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Continuous decomposition of quantum measurements via Hamiltonian feedback

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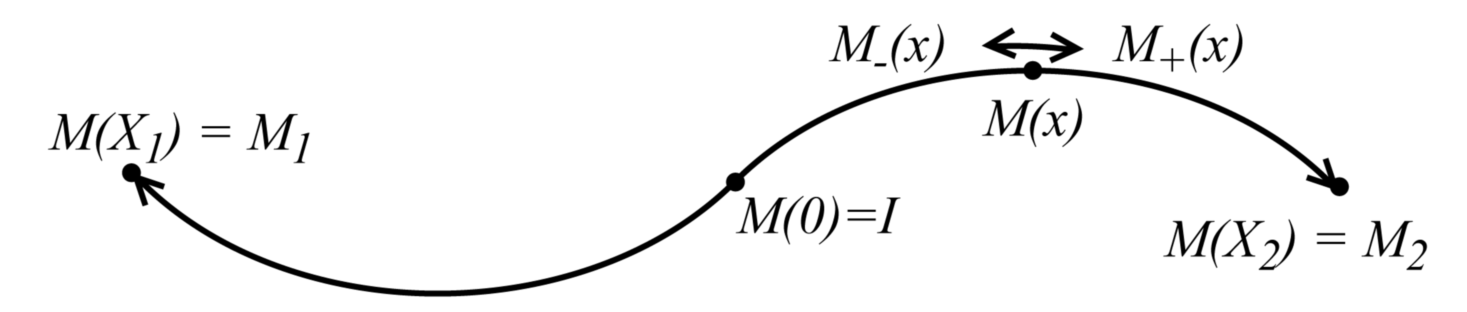
Continuous measurements

Quantum dynamics are reversible, deterministic and continuous. However, quantum measurements are irreversible, non-deterministic, and discontinuous. **Can we describe both dynamics and measurement continuously?**

Continuous decompositions as random walks

In [OB05] it was shown that any quantum measurement $\{M_1, M_2\}$ can be decomposed into a 1-dimensional continuous stochastic process

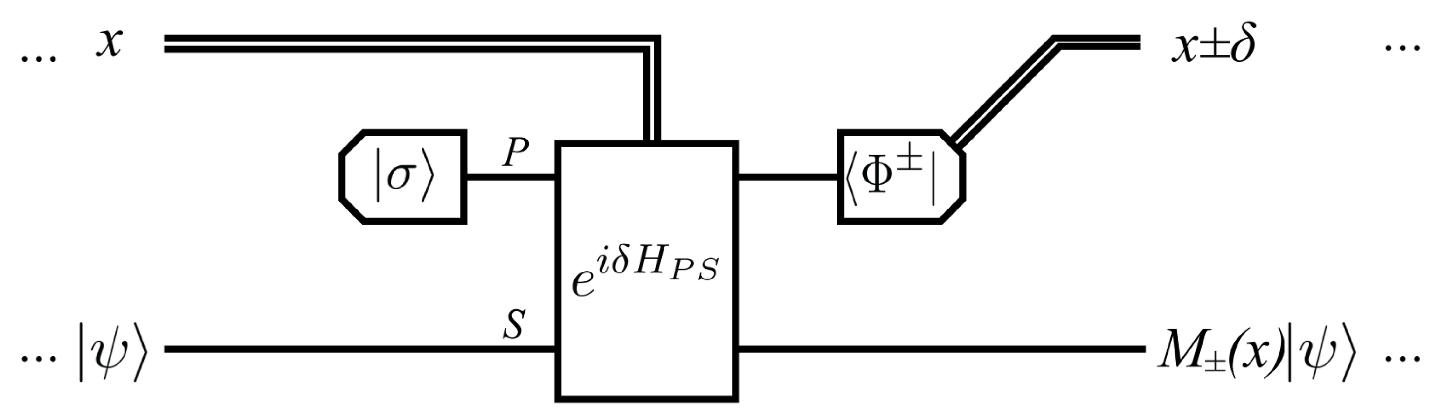


In [FB14] we've shown that any qubit-probe interacting in a fixed way with the system being measured could only decompose measurements of the form

$$M_1 = U_1\left(\alpha\Pi_S + \beta\Pi_{S^\perp}\right)V$$
$$M_2 = U_2\left(\sqrt{1-\alpha^2}\Pi_S + \sqrt{1-\beta^2}\Pi_{S^\perp}\right)V$$

Linear Hamiltonian control terms

New question: What can be done with a qubit-probe and a tunable interaction?



In the above circuit we define

Probe state	$\|\sigma(x)\rangle = \|0\rangle$
Detector states	$\langle\phi^\pm\| = \langle\pm\|$
Tunable interaction	$H_{PS} = Y_P \otimes \hat{\varepsilon}(x)$
Linear control terms	$\hat{\varepsilon}(x) = \sum_{i=0}^{d} p_i(x) H_i$
Algebra of the control set	$F = \text{span}\{H_1, \ldots, H_d\}$
Step operators	$M_\pm(x) = \frac{1}{\sqrt{2}}\mathbb{1} + i\delta\langle\pm\|H_{PS}(x)\|0\rangle$
Total walk operator	$M(x) \propto \lim_{\delta\to 0}\prod_{j=1}^{\lfloor\|x\|/\delta\rfloor} M_\pm(\pm j\delta)$ $M(x) = \sum_{i=1}^{D} H_i$
Endpoint operators	$M_{1,2} = \lim_{x\to X_{1,2}} M(x)$

The above must satisfy the following equations

Reversibility equation	$M_\mp(x\pm\delta)M_\pm(x) \propto \mathbb{1}$
Operator propagation	$\partial_x M(x) = -\hat{\varepsilon}(x)M(x)$

Quadratic systems of ODEs

The reversibiliy equation and the operator propagation equation can be rewritten as quadratic systems of ODEs

Quadratic ODE sys. (1)	$\sum_{k=0}^{d}\partial_x p_k(x)H_k = \frac{1}{2}\sum_{i,j=0}^{d} p_i(x)p_j(x)\{H_i, H_j\}$
Quadratic ODE sys. (2)	$\sum_{k=0}^{d}\partial_x a_k(x)H_k = -\frac{1}{2}\sum_{i,j=0}^{d} p_i(x)a_j(x)H_iH_j$

Closure lemma

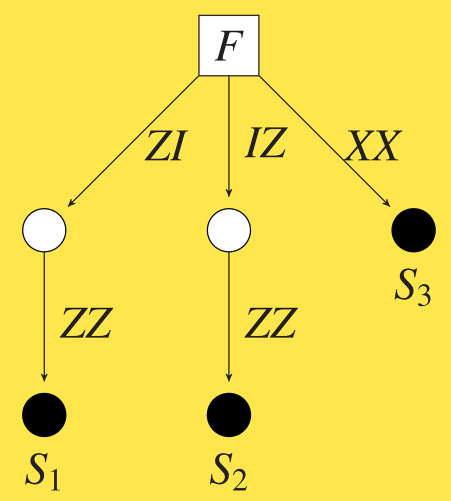
In order to satisfy the reversibility equation, the span of active linear control terms $F = \text{span}\{H_i\}$ must be closed under anti-commutation.

Restriction algorithm

```
function RESTRICT(F)
    if F^{∘2} = F then return F
    else
        S ← BASIS (F^{∘2})
        for all s ∈ S do
            s ← RESTRICT (F\s)
        end for
        return S
    end if
end function
```

Example

INPUT: $F = \{II, ZI, IZ, ZZ, XX\}$



RESTRICT$(F) = \{S_1, S_2, S_3\}$
$S_1 = \{II, IZ, XX\}$
$S_2 = \{II, ZI, XX\}$
$S_3 = \{II, ZI, IZ, ZZ\}$.

Extension algorithm

```
function EXTEND(F)
    while F^{∘2} ⊃ F do
        F ← BASIS (F^{∘2})
    end while
end function
```

Example

INPUT: $F = \{II, ZI, IZ, ZZ, XX\}$
EXTEND$(F) =$
$\{II, ZI, IZ, ZZ, XX, YY\}$

Discussion

- Using the control set $F = \text{span}\{I, X, Z\}$ we can only decompose measurements of the form achieved by qubit-probe feedback above.
- Quadratic ODE system (2) is completely determined by system (1) and the initial condition $M(0) = I$.
- Quadratic ODE system (1) contains no orbits [KS95].
- If the span of controls F is also closed under

$$H_1H_2H_3H_4 + H_4H_3H_2H_1 \in F$$

then by the *Cohn Reversible Theorem* F is the Hermitian part of the Free algebra generated by F (i.e.: the most general algebra). [McC04]

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