Fluctuation Theorems for Quantum Processes

T. Albash, D. Lidar, **M. Marvian**, P. Zanardi

Motivation & Introduction

Describing systems at non-equilibrium state, using *EQUALITIES*.

Classical Jarzynski equality: $\langle e^{-\beta(w-\Delta F)}\rangle = \gamma$

Relates average of the work done on the system to the free energy difference of final equilibrium state. More generally(Tasaki-Crooks):

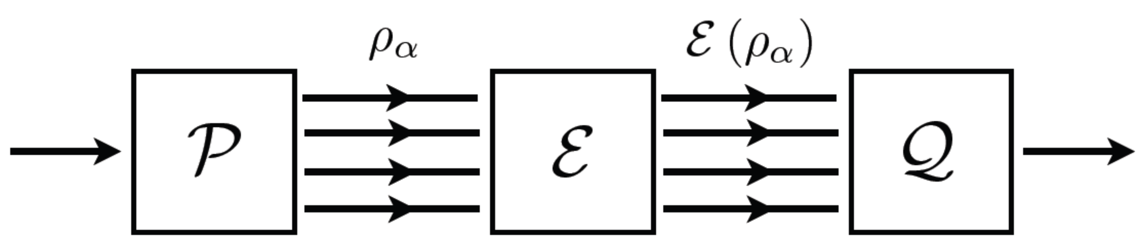
$$P_F(w)e^{-\beta(w-\Delta F)} = P_R(-w)$$

Relates PDF of work done in forward process to reverse process.

We present fluctuation theorems and moment generating function equalities for quantum dynamics (CPTP maps) and generalized measurements, with and without feedback.

Main Idea

Assume $\mathcal{E}$ is the most general quantum channel, and $\{P_\alpha\}$ and $\{Q_\beta\}$ are projective measurements with rank one.



$\{p_\alpha \to P_\alpha\}$ $\quad p_{\beta|\alpha} = Tr(Q_\beta \mathcal{E}(P_\alpha))$

Let V be a random variable parameterized by measurement outcomes $\{\alpha, \beta\}$.
Its PDF is:

$$P_\mathcal{E}(v) = \sum_{\alpha,\beta} \delta(v - V_{\alpha\beta})\, p_{(\alpha,\beta)} =$$

$$\sum_{\alpha,\beta} \delta(v - V_{\alpha\beta})\, Tr(Q_\beta \mathcal{E}(P_\alpha)) p_{(\alpha)}$$

Now if we assume: $V_{\alpha\beta} = \ln(\frac{p_{(\alpha)}}{q_{(\beta)}})$, then:

$$P_\mathcal{E}(v)e^{-v} = \sum_{\alpha,\beta} \delta(-v - V_{\beta\alpha})\, Tr\left(P_\alpha \mathcal{E}^*(Q_\beta)\right) q_{(\beta)} = P_{\mathcal{E}^*}(-v)$$

$\mathcal{E}^*$ is the dual map of $\mathcal{E}$. (Krauss operators complex conjucated)
Reverse process appears!
Now we can define efficacy γ : $\quad \gamma = \int \tilde{P}_{\mathcal{E}^*}(v)dv$

Quantum Jarzynski equality: $\quad \langle e^{-v}\rangle = \gamma.$

Also for moment generating functions: $\quad \chi_\mathcal{E}(\lambda - 1) = \tilde{\chi}_{\mathcal{E}^*}(-\lambda)$

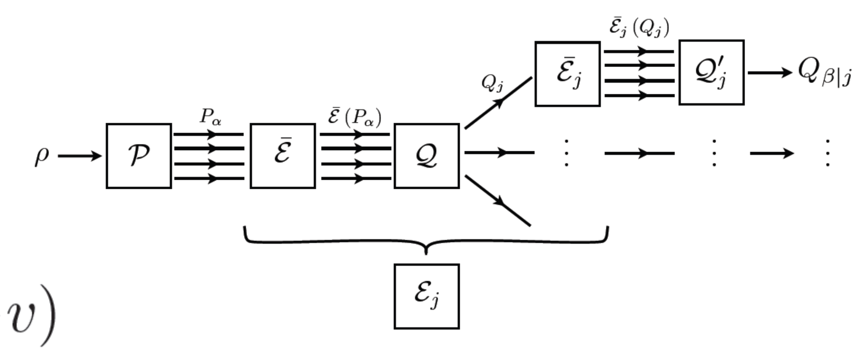
In this case $P_{\mathcal{E}^*}$ is PDF if channel is ***UNITAL.*** $\mathcal{E}(I) = I$.

The UNITALITY replaces MICRO-REVERSIBILITY

Generalization

Even using ***Generalized Measurements*** $(\sum P_\alpha^\dagger P_\alpha = \mathbf{I})$ and in presence of ***Feedback***:

Results hold:

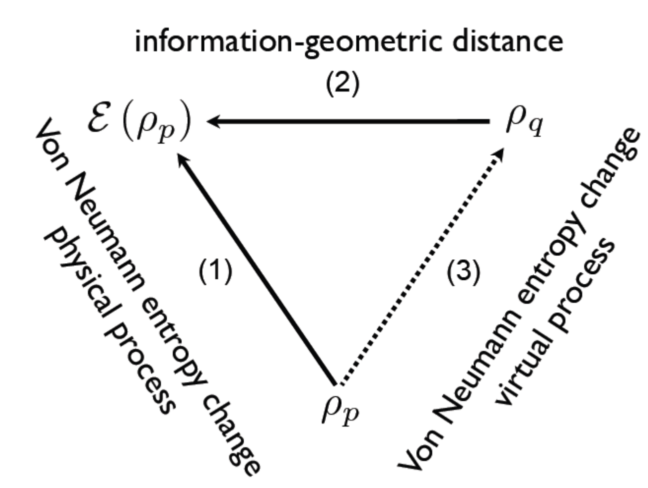


$$P_\mathcal{E}(v)e^{-v} = \tilde{P}_{\mathcal{E}^*}(-v)$$

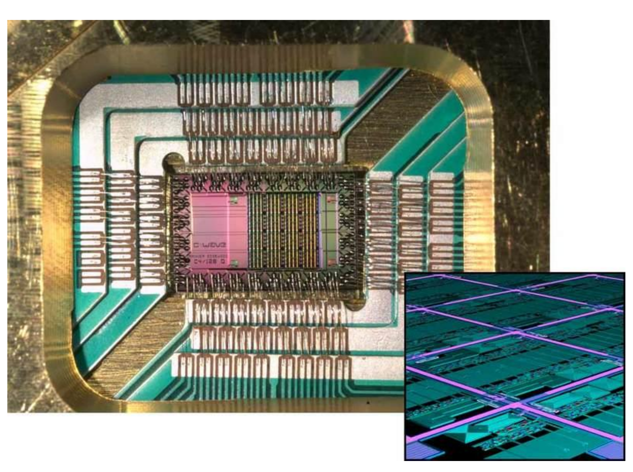
$$\chi_\mathcal{E}(\lambda - 1) = \tilde{\chi}_{\mathcal{E}^*}(-\lambda)$$

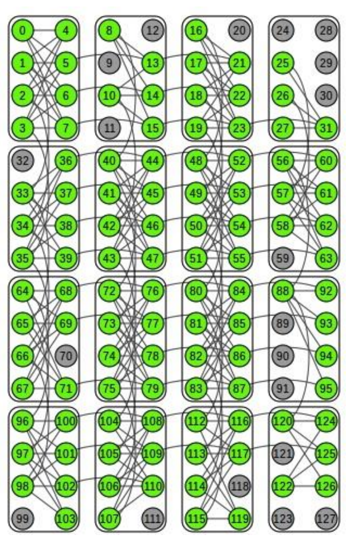
Nice Interpretation:

$$\langle v\rangle = S(\mathcal{E}(\rho_p)) - S(\rho_p) + S(\mathcal{E}(\rho_p)\|\rho_q)$$
$$= S(\rho_q) - S(\rho_p) + \mathrm{Tr}[(\rho_q - \mathcal{E}(\rho_p))\ln(\rho_q)]$$



Experiments

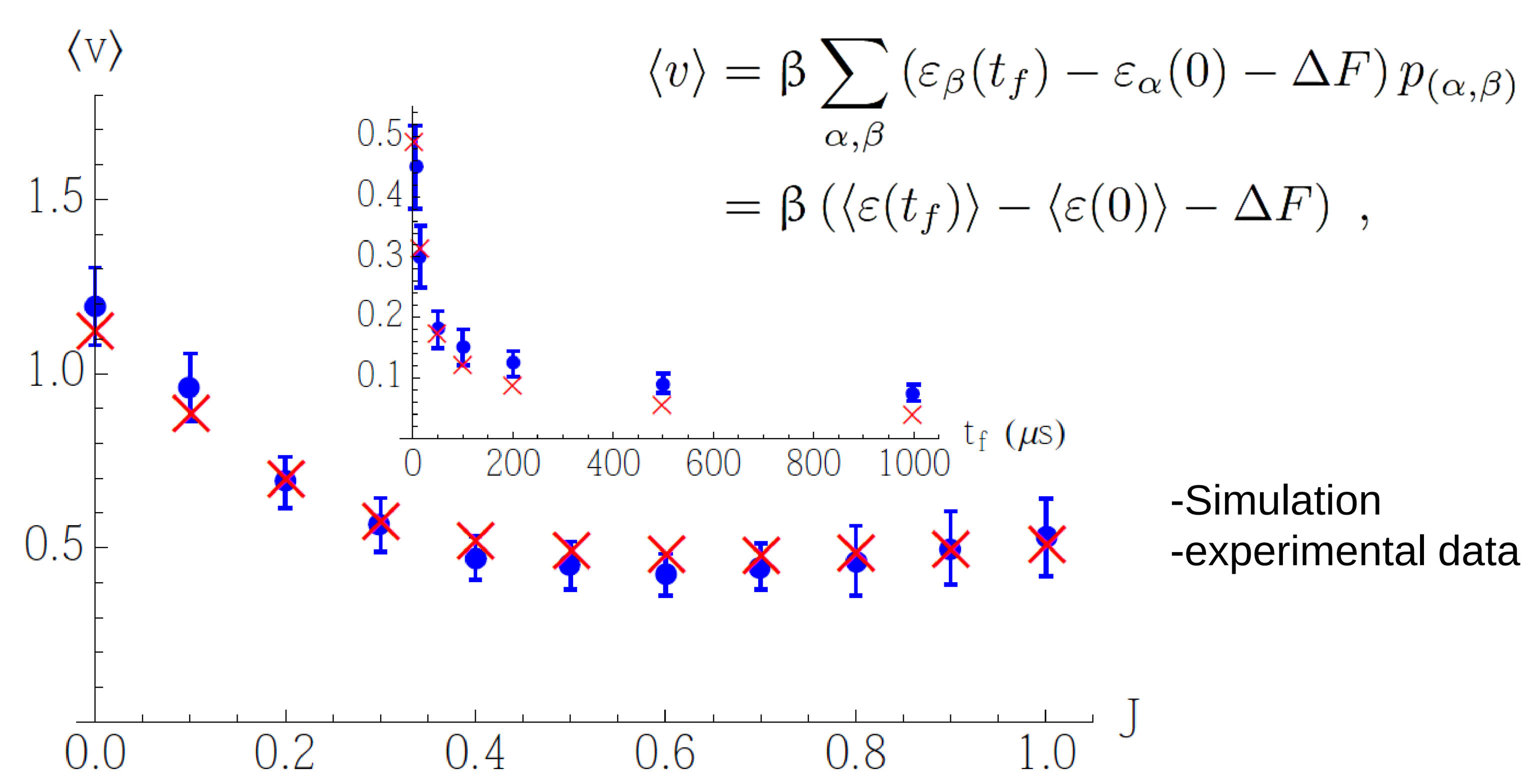




D-Wave1.

$$H = -A(t)(X_1 + X_2) - B(t)(\tfrac{1}{3}Z_1 + \tfrac{1}{3}Z_2 + J\, Z_1 Z_2)$$

Simulation using master equation to extract coupling to environment.



-Simulation
-experimental data

References

[1] "Fluctuation Theorems for Quantum Processes", [1212.6589], by T. Albash, D. Lidar, M. Marvian, P. Zanardi.
[2] C. Jarzynski, Phys. Rev. Lett. 78, 2690 (1997).
[3] T. Albash, S. Boixo, D. A. Lidar, and P. Zanardi, New J. of Phys. 14, 123016 (2012).

marvianm@usc.edu

Ming Hsieh Institute
Ming Hsieh Department of Electrical Engineering