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LDPC Code Design for Wireless Relay Channel with Inter-symbol Interference

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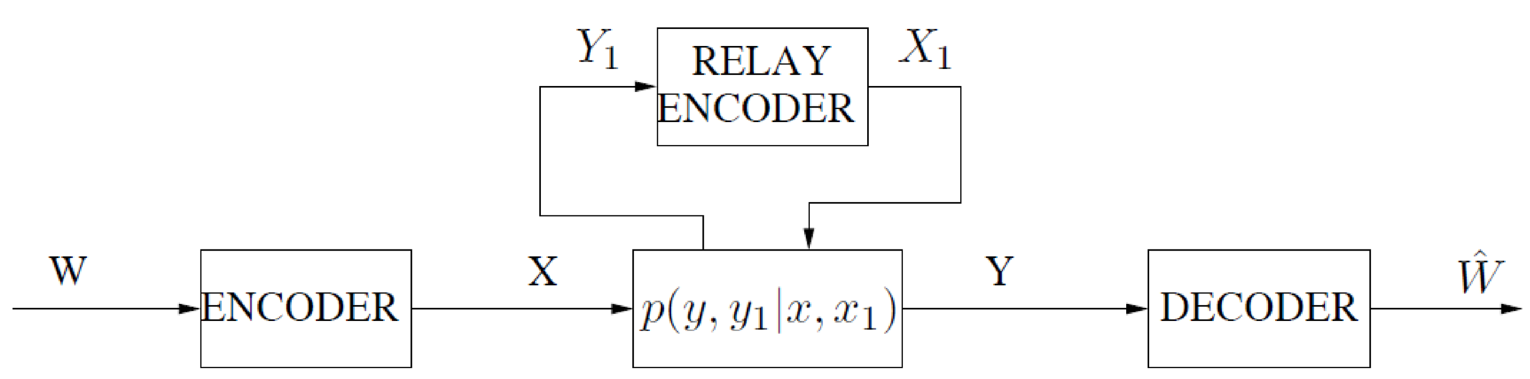
Degraded Relay Channel and Bi-layer LDPC Code

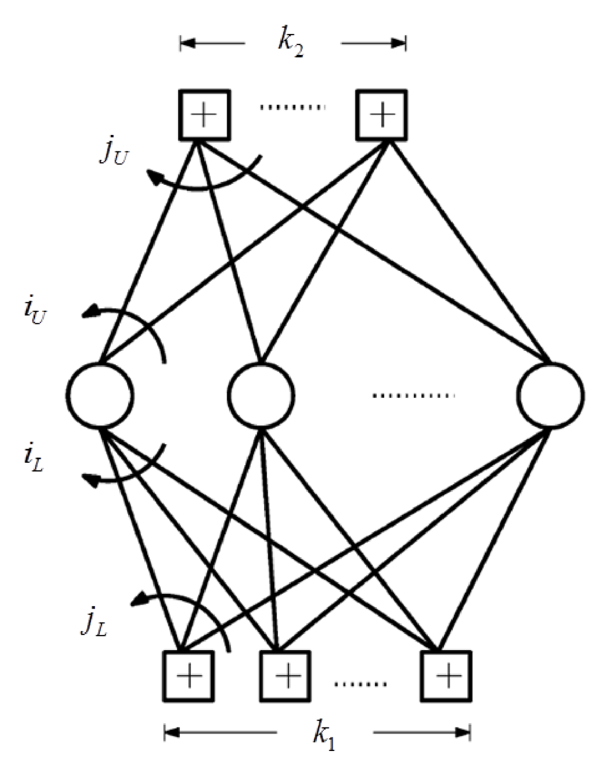
•Definition:

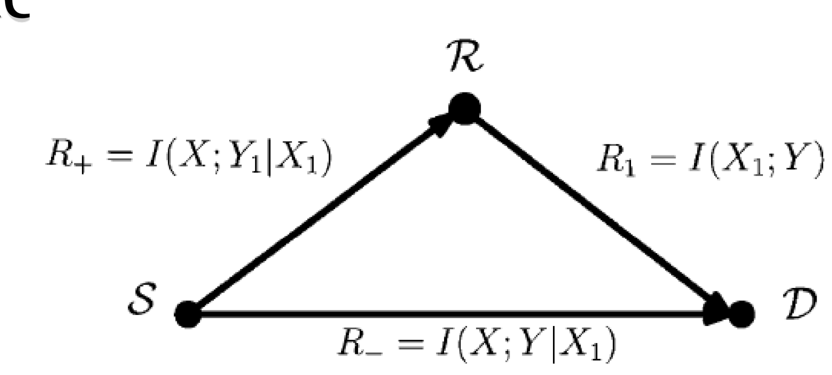
A Relay Channel is said to be degraded if $p(y, y_1 | x, x_1)$ can be written in the form

$$p(y, y_1 | x, x_1) = p(y_1 | x, x_1)\, p(y | y_1, x_1)$$



•Bi-layer LDPC Code·





$$C = \sup_{p(x,x_1)} \min\{I(X, X_1; Y), I(X; Y_1|X_1)\}$$

$$= \sup_{p(x,x_1)} \min\{I(X_1; Y) + I(X; Y|X_1), I(X; Y_1|X_1)\}$$

$$= \sup_{p(x,x_1)} \min\{R_1 + R_-, R_+\}$$

Degraded Relay Channel with ISI

•System Model:

$$Y_{1_k} = \sum_{i=0}^{m} h_{SR_i} X_{k-i} + n_{R_k}$$

$$Y_k = \sum_{i=0}^{m} \left(h_{SD_i} X_{k-i} + h_{RD_i} X_{1,k-i} \right) + n_{D_k}$$

•By adding cyclic prefix

$$Y_1 = \mathbf{H}_{SR} X + n_R$$

$$Y = \mathbf{H}_{SD} X + \mathbf{H}_{RD} X_1 + n_D,$$

•Decomposition by DFT:

$$\mathbf{F}Y_1 = \mathbf{F}\mathbf{H}_{SR}\mathbf{F}^{\dagger}\mathbf{F}X + \mathbf{F}n_R$$

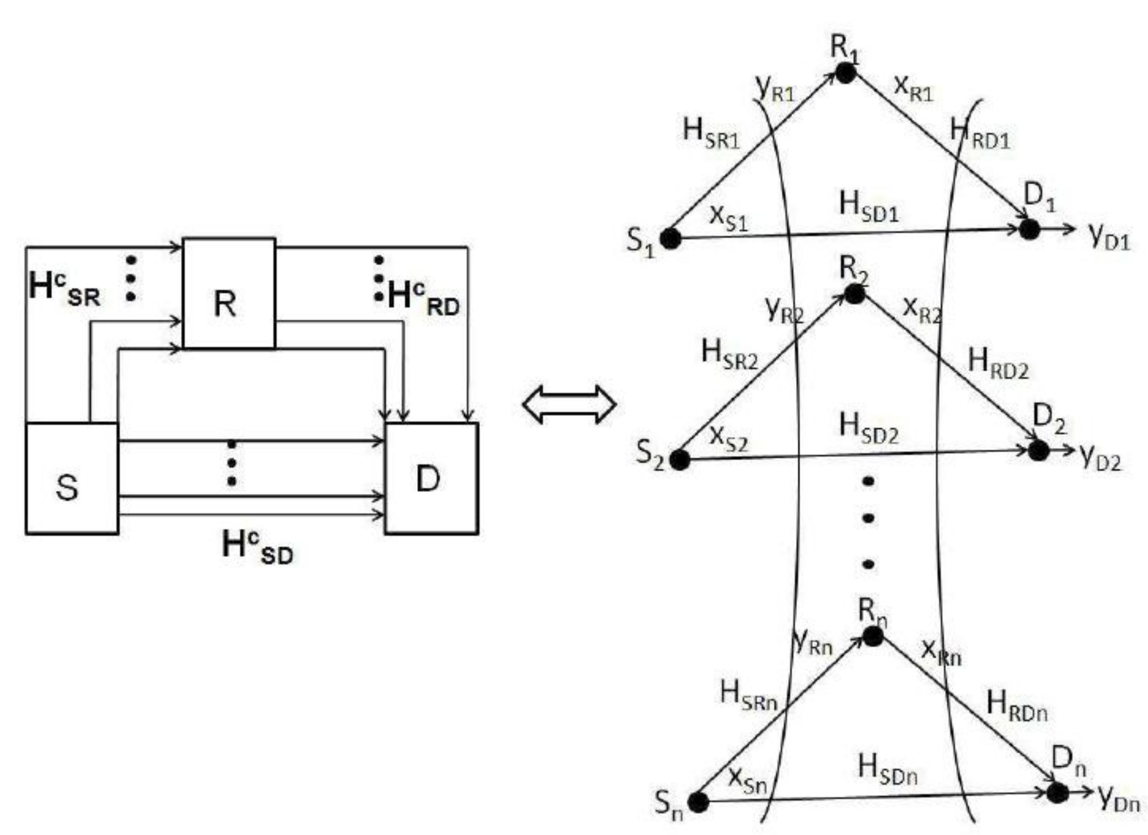
$$= \mathbf{D}_{SR}\mathbf{F}X + \mathbf{F}n_R$$

$$\mathbf{F}Y = \mathbf{F}\mathbf{H}_{SD}\mathbf{F}^{\dagger}\mathbf{F}X + \mathbf{F}\mathbf{H}_{RD}\mathbf{F}^{\dagger}\mathbf{F}X_1 + \mathbf{F}n_D$$

$$= \mathbf{D}_{SD}\mathbf{F}X + \mathbf{D}_{RD}\mathbf{F}X_1 + \mathbf{F}n_D.$$

Multi-path relay is decomposed to multiple sub-relay channels.

Decomposition



Power Allocation

•Define $a_{SRi} = |H_{SRi}|^2 / N_{Ri}$, $a_{SDi} = |H_{SDi}|^2 / N_{Di}$, $a_{RDi} = |H_{RDi}|^2 / N_{Di}$

The power allocation can be derived as follow

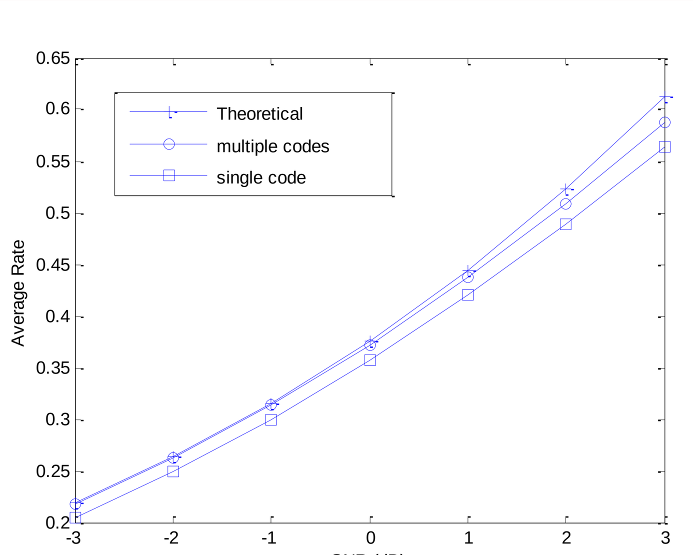
$$P_{Si} = \frac{\mu_{Si}}{\mu_i} P_{t_i}, \quad P_{Ri} = \frac{\mu_{Ri}}{\mu_i} P_{t_i},$$

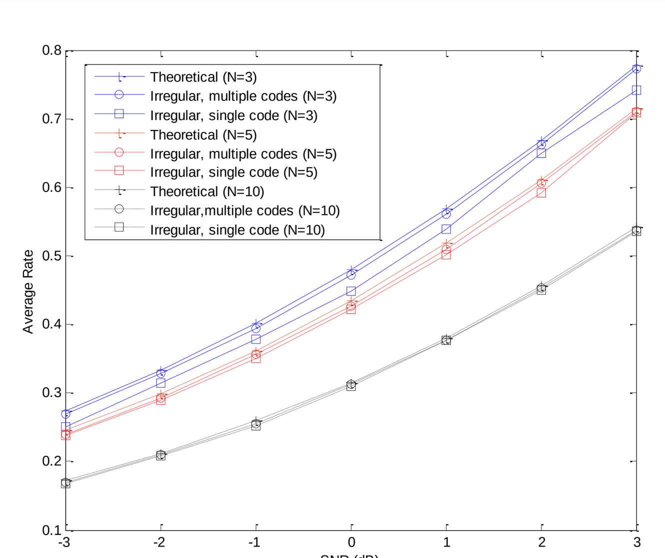
where

$$\mu_{Si} = 1 + a_{SDi} \frac{a_{SRi} - a_{SDi}}{(a_{SDi} + a_{RDi})^2} \qquad \mu_i = \mu_{Si} + \mu_{Ri} = 1 + \frac{a_{SRi} - a_{SDi}}{a_{SDi} + a_{RDi}}$$

$$\mu_{Ri} = a_{RDi} \frac{a_{SRi} - a_{SDi}}{(a_{SDi} + a_{RDi})^2} \qquad P_{t_i} = \mu_i P_i = \left(v_t - \frac{\mu_i}{a_{SRi}} \right)^+.$$

Simulation Results





Code Design for Multi-path Relay

•Define $R_+^i = \frac{1}{2}\log\left(1 + \frac{|H_{SRi}|^2 P_{S,i}}{N_R}\right)$, $R_-^i = \frac{1}{2}\log\left(1 + \frac{|H_{SDi}|^2 P_{S,i} + |H_{RDi}|^2 P_{R,n} + 2\sqrt{|H_{SDi} H_{RDi}|^2 P_{S,i} P_{R,i}}}{N_D}\right)$

1. Sort R_+^i, R_-^i, $\forall i = 1 \sim N$ in descending order
2. For the highest rate, the LDPC code is designed by iterative linear programming (LP) optimization

$$\max_{\lambda_i, \eta} \sum_{i \geq 2} \frac{\lambda_i}{i} \quad \text{s.t.} \quad \sum_{i \geq 2} \lambda_i = 1, \quad e^{(l)} < \mu_k e^{(l-1)}, \ \forall l = 1 \sim L$$

3. For the lower rates, the bi-layer LDPC code is designed by the following iterative LP optimization

$$\max_{\lambda_{i_L, i_U}} \eta \quad \text{s.t.} \quad \sum_{i_L \geq 2, i_U \geq 0} \lambda_{i_L, i_U} = 1$$

$$\sum_{i_U \geq 0} \lambda_{i_L, i_U} \frac{i_L}{i_L + i_U} - \eta \lambda_{i_L} = 0, \quad i_L \geq 2$$

$$e^{(l)} < \mu_k e^{(l-1)}, \quad \forall l = 1 \sim L$$

4. Repeat 3. iteratively until all rates have been done.

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