Convex Hull Convolutive NMF for Uncovering Temporal Patterns in Multivariate Time-Series Data

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Motivation

- Time-series data often contains rich structure
- **Goal**: automatically extract temporal patterns from a time-series

NMF Background

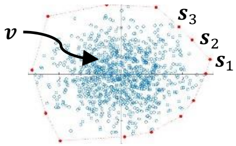
- **Non-negative Matrix Factorization**: decompose non-negative matrix V into product of two non-negative matrices WH

V input ≈ W dictionary × H encodings

- **Convolutive NMF**: consider temporal context in the input and learn a time-varying dictionary

V input ≈ $W(t)$ time-varying dictionary × H encodings

- Sparsity constraints typically imposed on encoding matrix (eg. CNMF-SC)
- **Convex Hull NMF**: form the dictionary from convex combinations of the points on the convex hull of the input
  - Allows mixed-sign input



$$\boldsymbol{v} = g_1\boldsymbol{s}_1 + g_2\boldsymbol{s}_2 + \cdots + g_p\boldsymbol{s}_p$$

V input ≈ $\underbrace{S \text{ convex hull points} \times G \text{ convex combinations}}_{W \text{ dictionary}}$ × H encodings

Convex Hull Convolutive NMF

- Incorporate temporal context in CH-NMF algorithm
- Key advantages over CNMF
  - Relax non-negativity constraint on input
  - Time-varying dictionary has same scale as input

V input ≈ $\underbrace{S \text{ convex hull points} \times G(t) \text{ time-varying convex combinations}}_{W(t) \text{ time-varying dictionary}}$ × H encodings

- Find K temporal patterns with duration T
- Algorithm:
  1. Input: V, K, T, λ (sparsity level of encoding matrix)
  2. Find convex hull points S from data in V
  3. Iteratively update G and H

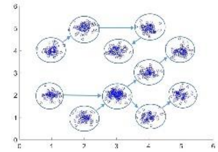
$$G(t) \leftarrow G(t) \otimes \frac{\left(\left[S^\top V\right]^+ + \left[S^\top S\right]^- F\right) \overset{t\rightarrow}{H}{}^\top}{\left(\left[S^\top V\right]^- + \left[S^\top S\right]^+ F\right) \overset{t\rightarrow}{H}{}^\top},\ \forall t$$

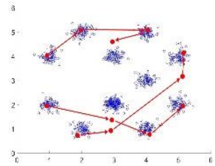
$$H \leftarrow H \otimes \frac{\sum_{t=0}^{T-1} G^\top(t) \left(\left[S^\top V\right]^+ \overset{\leftarrow t}{I_n} + \left[S^\top S\right]^- \overset{\leftarrow t}{F}\right)}{\sum_{t=0}^{T-1} G^\top(t) \left(\left[S^\top V\right]^- \overset{\leftarrow t}{I_n} + \left[S^\top S\right]^+ \overset{\leftarrow t}{F}\right) + \lambda}$$

  4. Return S, $G(t)$, and H

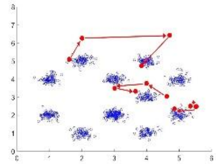
Synthetic Data Experiment

- Validate the algorithm
- Create time-series data from 3 Markov chains with 4 states
  - 3 patterns → $K = 3$
  - Pattern length of 4 → $T = 4$





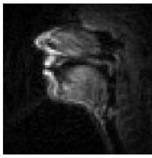
CH-CNMF basis



CNMF-SC basis

Articulatory Data Experiment

- Automatically find gestures from vocal tract measurements during speech production

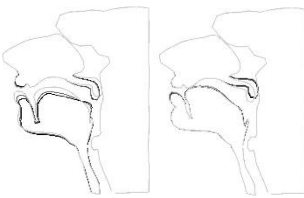


MRI frame

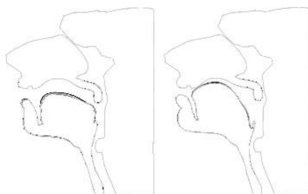
Contour tracking →

Vocal tract contours

→ V constriction degrees



CH-CNMF basis examples



CNMF-SC basis examples

- Analyzed RMSE and correlation of reconstructed contours with input
  - Reconstructed with calculated encoding matrix H_{test} and random encoding matrix H_{rand}

Algorithm	Encodings	RMSE (mm)	Correlation
CH-CNMF	H_{test}	0.824	0.964
	H_{rand}	3.419	-0.002
CNMF-SC	H_{test}	6.058	0.619
	H_{rand}	8.127	0.168

Conclusion & Future Work

- Developed an algorithm to automatically extract temporal patterns from time-series data
- CH-CNMF recovers patterns more faithfully and approximates the input with lower error and higher correlation than CNMF-SC
- Allow T to vary for different patterns
- Use encoding matrix as features for classification
- Explore joint learning of multiple modalities

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