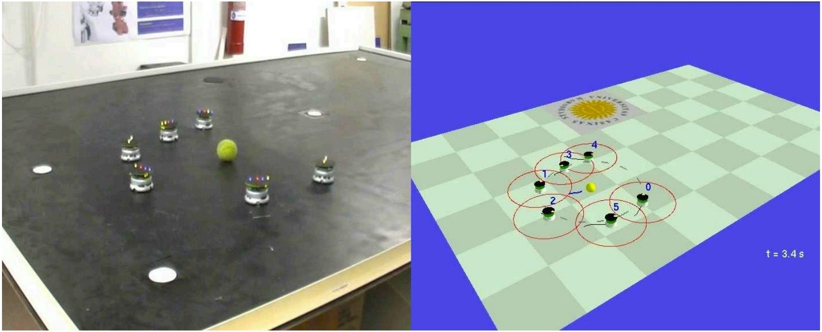
Target Entrapment/Escorting



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Any questions?

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