Statistical Analysis of Phase Locking Value

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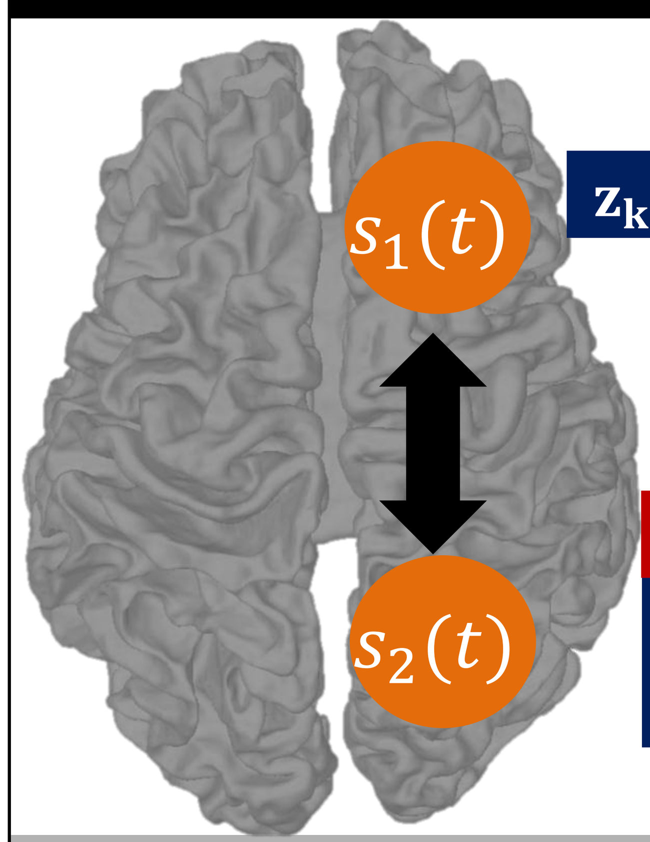
Synchronization in Brain Signals



Analytical Form:

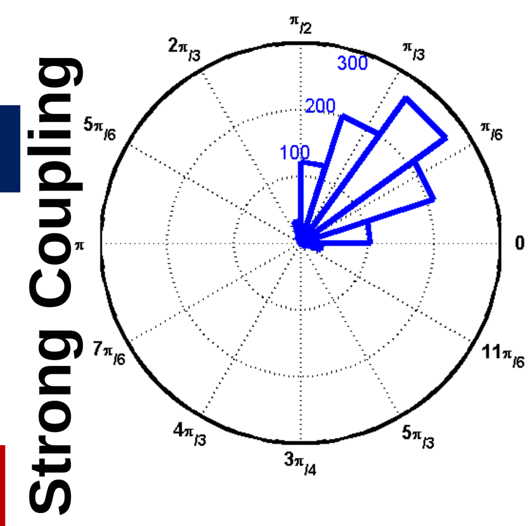
$$z_k(t,f) = s_k(t) + j\,\mathrm{Hilbert}(s_k(t))$$

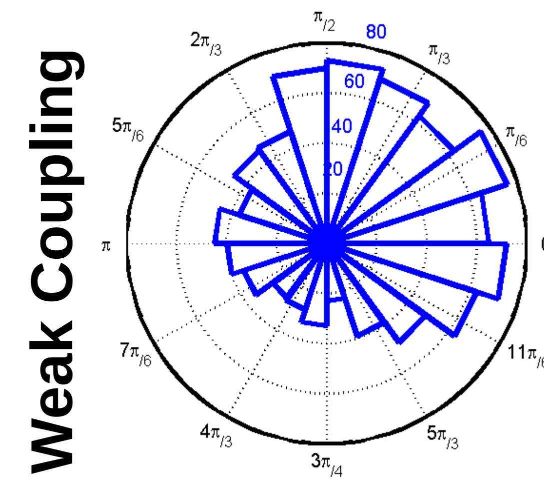
Polar Form:

$$z_k(t,f) = A_k(t)e^{j\,\Phi_k(t)}$$

Phase Locking Value

$$PLV(t,f) = \left|E\left[e^{j(\Phi_1(t)-\Phi_2(t))}\right]\right|$$





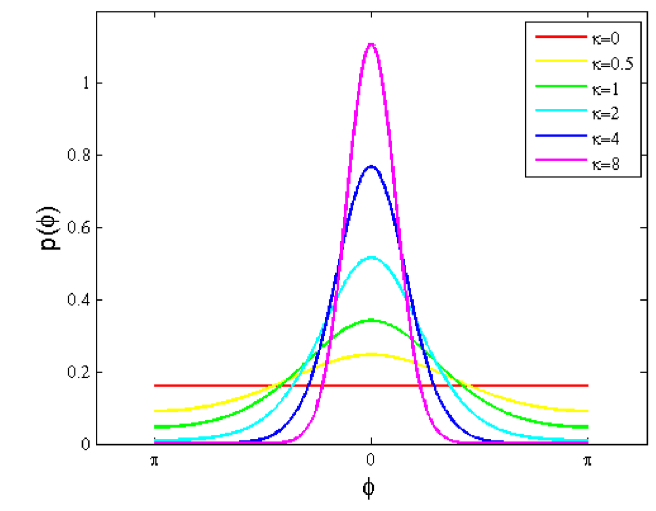
$$PLV_{sample} \triangleq \left|\frac{1}{N}\sum_{n=1}^{N} e^{j\Delta\phi^{(n)}(t)}\right|$$

Trial Index

PLV is preferred as an indicator of synchronization because it is believed not to be sensitive to evoked response changes.

Von Mises Model

$$\Delta\phi^{(n)}(t) \sim VM(\phi \mid \mu,\kappa) = \frac{1}{2\pi I_0(\kappa)} e^{j\kappa\cos(\phi-\mu)}$$



Maximum Likelihood Estimator:

$$PLV_{vonMises} = PLV_{sample} = \left|\frac{1}{N}\sum_{n=1}^{N} e^{j\Delta\phi^{(n)}(t)}\right|$$

Circularly Symmetric Complex Gaussian Model

- **If $s_1(t)$ and $s_2(t) \sim N(0,1)$, the complex random vector $z(t) = [z_1(f,t) \quad z_2(f,t)]^T$ follows a circular complex Gaussian distribution:**

$$p(z(t)) = \frac{1}{\pi^2|K_z|}\exp\{-z(t)K_z^{-1}z(t)\}$$

where $K_z^{-1} = \begin{bmatrix} \kappa_{11} & \kappa_{12}e^{j\mu_{12}} \\ \kappa_{21}e^{j\mu_{21}} & \kappa_{22} \end{bmatrix}$ and $\kappa_{12} = \kappa_{21}, \mu_{12} = -\mu_{21}$

- **Conditional distribution of phase difference**

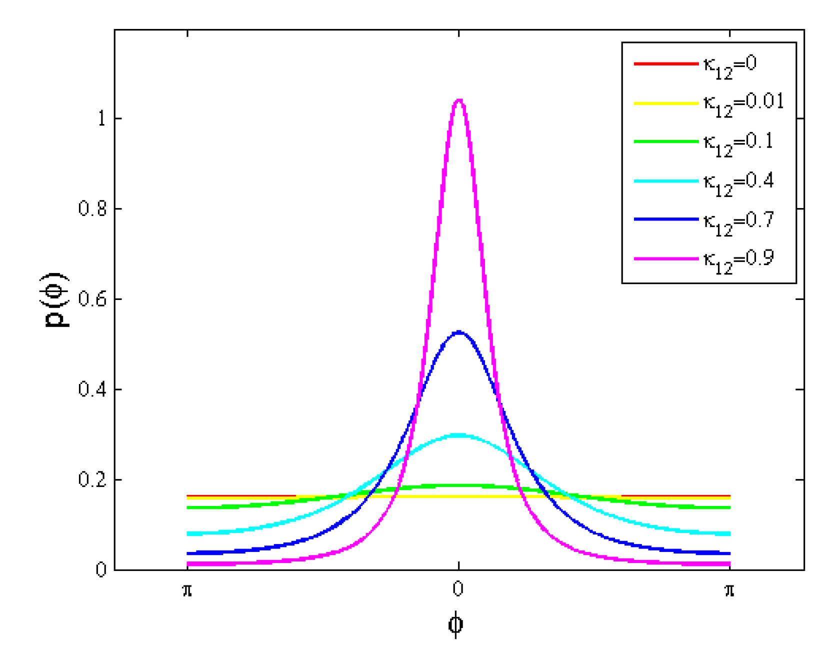
$$p(\Delta\phi(t)|A(t)) = \frac{1}{2\pi I_0(-2\kappa_{12}A_1(t)A_2(t))} e^{-2\kappa_{12}A_1(t)A_2(t)\cos(\Delta\phi(t)-\mu_{12})}$$

➢ **Note: von Mises but amplitude an phase are NOT independent.**

- **Marginal distribution of phase:**

$$p(\Delta\phi(t)) = \frac{1}{2\kappa^2\pi|K_z|}\left(\frac{1}{1-a^2} - \frac{a(\cos^{-1}(a))}{\sqrt{(1-a^2)^3}}\right)$$

where $a \triangleq \frac{\kappa_{12}}{\kappa_{11}}\cos(\Delta\phi(t) - \mu_{12})$



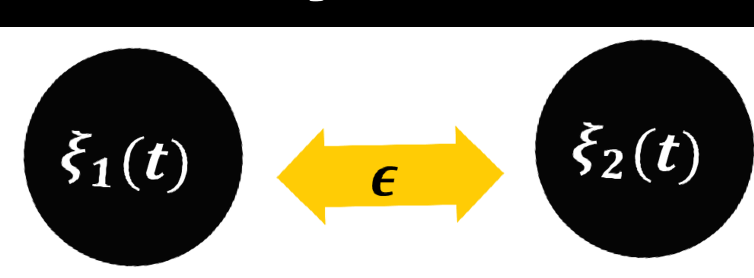
- $PLV_{circgauss} \triangleq \left|E\left[e^{j\Delta\phi(t)}\right]\right| = \left|\frac{\pi}{\sqrt{2}}\left(\frac{3-2\omega^2}{2\omega+2}\right)\left[\omega^3\ {}^4{}_2F_1\left(\frac{3}{4},\frac{5}{4},1\ \omega^2\right) + \frac{3}{4}\omega^5\ {}^2{}_2F_1\left(\frac{5}{4},\frac{7}{4},2\ \omega^2\right)\right]\right|$ where $\omega = \frac{\kappa_{12}}{2\kappa_{11}\kappa_{22}-\kappa_{12}}$ which is a function of square coherence, namely $\kappa_{12}\ \kappa_{11}\kappa_{22}$.

Simulations: ROC Analysis of Roessler Oscillators

2 Roessler oscillators ξ_j **where** $j \in \{1,2\}$

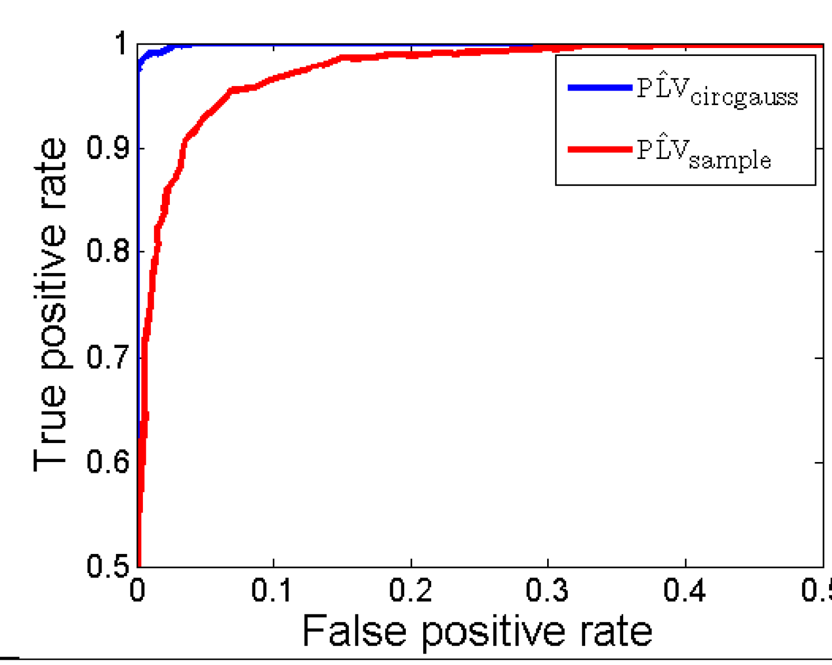
$$\xi_j = \begin{pmatrix} X_j \\ Y_j \\ Z_j \end{pmatrix} = \begin{pmatrix} -\omega_j Y_j - Z_j + \left[\sum_{i\neq j} \epsilon(X_i - X_j)\right] + \sigma_j\eta_j \\ \omega_j X_j + aY_j \\ b + (X_j - c)Z_j \end{pmatrix}$$

Controls the amount of coupling



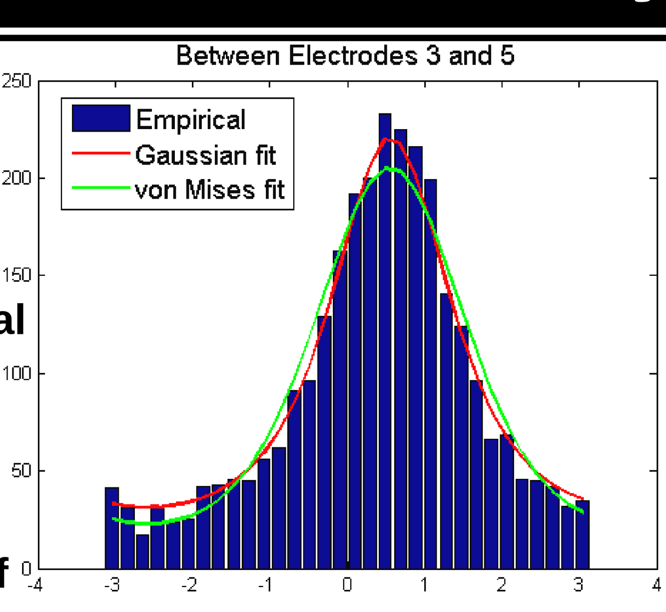
True Positive : if PLV> τ and $\epsilon > 0$

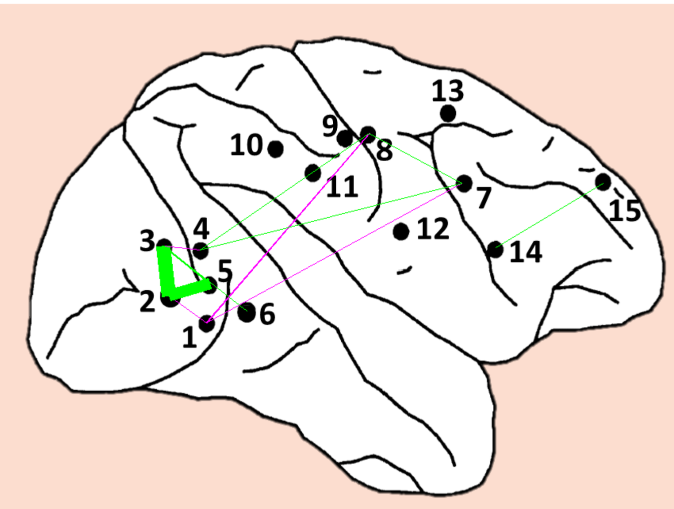
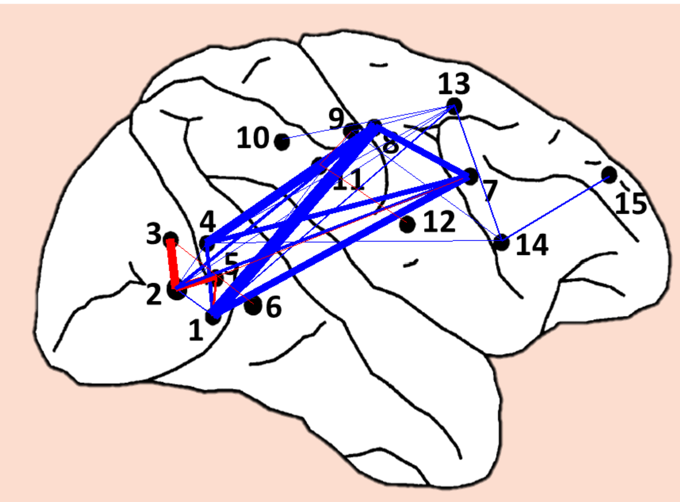
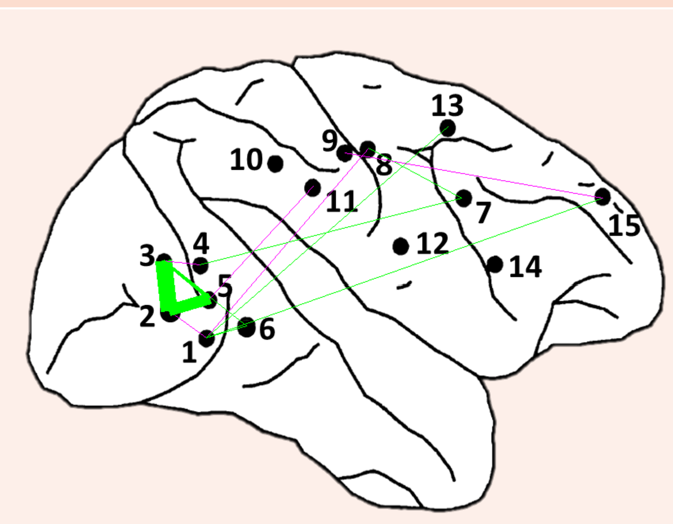
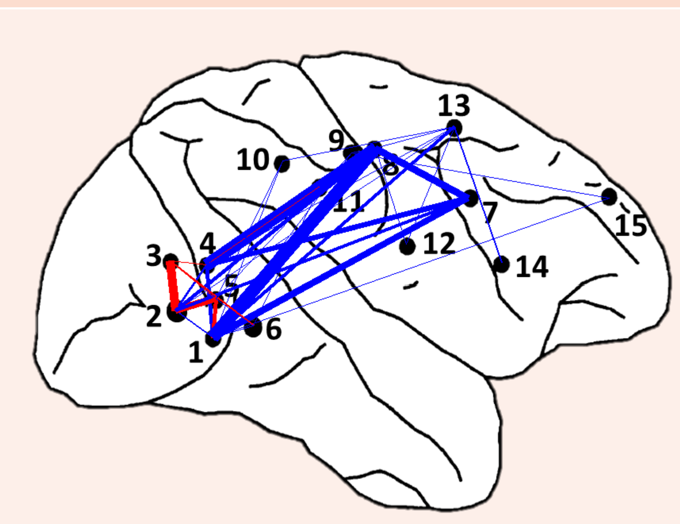
False Positive : if PLV> τ and $\epsilon = 0$



Analysis of LFP Data

- **Local field potential (LFP) time series, sampled at 200 Hz, from a macaque monkey implanted with transcortical bipolar electrodes at 15 sites in the right hemisphere.**
- **Monkey performed a GO, NO-GO visual pattern discrimination task.**
- **We used 10,178 trials taken from 18 sessions focusing on 120±25 msec and 260±25 msec after stimulus presentation in the frequency range of beta [13-30] Hz.**



PLV Type	Diamond vs Line (Early Response at 120±25 msec)	GO vs NO-GO (Late Response at 260±25 msec)
PLV_{sample}		
$PLV_{circgauss}$		

Conclusion

- **For Gaussian signals, phase and amplitude are NOT statistically independent. Phase synchronization is a function of coherence.**
- **LFP networks verify strong relationship between coherence and PLV.**
- **For approximate Gaussian data, this result clarifies the relationship between coherence and PLV.**
- **We observe very little difference between Gaussian and Von Mises based methods. Gaussian based method has some advantages as seen from ROC curves.**

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