Ming Hsieh Department of Electrical Engineering

USC Viterbi School of Engineering

Canonical Granger Causality between Sets

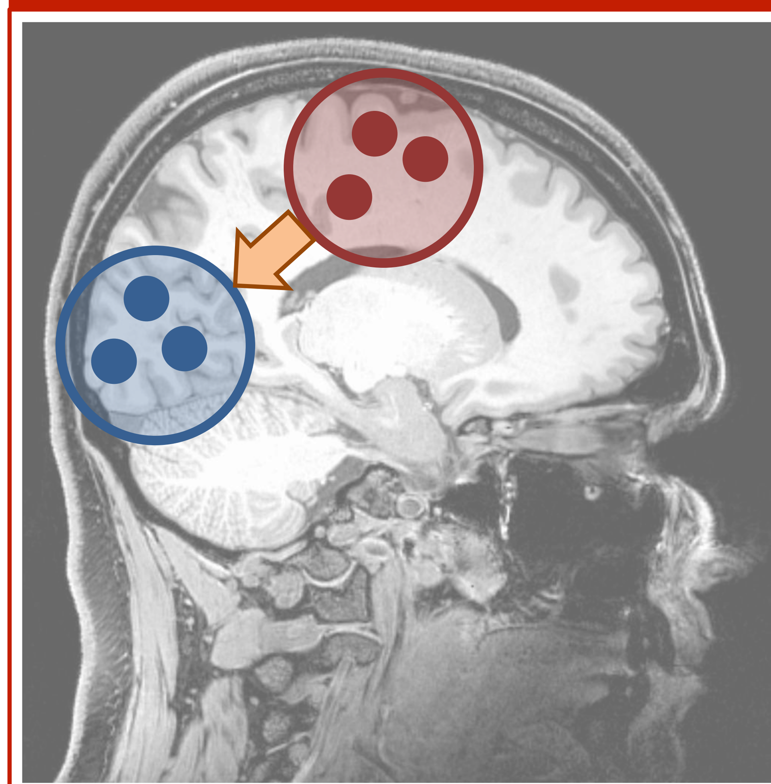
Syed Ashrafulla, Justin P. Haldar, Anand A. Joshi, Richard M. Leahy

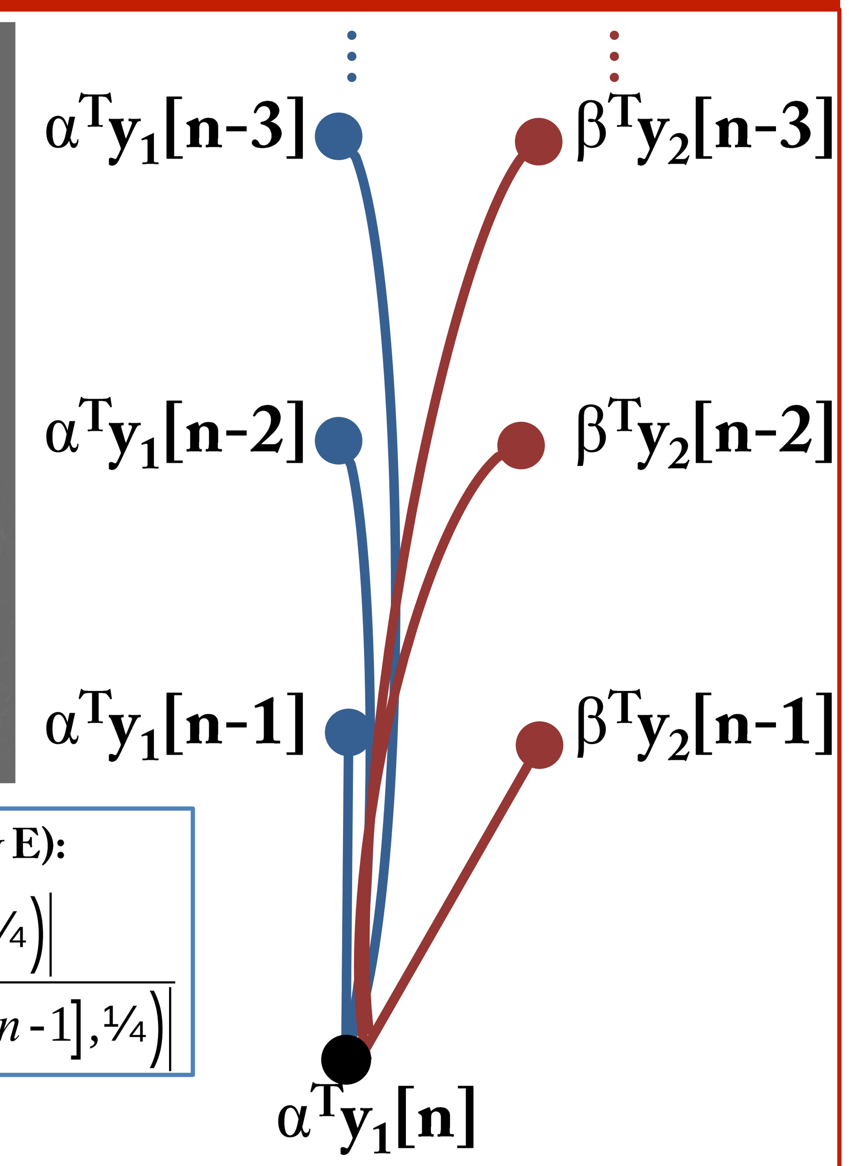
Biomedical Imaging Research Group, Signal & Image Processing Institute

Abstract

Biological imaging such as electroencephalography (EEG) records multiple channels from a region of interest. However, biological signals are not specific to single channels. Here, we determine causality between sets of channels, each representing a region. We first collapse each set to a univariate signal by a weighted sum. Then, we estimate the causality between the representative signals of each set. The causality measure is maximized by optimizing the weights over both regions. In addition, the optimal weights provide us with a topography of interacting signals within this network. We show how this "canonical Granger causality (GC)" discriminates directed relationships with short time records better than other multivariate techniques. Applied to an epileptic spike, we show times where canonical GC finds transfer of spike information to the surface of the brain.

Canonical Granger Causality





Multivariate GC (Barrett2010, Phys Rev E):

$$M_{2\to1} = \ln \frac{\left|Var\left(y_1[n] \middle| y_1[n-1], \ldots\right)\right|}{\left|Var\left(y_1[n] \middle| y_1[n-1], \ldots, y_2[n-1], \ldots\right)\right|}$$

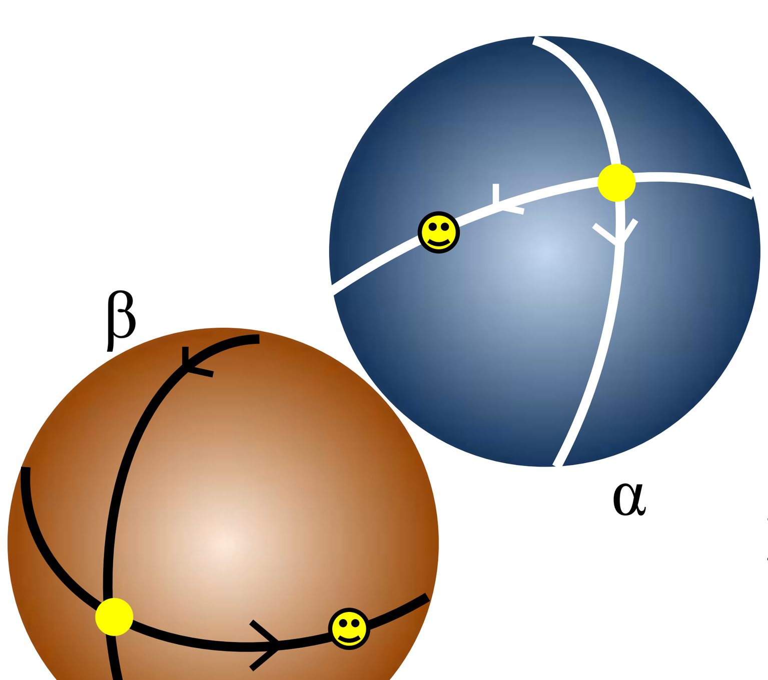
Proposed Canonical GC:

$$C_{2\to1} = \max_{\substack{\alpha,\beta \\ \|\alpha\|=\|\beta\|=1}} \ln \frac{Var\left(\alpha^T y_1[n] \middle| \alpha^T y_1[n-1], \ldots\right)}{Var\left(\alpha^T y_1[n] \middle| \alpha^T y_1[n-1], \ldots, \beta^T y_2[n-1], \ldots\right)}$$

Optimization

→ Modify conjugate gradient descent

Submanifold: $S = S^{A-1} \times S^{B-1}$



Gradient

$$\nabla C = \begin{bmatrix} \nabla_\alpha C \\ \nabla_\alpha C \end{bmatrix}$$

Modified Gradient

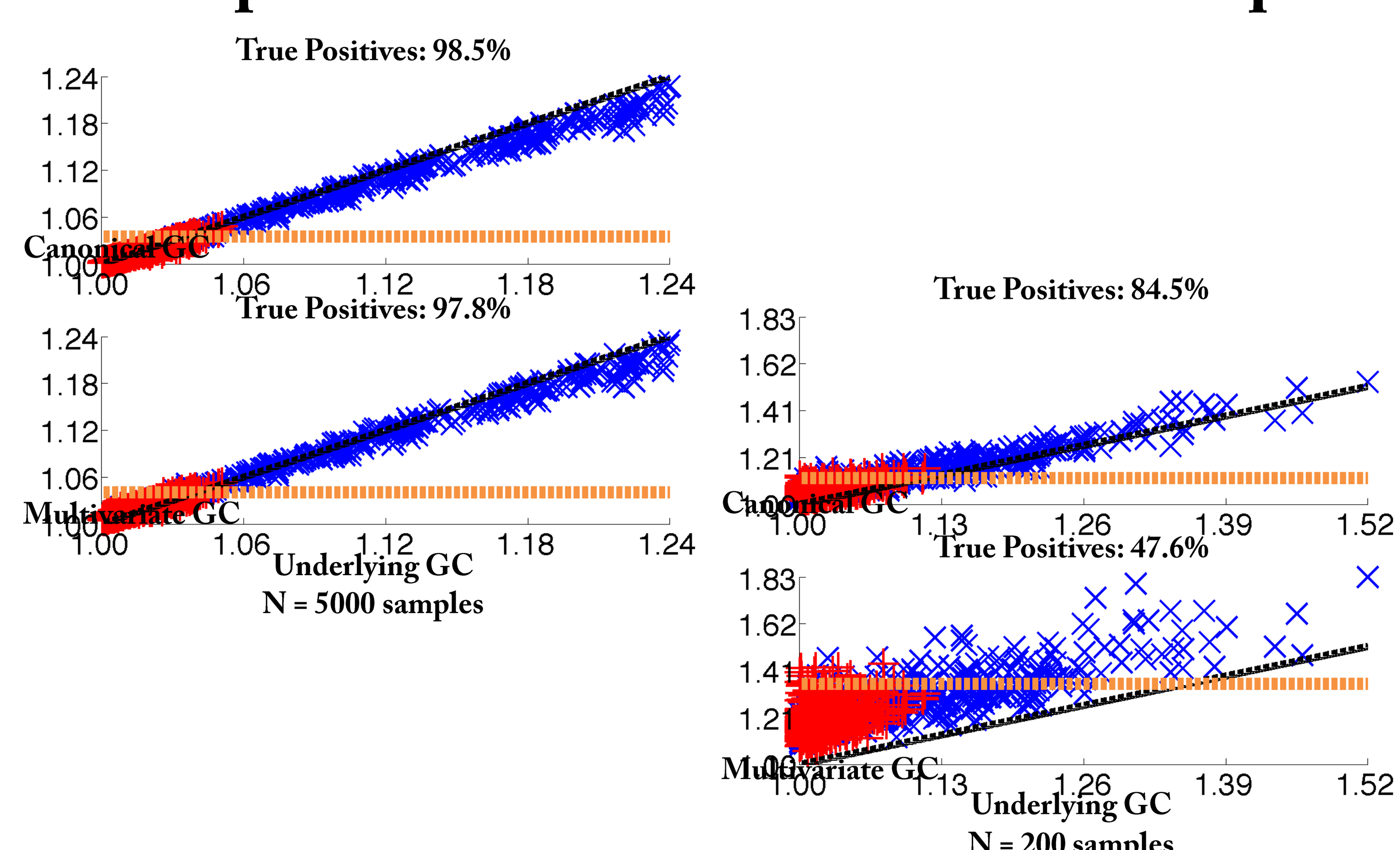
$$\nabla^S C = \begin{bmatrix} (I - \alpha\alpha^T)\nabla_\alpha C \\ (I - \beta\beta^T)\nabla_\beta C \end{bmatrix}$$

Line search → Geodesic search

$$\begin{bmatrix} \alpha(t_\alpha) \\ \beta(t_\beta) \end{bmatrix} = \begin{bmatrix} \alpha^k \cos(t_\alpha) + \mathbf{D}_\alpha^k \sin(t_\alpha) \\ \beta^k \cos(t_\beta) + \mathbf{D}_\beta^k \sin(t_\beta) \end{bmatrix}$$

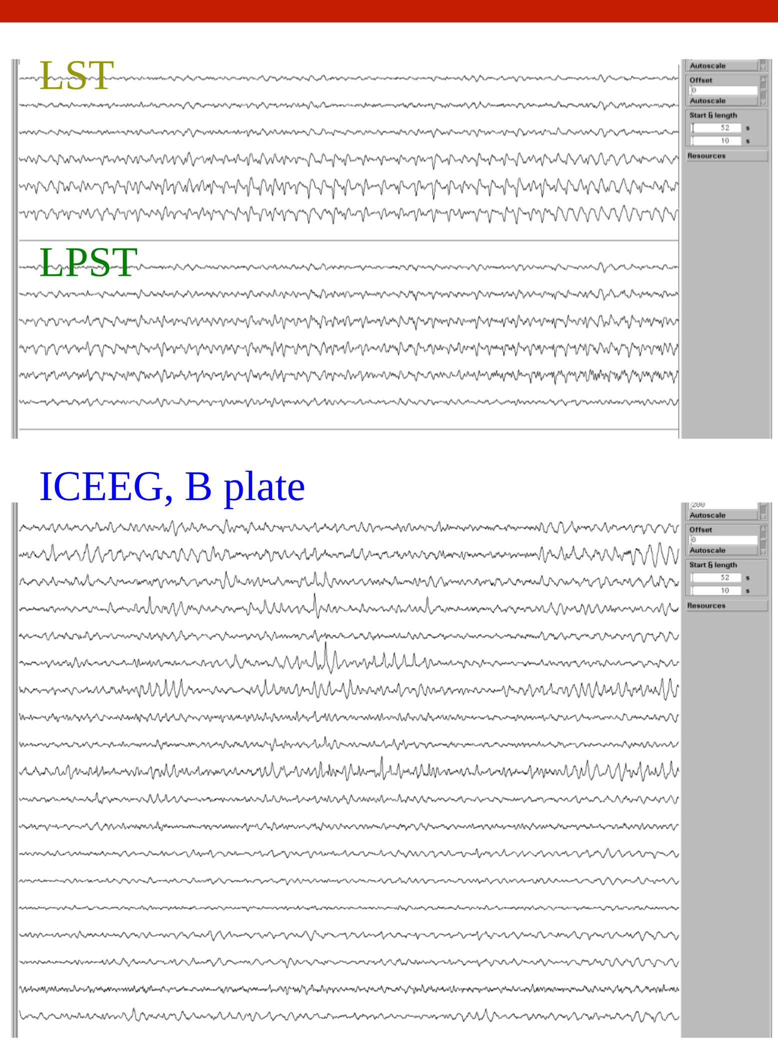
Simulations

→ Compare with other multivariate techniques

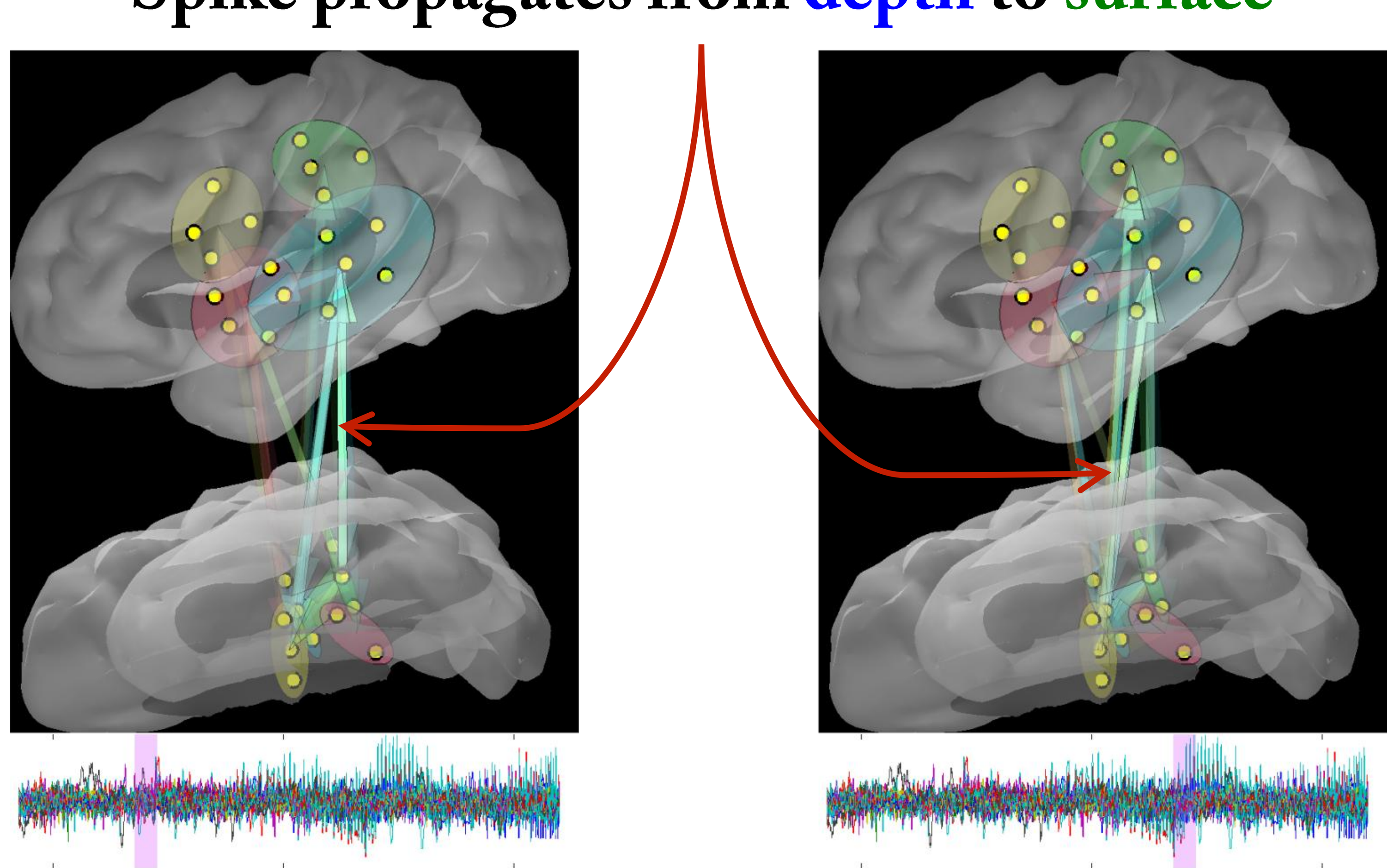


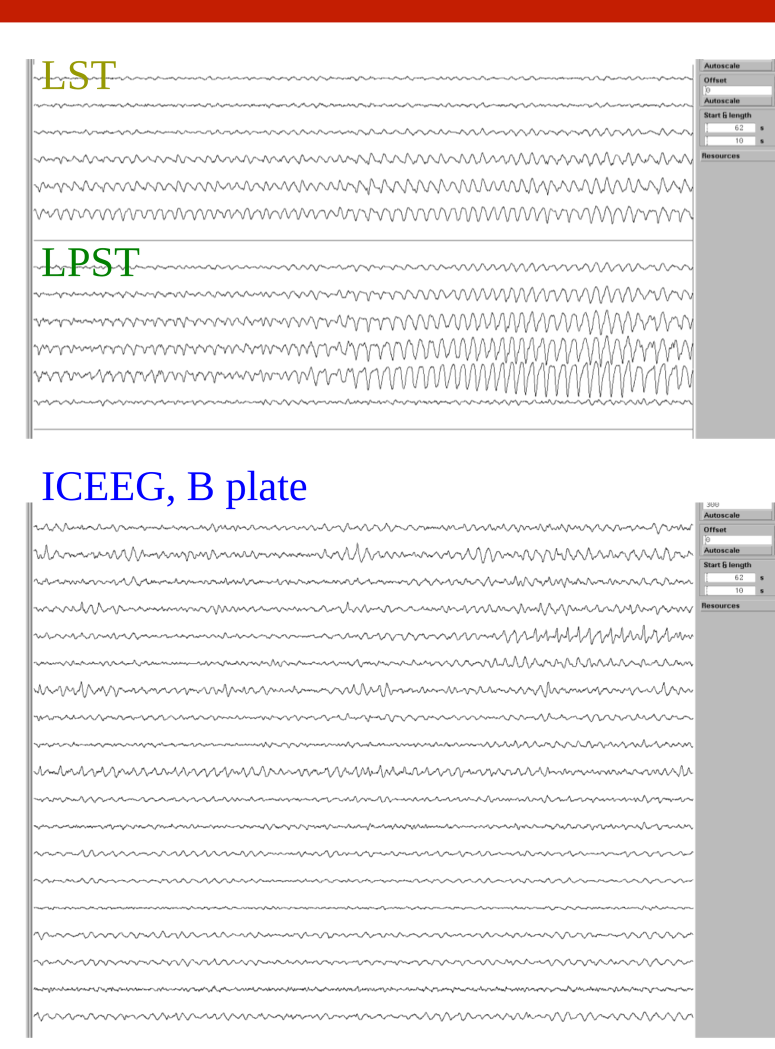
→ More powerful with short time windows

Results: Epileptic Spike



Spike propagates from depth to surface





Corresponding author: ashraful@usc.edu

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