USC University of Southern California

USC Viterbi School of Engineering

Ming Hsieh Department of Electrical Engineering

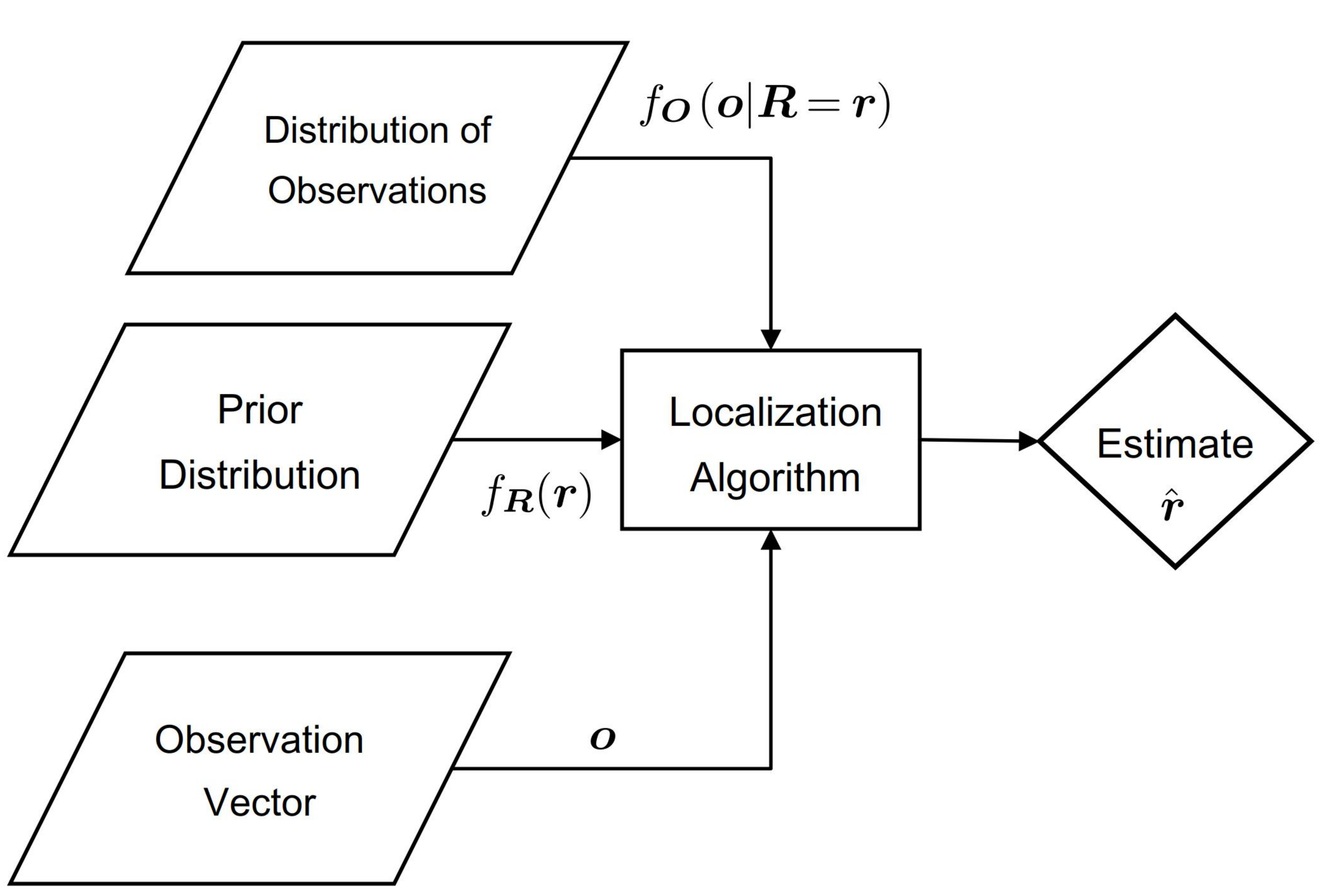
A Unifying Bayesian Optimization Framework for Radio Frequency Localization

Nachikethas A. Jagadeesan and Bhaskar Krishnamachari

Table I. Literature Survey on Localization Methods and Metrics

Study	*Algorithm*	*Model*	*Metric*
WLAN location determination via clustering and probability distributions [Youssef et al. 2003]	MLE	Fingerprinting	$P(d)$
Maximum likelihood localization estimation based on received signal strength [Waadt et al. 2010]	MLE	Log-Normal	MSE
Experimental comparison of RSSI-based localization algorithms for indoor wireless sensor networks [Zanca et al. 2008]	Multilateration and MLE	Log-Normal	EDE
A Bayesian sampling approach to in-door localization of wireless devices using RSSI [Seshadri et al. 2005]	Weighted MLE and Error CDF	Fingerprinting	EDE
RADAR: An In-Building RF-based user location and tracking system [Bahl and Padmanabhan 2000]	Clustering	Fingerprinting	EDE
The Horus WLAN location determination system [Youssef and Agrawala 2005]	MLE	Fingerprinting	EDE

- The current RF localization literature is disconnected and disorganized, with no theoretical understanding of how we can compare different algorithms.
- Often first an algorithm is proposed, arrived at chiefly out of experience or intuition.
- Algorithm evaluation is often based on metrics chosen *after* the design of the said algorithm.
- Instead, we advocate an optimization based approach where the objective specifies the desired performance characteristics of the derived algorithm.
- An optimization based approach allows us to define a partial ordering over the set of algorithms, allowing for fair and meaningful algorithm evaluation.



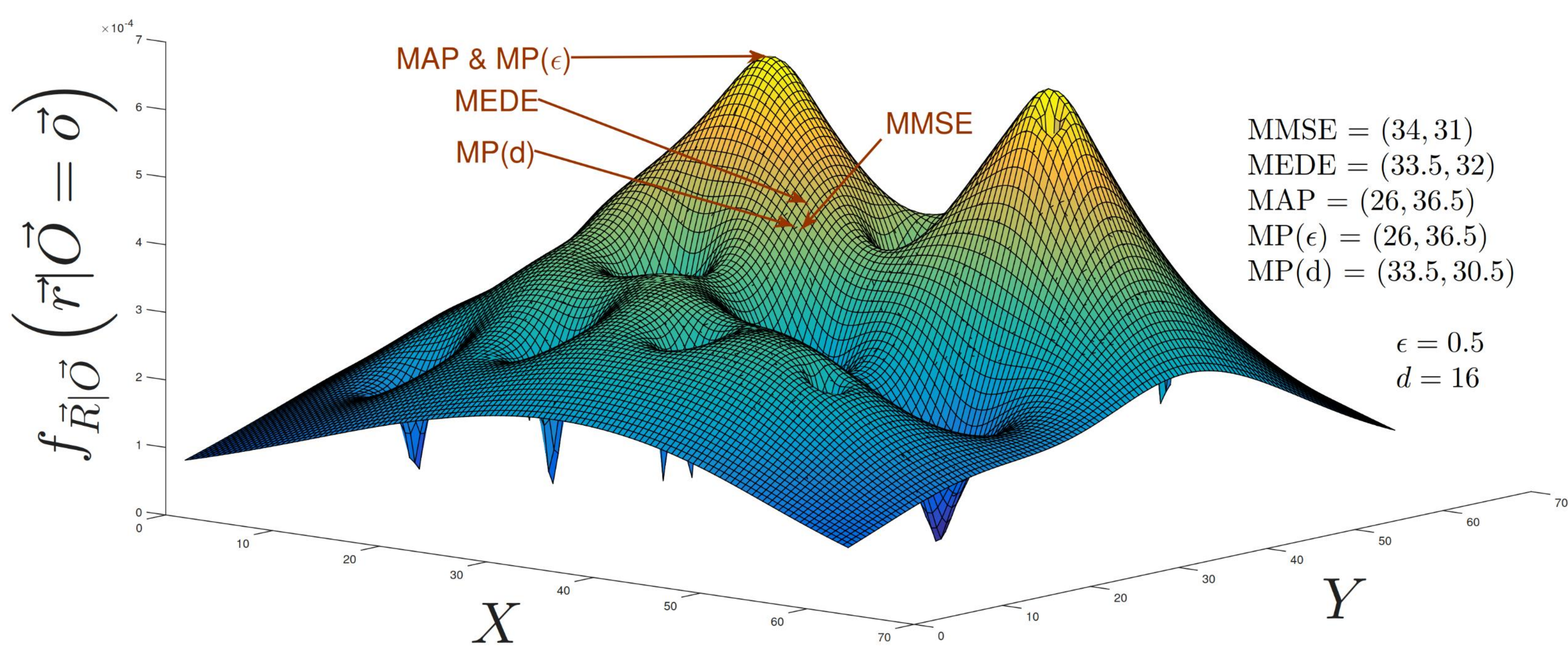
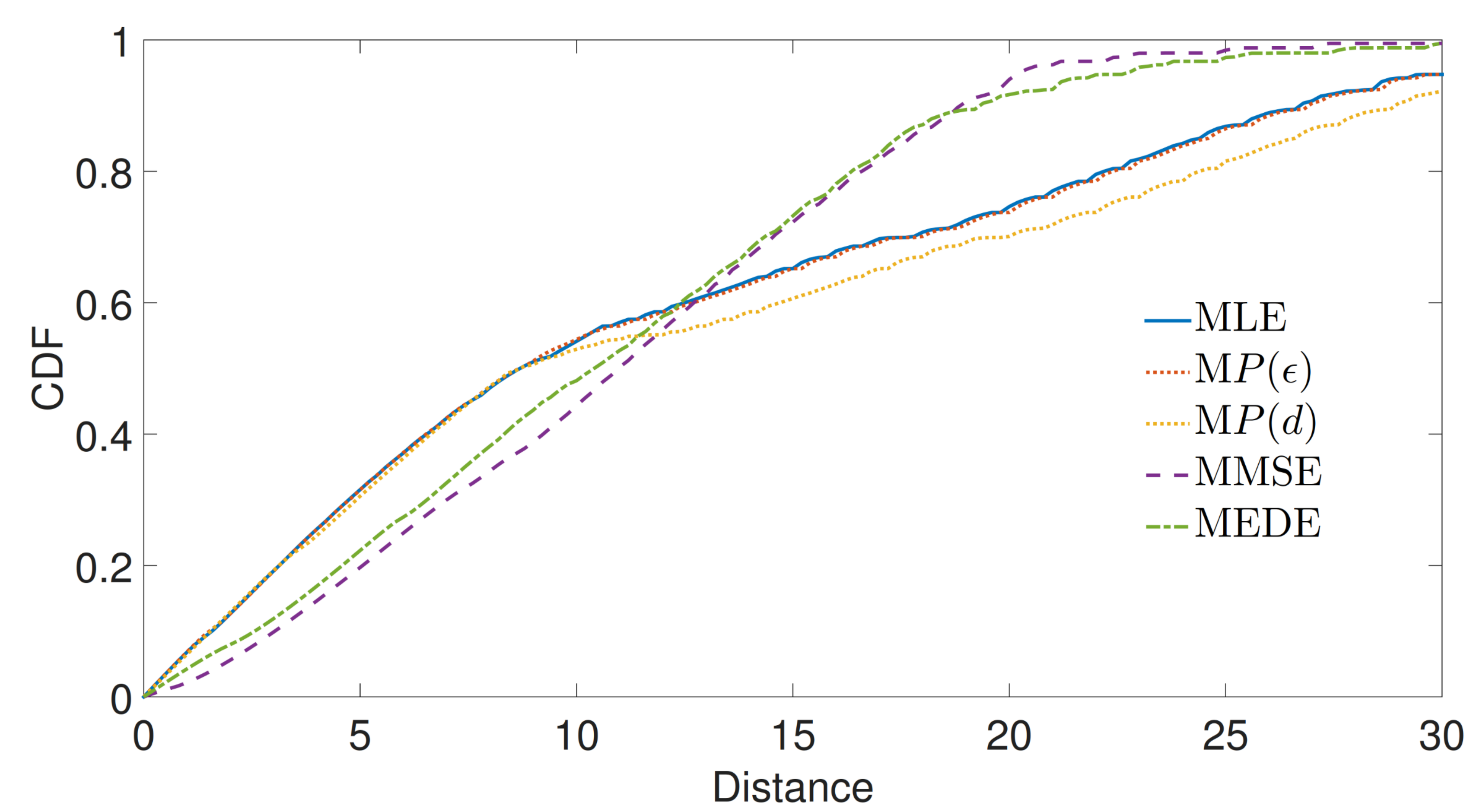


Table II. Recovering existing algorithms in our framework

Algorithm	*Cost function*	*Optimization*
MMSE	$\mathcal{C}(\boldsymbol{r},\tilde{\boldsymbol{r}},\boldsymbol{o}) = (\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2)^2$	$\boldsymbol{r}_{MMSE} = \arg\min_{\tilde{\boldsymbol{r}}} \mathrm{E}\left[(\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2)^2\right]$
MEDE	$\mathcal{C}(\boldsymbol{r},\tilde{\boldsymbol{r}},\boldsymbol{o}) = \|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2$	$\boldsymbol{r}_{MEDE} = \arg\min_{\tilde{\boldsymbol{r}}} \mathrm{E}\left[\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2\right]$
MP(d)	$\mathcal{C}(\boldsymbol{r},\tilde{\boldsymbol{r}},\boldsymbol{o}) = -P\left(\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2 \le d\right)$	$\boldsymbol{r}_{MP(d)} = \arg\max_{\tilde{\boldsymbol{r}}} \mathrm{E}\left[P\left(\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2 \le d\right)\right]$
MLE	$\mathcal{C}(\boldsymbol{r},\tilde{\boldsymbol{r}},\boldsymbol{o}) = -P\left(\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2 \le \epsilon\right)$	$\boldsymbol{r}_{MLE} = \lim_{\epsilon\to 0} \arg\max_{\tilde{\boldsymbol{r}}} \mathrm{E}\left[P\left(\|\tilde{\boldsymbol{r}}-\boldsymbol{r}\|_2 \le \epsilon\right)\right]$

Definition 4.4. Let $g : \mathcal{Q} \subseteq \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ be a monotonically increasing function. Denote the set of all such functions by $\mathcal{G}$. For a localization algorithm A, $g(D_A)$ is the *distance error localization cost function*. $\mathrm{E}\left[g(D_A)\right]$ is the *expected cost* of the algorithm A.



The error CDFs of MLE, MP(d), MMSE and MEDE. For this illustration, nine transmitters were placed evenly in a line and log-normal fading was assumed. Note that none of the given algorithms are strictly stochastically dominated.

THEOREM: *For any two localization algorithms $A_1, A_2 \in \mathcal{A}$, if A_1 stochastically dominates A_2, then*

$$\mathrm{E}\left[g(D_{A_1})\right] \le \mathrm{E}\left[g(D_{A_2})\right] \quad \forall g \in \mathcal{G}.$$

If A_1 strictly stochastically dominates A_2, then

$$\mathrm{E}\left[g(D_{A_1})\right] < \mathrm{E}\left[g(D_{A_2})\right] \quad \forall g \in \mathcal{G}.$$

THEOREM: *For any two localization algorithms A_1 and A_2, if A_2 does not stochastically dominate A_1 and vice versa, then there exits distance based cost functions $g_1, g_2 \in \mathcal{G}$ such that*

$$\mathrm{E}\left[g_1(D_{A_1})\right] < \mathrm{E}\left[g_1(D_{A_2})\right],$$

and

$$\mathrm{E}\left[g_2(D_{A_2})\right] < \mathrm{E}\left[g_2(D_{A_1})\right].$$

nanantha@usc.edu

Ming Hsieh Institute
Ming Hsieh Department of Electrical Engineering