Non-Iterative Parameter Estimation for Total Variability Model Using Randomized SVD

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Total Variability Model

Parametric model for explaining changes in feature space distribution across utterances
Used in tasks like speaker and language identification

Model Formulation

Feature vectors : $\mathbf{X}_u = \{\boldsymbol{x}_{ut}\}_{t=1}^{T_u}$, **ivector** : $\boldsymbol{w}_u$

$$\boldsymbol{W}_u \sim \mathcal{N}(\mathbf{0}, \mathbf{I}), \quad \boldsymbol{x}_{ut} | \boldsymbol{w}_u \sim \text{GMM} \left\{ p_c,\ \boldsymbol{\mu}_{uc} = \boldsymbol{\mu}_c + \mathbf{T}_c \boldsymbol{w}_u,\ \Sigma_c \right\}_{c=1}^{C}$$

$$M_0 = \begin{bmatrix} \boldsymbol{\mu}_1 \\ \vdots \\ \boldsymbol{\mu}_C \end{bmatrix}, \quad M_u = \begin{bmatrix} \boldsymbol{\mu}_{u1} \\ \vdots \\ \boldsymbol{\mu}_{uC} \end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix} \mathbf{T}_1 \\ \vdots \\ \mathbf{T}_C \end{bmatrix} \qquad M_u = M_0 + \mathbf{T}\boldsymbol{w}_u$$

Conventional Training Method

Expectation Maximization

Expectation : Evaluate the posterior distribution for ivectors

$$\boldsymbol{w}_u | \mathbf{X}_u, \Theta^{(n-1)} \sim \mathcal{N}\left(\left(\mathbf{I} + \mathbf{T}^\mathsf{T}\Sigma^{-1}\mathbf{N}_u\mathbf{T}\right)^{-1} \mathbf{T}^\mathsf{T}\Sigma^{-1}\boldsymbol{F}_u, \left(\mathbf{I} + \mathbf{T}^\mathsf{T}\Sigma^{-1}\mathbf{N}_u\mathbf{T}\right)^{-1} \right)$$

Maximization : Maximize the following objective function

$$\mathcal{L}(\Theta) = \sum_{u=1}^{U} \mathbb{E}_{\boldsymbol{w}_u | \mathbf{X}_u, \Theta^{(n-1)}} \left[\log P(\mathbf{X}_u, \boldsymbol{w}_u | \mathbf{C}_u, \Theta^{(n)}) \right]$$

Expensive to evaluate all ivectors every iteration

Key Result

Assuming

$$\boldsymbol{N_{uc}} \approx \boldsymbol{T_u p_c}, \quad \mathbf{T}^\mathsf{T}\Sigma^{-1}\mathbf{P}\mathbf{T} + \tfrac{1}{T_u}\mathbf{I} \approx \mathbf{T}^\mathsf{T}\Sigma^{-1}\mathbf{P}\mathbf{T} + \tfrac{1}{T}\mathbf{I}$$

Given

Covariance matrices Σ_c, UBM posteriors $\mathbf{C}_u$

Objective Function : Marginalized Likelihood

$$\sum_{u=1}^{U} \log P(\mathbf{X}_u | \mathbf{C}_u, \Theta) = \sum_{u=1}^{U} \log \left(\int_{\boldsymbol{w}_u} P(\mathbf{X}_u, \boldsymbol{w}_u | \mathbf{C}_u, \Theta) \right)$$

Is maximized for $\mathbf{T}$ = Partial SVD of normalized statistics

Proposed Training Method

Randomized SVD of Normalized Statistics

$$\Sigma_c^{-1} = \mathbf{L}_c \mathbf{L}_c^\mathsf{T}, \quad \widetilde{F}_u = \begin{bmatrix} \frac{1}{\sqrt{N_{u1}}} \mathbf{L}_1{}^\mathsf{T} \boldsymbol{F}_{u1} \\ \vdots \\ \frac{1}{\sqrt{N_{uC}}} \mathbf{L}_\mathbf{C}{}^\mathsf{T} \boldsymbol{F}_{uC} \end{bmatrix}, \quad \widetilde{\mathbf{F}} = \begin{bmatrix} \widetilde{F}_1 & \dots & \widetilde{F}_U \end{bmatrix}$$

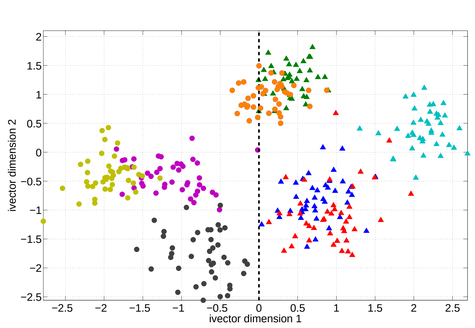
$$\widetilde{\mathbf{F}} = \mathbf{U}_F \mathbf{D}_F \mathbf{V}_F^\mathsf{T}, \quad \widetilde{\mathbf{T}} = \mathbf{U}_F \mathbf{D}_F, \quad \mathbf{T} = \Sigma \mathbf{L} \mathbf{P}^{-\frac{1}{2}} \widetilde{\mathbf{T}}$$

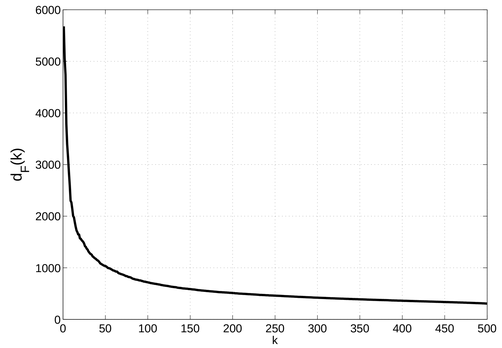
Key Advantages

Non-iterative, Efficient, Parallelizable, Interpretable

No need for iterations like in EM

i-vector dimensions ranked by singular values, no re-training for different i-vector dimensionality





Experimental Results

RATS language identification data : 6 languages, 16000 utterances per language, 30s each

	EM		**SVD-1**		**SVD-2**	
T_θ	33.5 hrs		**30 min**		**30 min**	
T_w	1.13 sec		**0.12 sec**		1.13 sec	
Dur	*EER*	*ACC*	*EER*	*ACC*	*EER*	*ACC*
10	**8.30**	83.50	8.78	**83.60**	9.26	81.15
5	**10.04**	**80.80**	10.40	80.55	10.94	78.05
3	**13.41**	72.75	13.59	**74.25**	14.97	70.55
1	**21.59**	56.45	22.13	**58.35**	22.61	54.65