Ming Hsieh Department of Electrical Engineering

The Network Concert Queueing Game

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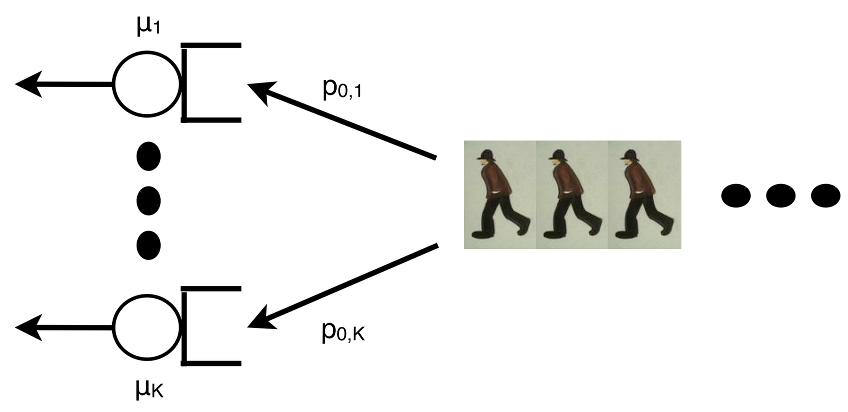
Introduction

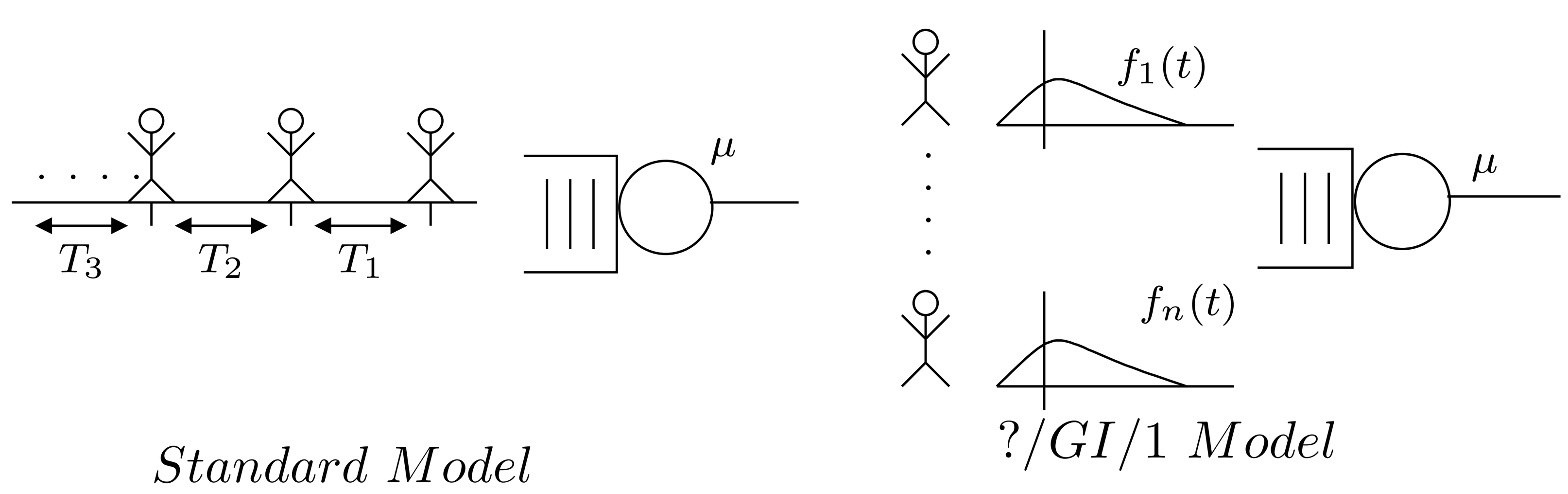


- Consider arriving at a concert, where the gates open at a fixed time. Should one arrive early and be the first in line? Or should one arrive later but risk missing out on the concert? On arrival, which gate should one enter?
- Standard queueing models do not capture this behavior. So, **what is the arrival process in this case? How does one model this behavior?**

Two parts to the model:

- A model of the parallel queueing network
- Modeling the strategic arrival behavior as a game

Network of ?/GI/1 Queues



- The arrival process is unknown *a priori* and is determined in equilibrium. The service capacity is modeled as a renewal process, with known mean service rate
- We call this a *?/GI/1* queue, and develop pathwise fluid and diffusion approximations to the queue length and virtual waiting time processes.
- Consider a parallel network of *K ?/GI/1* queues, with heterogeneous service rates and start times.

***Theorem*:** The fluid queue length state of the network is

$$\bar{\mathbf{Q}}(\mathbf{t}) = \bar{\mathbf{X}}(\mathbf{t}) + \sup_{-T_0 \le s \le t} \{-\bar{\mathbf{X}}(\mathbf{s})\}^+$$

$$\bar{\mathbf{X}}(t) = F(t)\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \end{bmatrix} - \begin{bmatrix} \mu_1(t - T_{s,1})\mathbf{1}_{\{t \ge T_{s,1}\}} \\ \mu_2(t - T_{s,2})\mathbf{1}_{\{t \ge T_{s,2}\}} \\ \vdots \\ \mu_K(t - T_{s,K})\mathbf{1}_{\{t \ge T_{s,K}\}} \end{bmatrix}$$

The virtual waiting time snapshot process is:

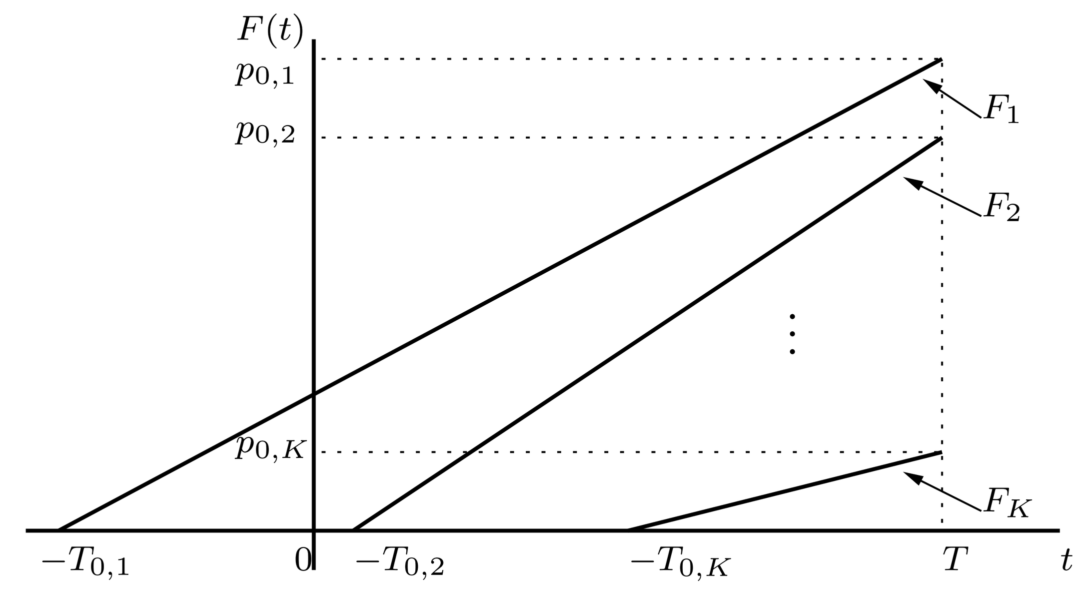
$$\bar{\mathbf{W}}(t) = \mathbf{M}\bar{\mathbf{Q}}(t) - \mathbf{t}_{\mathbf{s},-}(t)$$

$$\mathbf{M} = \text{diag}\left(\frac{1}{\mu_1}, \ldots, \frac{1}{\mu_K}\right) \quad \mathbf{t}_{\mathbf{s},-}(t) = \begin{bmatrix} (t - T_{s,1})\mathbf{1}_{\{t \le T_{s,1}\}} \\ (t - T_{s,2})\mathbf{1}_{\{t \le T_{s,2}\}} \\ \vdots \\ (t - T_{s,K})\mathbf{1}_{\{t \le T_{s,K}\}} \end{bmatrix}$$

Model of Strategic Arrivals

- Each arriving user chooses a *mixed strategy* over a set of possible arrival times, i.e., a probability distribution function, and a routing probability
- The *Nash equilibrium strategy* minimizes a cost, $C(t) = (\alpha + \beta)W(t) + \beta t$, given the strategies of other arriving users, compared to other possible strategies
  - (α, β) characterizes the population of arriving users, and we define $\gamma = \frac{\alpha}{(\alpha + \beta)}$
- We study the game in the fluid/large population setting, and consider a *non-atomic* game formulation

Single Population of Arriving Users



Theorem: The unique equilibrium arrival profile is $F^* = (F_1^*, \ldots, F_{K'}^*)$ where

$$F_l^*(t) = \frac{p_l \times (t + T_{0,l})}{(T + T_{0,l})} \quad \forall t \in [-T_{0,l}, T],$$

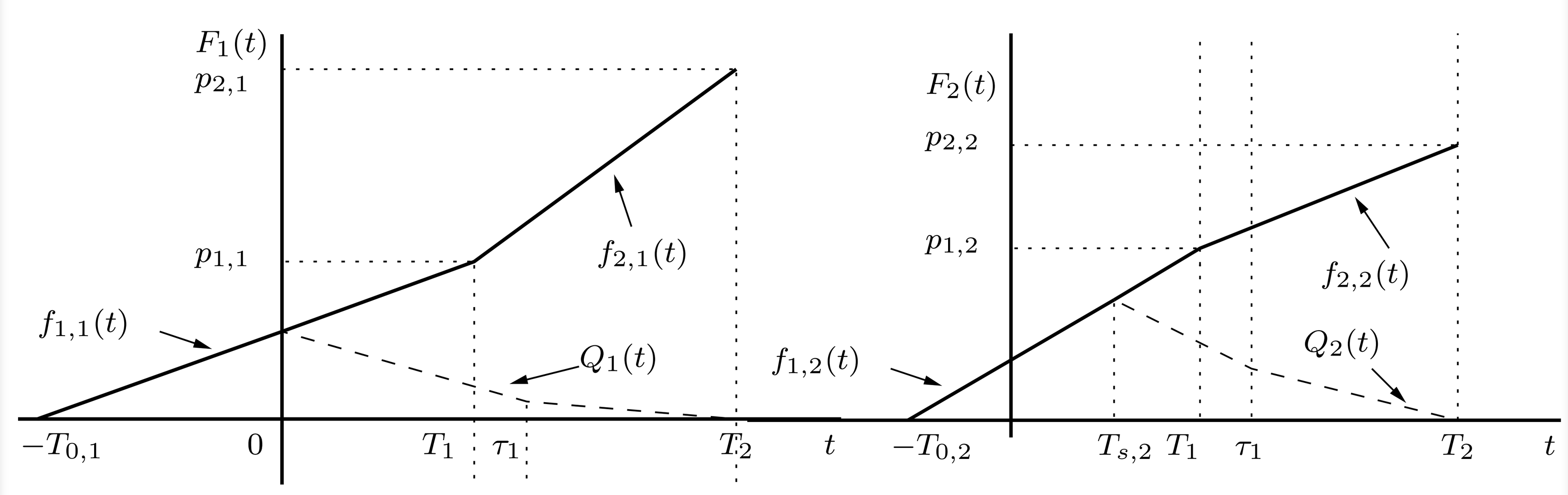
$$T = \frac{1 + \sum_{k=1}^K \mu_k T_{s,K}}{\sum_{k=1}^K \mu_k}, \; -T_{0,l} = (1 - \frac{1}{\gamma})T + \frac{T_{s,l}}{\gamma},$$

and

$$p_l = \frac{\mu_l}{\sum_{k=1}^K \mu_k}(1 - \sum_{k \ne l} \mu_l (T_{s,l} - T_{s,k})).$$

Theorem: The Price of Anarchy of the equilibrium solution is bounded above by 2.

Multiple Populations of Arriving Users



Theorem: Consider *N* distinct populations, and assume $\gamma_1 \le \gamma_2 \le \ldots \gamma_N$. Then, at equilibrium population *i* users arrive before population *j* users, for *i* < *j*. Furthermore, the arrivals are over disjoint intervals, without any gaps.

Theorem: Suppose the queues offer the same service rate, and start at uniformly staggered times (i.e., queue *k* starts at time *(k-1)* τ), then the Price of Anarchy of the equilibrium is bounded above by 2.

Conclusion

- We proved the existence and uniqueness of a Nash equilibrium solution to a large population game of strategic arrivals to a parallel network of queues.
- We showed that the arrival distribution, both in the case of a single population and multiple arriving populations, is a uniform distribution function
- The equilibrium solution will be furthered refined in a Functional Central Limit Theorem setting in future work

References

[1] H. Honnappa and R. Jain, "Strategic Arrivals into Queueing Networks: The Network Concert Queueing Game," Operations Research (Submitted), 2011.

[2] H. Honnappa, R. Jain and A. R. Ward, "The F/GI/1 Queue: Fluid and Diffusion Approximations, Draft Preparation, 2012.

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