

# Online Learning Algorithms for Network Optimization with Unknown Variables

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April 20, 2012

# Summary of My PhD Research (1)

Bio: BS'05, Tsinghua Univ. -> MS'07, Tsinghua Univ. -> Join ANRG, USC in Fall 2007

## A. Online Learning Algorithms:

### 1 MAB with Linear Rewards

- i.i.d. and Markovian formulation
- DySPAN'10, IEEE/ACM Trans. Networking, Globecom'11, Machine Learning (under submission), Infocom'12 (mini-conf), SECON'12
- joint work with Bhaskar Krishnamachari, Rahul Jain, Mingyan Liu.

### 2 Learning in Decentralized Settings

- Globecom'11
- joint work with Bhaskar Krishnamachari

### 3 Learning with Non-Linear Rewards

- ITA'12 (under submission)
- joint work with Bhaskar Krishnamachari

### 4 Non-Bayesian Restless Multi-Armed Bandits

- ICASSP'11, IEEE Trans. Information Theory (under submission), Allerton'11
- joint work with Bhaskar Krishnamachari, Qing Zhao, Wenhan Dai, Naumaan Nayyar.

# Summary of My PhD Research (2)

## B. Network Game Theory, Algorithmic Game Theory and Economics

- ① Incentive Mechanisms for M/M/1 Queueing Game
  - Infocom'11, IEEE Trans. Automatic Control (under submission)
  - joint work with Bhaskar Krishnamachari, Hua Liu.
- ② Funding Games
  - EC'12 (under submission)
  - joint work with Bhaskar Krishnamachari, Amotz Bar-Noy, Matthew Johnson, George Rabanca

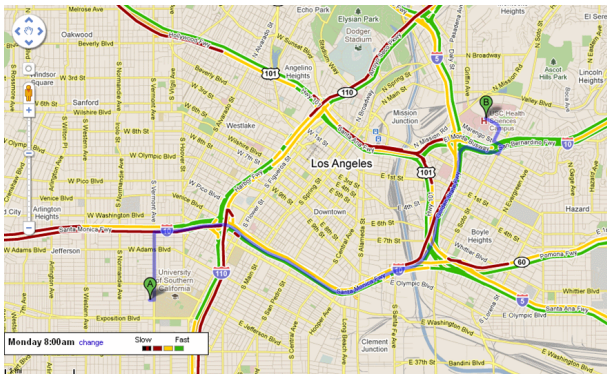
## C. Wireless Networks and Communications

- ① The Saturation Throughput Region of  $p$ -Persistent CSMA
  - ITA'11
  - joint work with Bhaskar Krishnamachari, Shankar Ganesan.
- ② Subcarrier Allocation in Multiuser OFDM Systems
  - WCNC'10
  - joint work with Bhaskar Krishnamachari, Pai-Han Huang, Ashwin Sridharan.

# Today's focus: Online Learning Algorithms

# Online Learning Algorithms: Motivating Example 1

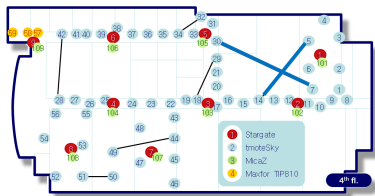
- Finding the lowest expected delay path through traffic using prior observations.



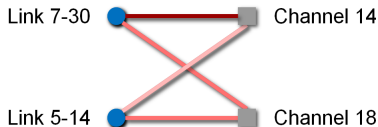
A sample path from Google Maps.

# Motivating Example 2

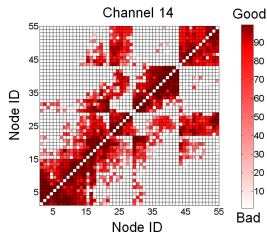
- Channel allocation for wireless links.



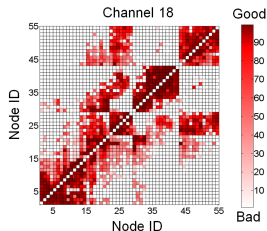
The TutorNet testbed at USC.



Bipartite link channel allocation graph.



Link qualities on channel 14.



Link qualities on channel 18.

# Online Learning for Stochastic Network Optimization

- Common theme: find an optimal network structure (best path / matching), assuming the underlying edge weights are unknown random variables.
- Problem formulation:

$$\begin{aligned} \max \quad & \mathbb{E}\left[\sum_{\tau=1}^t f(\mathbf{a}(\tau), \mathbf{X}(\tau))\right] \\ \text{s.t.} \quad & \mathbf{a}(\tau) \in \mathcal{F} \end{aligned} \tag{1}$$

where  $\mathbf{X}$  are unknown random variables;  $\mathbf{a}(\tau)$  is action at time  $\tau$ ;  $\mathcal{F}$  is a finite set.

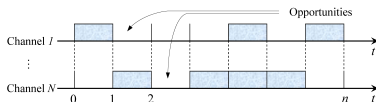
## Our focus on this topic

- Develop online learning algorithms for stochastic network optimization.

# Multi-Armed Bandits (MAB)

- Multi-armed bandit (MAB) problems provide a fundamental approach to learning under stochastic rewards.

- It has rich applications in networking contexts



Cognitive Radio Networks [e.g. Anandkumar *et al.*'10]

## Trade-off

- Exploration vs Exploitation



Internet advertising [e.g. Pandey *et al.*'07]



# Summary of Proposed Algorithms

Problems	Random Process	Proposed Algorithms	Regret Bound*
MAB with Linear Rewards	i.i.d.	LLR	$O(N^4 \ln t)$
		LLR-K	$O(N^4 \ln t)$
		LLR with a $\beta$ -approximation algorithm	$O(N^4 \ln t)$ ‡
MAB with Linear Rewards	Restless Markovian	MLMR	$O(N^4 \ln t)$ ‡
	Restless Markovian		$O(L(t)N^4 \ln t)$ †
MAB with Linear Rewards	Restless Markovian	CLRM	$O(N^4 \ln t)$ ‡
	Restless Markovian		$O(L(t)N^4 \ln t)$ †
Distributed Learning with Prioritization	i.i.d.	DLP	$O(M(N + M) \ln t)$
Distributed Learning with Fairness	i.i.d.	DLF	$O(M(N - M) \ln t)$
Selective learning of the $K$ -th largest arm	i.i.d.	SL( $K$ )	$O(N \ln t)$
MAB with Non-Linear Rewards	i.i.d.	CWF1	$O(N^4 \ln t)$
		CWF2	$O(\frac{N^2}{B(N)^2} \ln t)$
Non-Bayesian Restless MAB with identical transition matrices	Restless Markovian	SPUDC	$O(L(t) \ln t)$ †
Non-Bayesian Restless MAB with non-identical transition matrices	Restless Markovian	R2PC	$O(L(t) \ln t)$ †

Notes:

\*. Upper bounds on regret are achieved uniformly.

‡.  $\beta$ -approximation regret.

‡. weak regret; an upper bound on  $L$  is known.

†.  $L(t)$  is any arbitrarily slowly diverging non-decreasing sequence.

Thanks!

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