USC University of Southern California

USC Viterbi School of Engineering

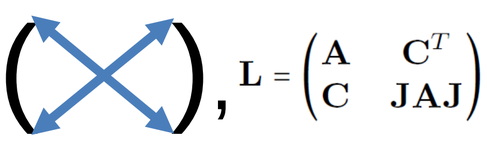
Ming Hsieh Department of Electrical Engineering

Symmetric Line Graph Transforms for Inter Predictive Video Coding

Keng-Shih Lu and Antonio Ortega, Signal and Image Processing Institute

Bisymmetry Property for Speedup of GFTs

Bisymmetric graph Laplacian matrix **L**



$$\mathbf{L} = \begin{pmatrix} \mathbf{A} & \mathbf{C}^T \\ \mathbf{C} & \mathbf{JAJ} \end{pmatrix}$$

$\Leftrightarrow$

Even/odd symmetric eigenvectors

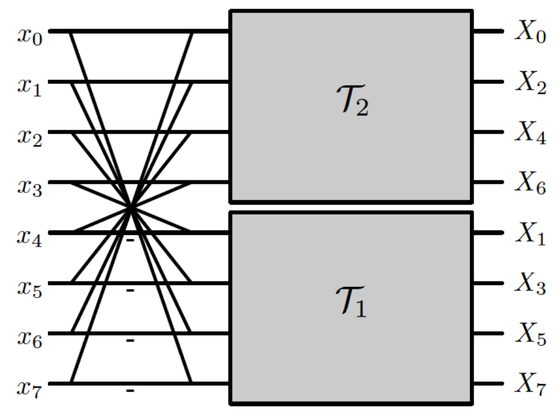
$$\mathbf{L} = \mathbf{E}\Lambda\mathbf{E}^T$$

$$\mathbf{E} = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{E}_1 & \mathbf{E}_2 \\ -\mathbf{JE}_1 & \mathbf{JE}_2 \end{pmatrix}$$

$\Rightarrow$

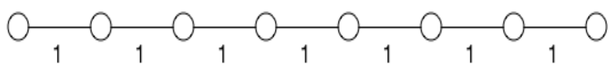
Computation reduction in the Graph Fourier Transform (GFT)

$$\underbrace{\mathbf{E}^T}_{\text{large GFT}}\mathbf{x} = \underbrace{\begin{pmatrix} \mathbf{O} & \mathbf{E}_1^T\mathbf{J}/\sqrt{2} \\ \mathbf{E}_2^T/\sqrt{2} & \mathbf{O} \end{pmatrix}}_{\text{2 smaller \& parallel transforms}} \underbrace{\begin{pmatrix} \mathbf{I} & \mathbf{J} \\ \mathbf{J} & -\mathbf{I} \end{pmatrix}}_{\text{butterfly}} \mathbf{x}.$$



Symmetric Line Graph Transform (SLGT)

- Line graphs can characterize 1st order Gaussian Markov Random Fields, which are suitable for modeling video pixels
- Many known fast algorithms are GFTs of line graphs
  e.g. type-2 DCT is the GFT of



- **Tridiagonal** and **bisymmetric** graph Laplacian matrix

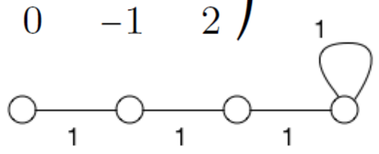
$$\mathbf{P} = \left(\begin{array}{cccc|cccc} k_1 & -w_1 & & & & & & \\ -w_1 & k_2 & -w_2 & & & & & \\ & \ddots & \ddots & \ddots & & & & \\ & & -w_{N/2-1} & k_{N/2} & -w_{N/2} & & & \\ \hline & & & -w_{N/2} & k_{N/2} & -w_{N/2-1} & & \\ & & & & \ddots & \ddots & \ddots & \\ & & & & & -w_2 & k_2 & -w_1 \\ & & & & & & -w_1 & k_1 \end{array}\right) = \left(\begin{array}{c|c} \mathbf{A} & \mathbf{C}^T \\ \hline \mathbf{C} & \mathbf{JAJ} \end{array}\right)$$

Example of a Fast SLGT

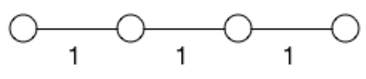
- Choose 2 graph Laplacian matrices with fast GFT algorithms (and some additional conditions)

$$\mathbf{L}_1 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix} \qquad \mathbf{L}_2 = \begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

$\mathcal{G}_1$:



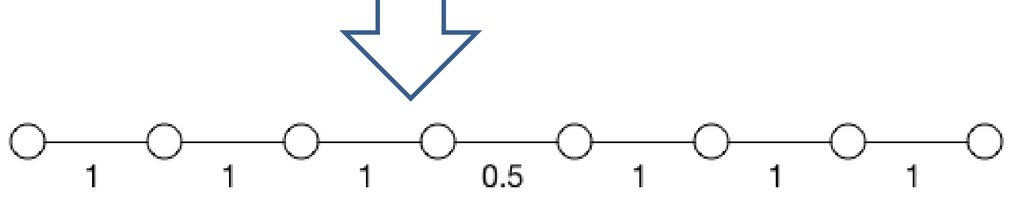
$\mathcal{G}_2$:



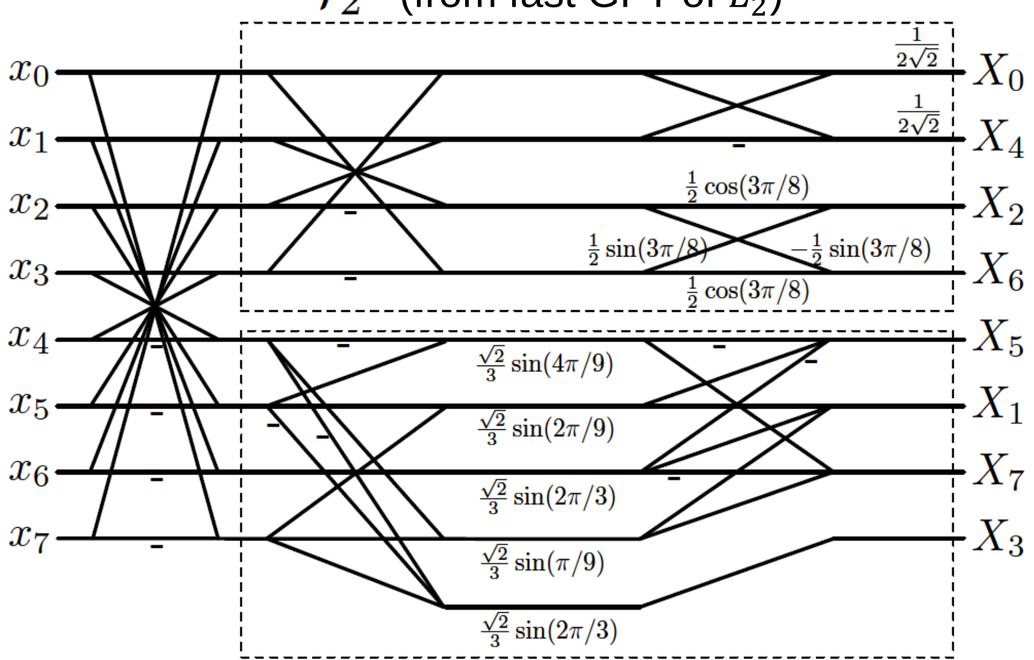
- Obtain $\hat{\mathbf{L}} = \begin{pmatrix} \mathbf{A} & \mathbf{C}^T \\ \mathbf{C} & \mathbf{JAJ} \end{pmatrix}$, where

$$\mathbf{A} - \mathbf{JC} = \mathbf{L}_1$$
$$\mathbf{A} + \mathbf{JC} = \mathbf{L}_2$$

$\hat{\mathcal{G}}$:



$\mathcal{T}_2$ (from fast GFT of L_2)



$\mathcal{T}_1$ (from fast GFT of L_1)

Data-Driven SLGT for Video Coding

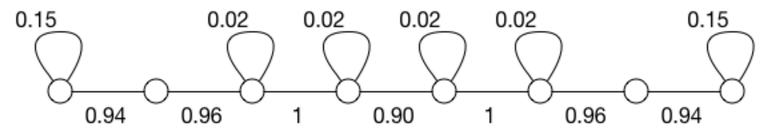
- Data: 8x8 inter predictive residual blocks

Convex problem

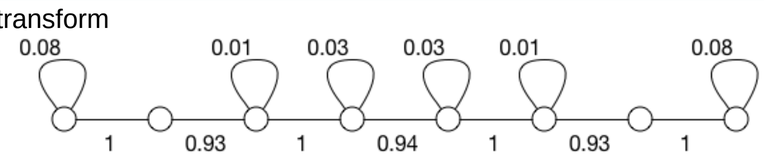
$$\begin{aligned} \text{minimize} \quad & -\log(\det \mathbf{P}) + \text{trace}(\mathbf{KP}) \\ \text{subject to} \quad & \mathbf{P} \text{ is positive semidefinite}, \quad p_{ij} \le 0 \text{ if } i \ne j \\ & \mathbf{P} \text{ is bisymmetric and tridiagonal.} \end{aligned}$$

Learned symmetric graphs

For column transform



For row transform



Percentage of bit rate gain (compared with DCT)

	32dB	34dB	36dB	38dB	Average
BQMall	0.37	0.12	0.05	-0.05	0.10
BasketballDrill	0.53	0.23	0.13	-0.01	0.19
Kimono1	1.14	1.67	1.48	0.83	1.40
Cactus	0.33	0.13	0.11	0.03	0.20
ParkScene	0.98	0.64	0.46	0.41	0.58
BQTerrace	0.56	0.25	0.09	0.01	0.17
Average	0.70	0.59	0.46	0.26	0.51

Conclusions & Future Work

- Potentially useful SLGTs with fast algorithms can be explored based on the bisymmetry property
- We can learn SLGTs close to the KLTs that provide compression gains as compared to DCT. The number of multiplications they require is half of that required by KLT
- Future Work: Symmetries in 2D grids, general symmetric graphs

kengshil@usc.edu

STAC Group http://biron.usc.edu/wiki/index.php/CompressionGroup

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