A. Network Primitives

By assumption the nodes in the network are non-flidling and of $D = \mu$ including both arrivals from the external stream and from internal routing and the amount of time the $i$ would have to wait known mean service rate equilibrium. The service capacity is modeled as a renewal process, with Two parts to the model:

- The arrival process is unknown
- We call this a $\gamma/GI/1$ queue, and develop pathwise fluid and diffusion approximations to the queue length and virtual waiting time processes.
- Consider a parallel network of $K \gamma/GI/1$ queues, with heterogeneous service rates and start times.

Theorem: The fluid queue length state of the network is

$$Q(t) = \bar{X}(t) + \sup_{-T \leq s \leq t} \{-X(s)\}^+$$

$$\bar{X}(t) = F(t) \begin{bmatrix} p_1 - \mu_1(T_{1,t}) 1_{1 \leq t \leq T_{1,t}} \\ p_2 - \mu_2(T_{2,t}) 1_{2 \leq t \leq T_{2,t}} \\ \vdots \\ p_K - \mu_K(T_{K,t}) 1_{K \leq t \leq T_{K,t}} \end{bmatrix}.$$  

The virtual waiting time snapshot process is:

$$W(t) = MQ(t) - t$$

$$M = \text{diag}(\frac{1}{\mu_1}, \ldots, \frac{1}{\mu_K})$$  

Model of Strategic Arrivals

- Each arriving user chooses a mixed strategy over a set of possible arrival times, i.e., a probability distribution function, and a routing probability
- The Nash equilibrium strategy minimizes a cost, $C(t) = (\alpha + \beta)W(t) + \beta t$, given the strategies of other arriving users, compared to other possible strategies
  
- $(\alpha, \beta)$ characterizes the population of arriving users, and we define $\gamma = \frac{\alpha}{\alpha + \beta}$
- We study the game in the fluid/large population setting, and consider a non-atomic game formulation

References