

# Capacity and Power Allocation for Degraded Decode-and-Forward Relay Channel with ISI

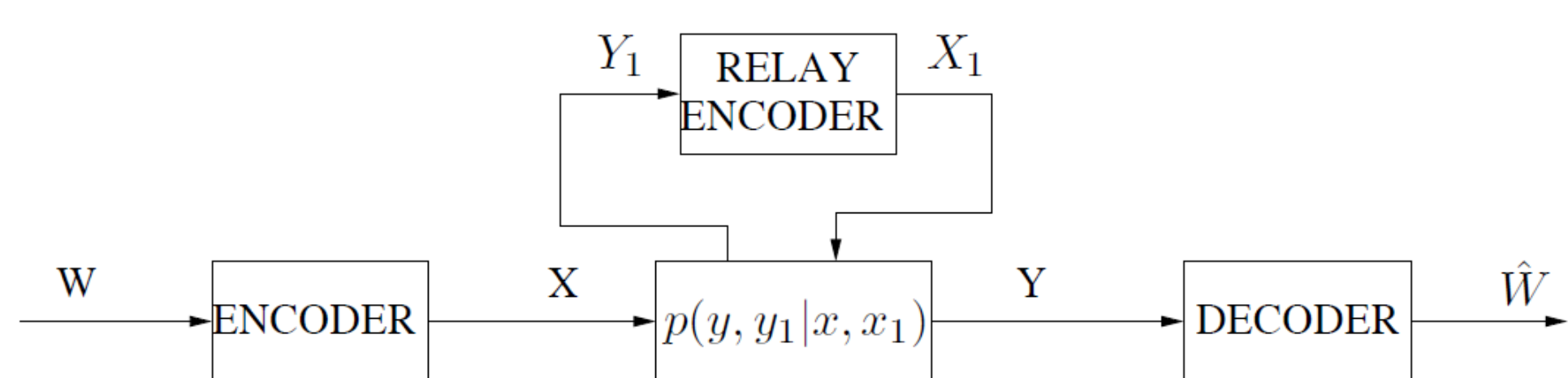
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## Degraded Relay Channel

### •Definition:

A Relay Channel is said to be degraded if  $p(y, y_1 | x, x_1)$  can be written in the form

$$p(y, y_1 | x, x_1) = p(y_1 | x, x_1) p(y | y_1, x_1)$$



### •Capacity:

From the definition, since  $y$  depends on  $x$  only through  $y_1$  and  $x_1$ , we can define

$$X = \sqrt{(1-\alpha)P_S / P_R} X_1 + X_{10}$$

and therefore the capacity for a degraded channel is

$$C^* = \max_{0 \leq \alpha \leq 1} \min \left\{ \frac{1}{2} \log \left( 1 + \frac{P_S + P_R + \sqrt{2(1-\alpha)P_S P_R}}{N_D} \right), \frac{1}{2} \log \left( \frac{\alpha P_S}{N_R} \right) \right\},$$

where  $X_1 \sim N(0, P_R)$  and  $X_{10} \sim N(0, \alpha P_S)$ .

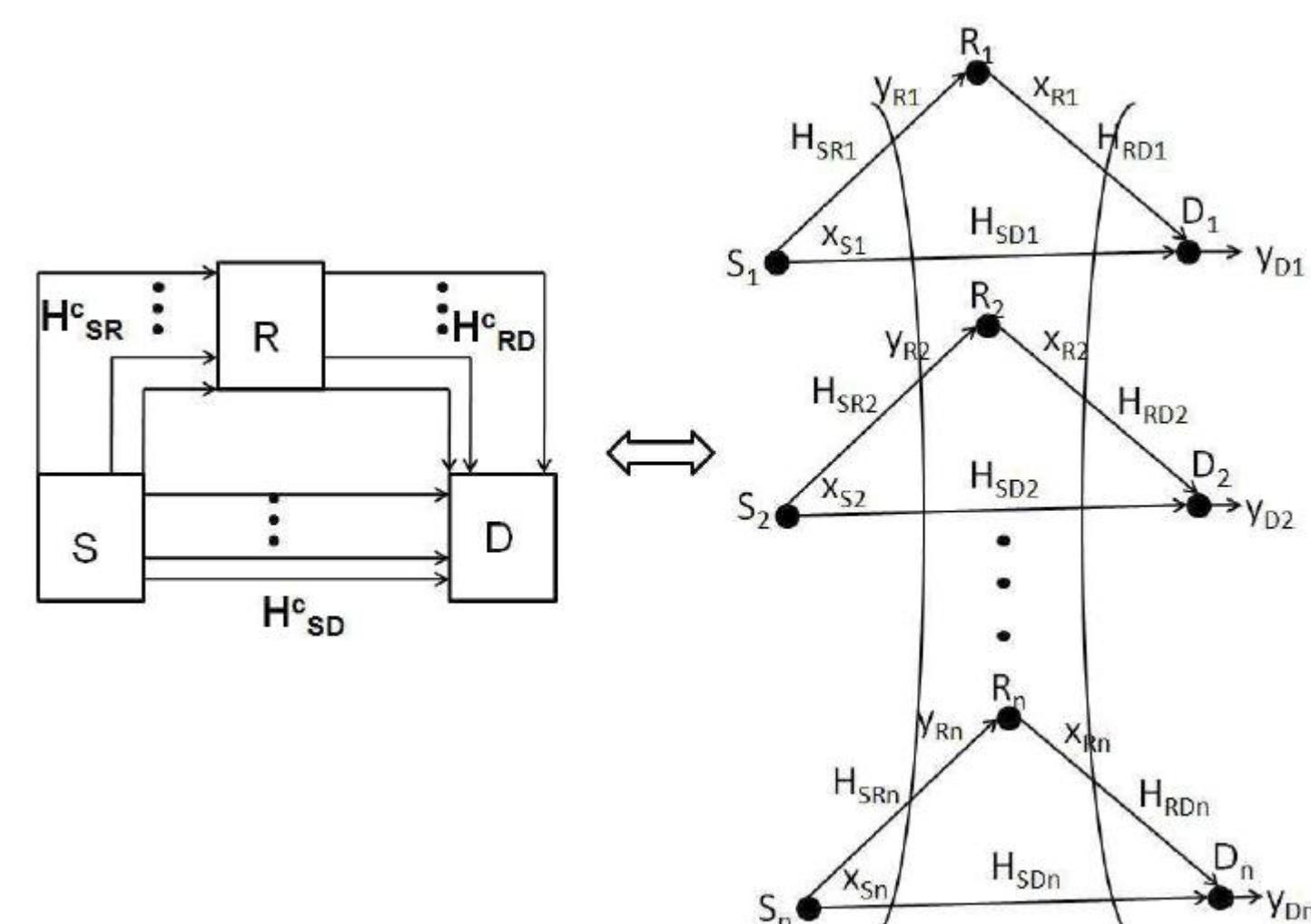
## Degraded Relay Channel with ISI

### •Capacity

The capacity region of circular Gaussian relay channel (CGR C) and linear Gaussian relay channel (LGRC) is the same when the input block size  $N$  goes to infinity.

•By DFT, a multi-path relay channel can be decomposed as a set of  $N$  parallel and independent scalar relay channels and is optimal for the computation of DF rate.

## Decomposition



## Power Allocation for Parallel Channels

### •Why?

- To appropriately assign power on the sub-bands that can provide higher rates.
- To design codes with rates approaching capacity of multi-path relay channel

### •How?

1. Find the capacity of each sub-band under equal power allocation.

$$C_n^* = \max_{0 \leq \alpha_n \leq 1} \min \{ C_{1,n}, C_{2,n} \}$$

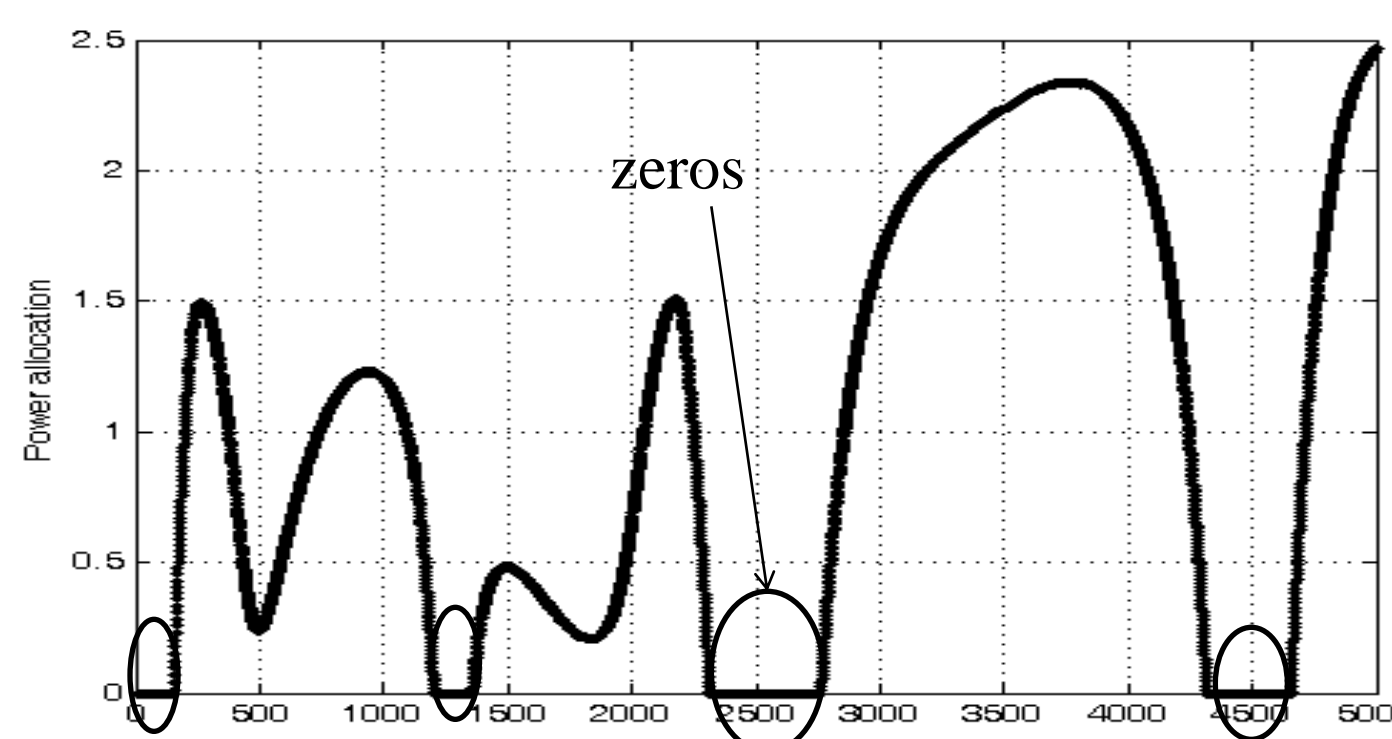
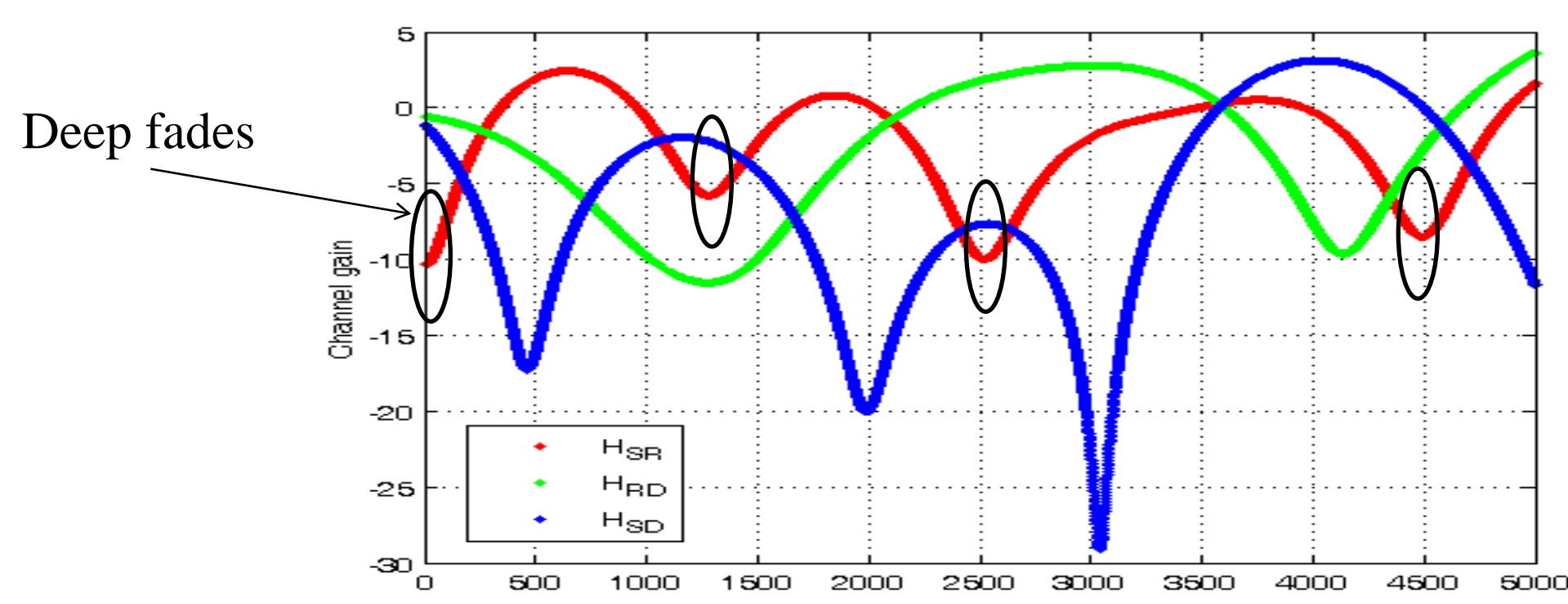
2. Find the corresponding SNR  $\gamma_n^*$  by

$$C_n^* = 1/2 \log(1 + \gamma_n^*),$$

3. Equivalent to power allocation issue on OFDM

$$P_{S,n} = P_{R,n} = (v_t - 1/\gamma_n^*)^+$$

## Simulation Results



### NOTE:

Since  $H_{SD}$  is degraded, when  $H_{SR}$  is in a deep fade, capacities of both paths are low and thus zero power is assigned.