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Computing encounter distributions of multiple random walkers

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Introduction

- ***Goal***: characterize the distribution of encounter times of walkers on random graphs including : exact computation of **pairwise inter-encounter time** distribution for a particular pair of random walkers, and approximate computation of **individual-to-any inter-encounter time** (i.e., the time between contacts of a particular walker with any of the other walkers in the population).
- ***Motivation***: exploit potential of opportunistic networks (inter encounter time)

Problem Formulation And Modeling

- N walkers walking on the connected graph characterized by V vertexes and S edges.
- For every timeslot, all walkers move on the graph following transition probability matrix **P.**

$$A = \{1,2,3,..,|V|\} \quad B = \{(1,1),(1,2),(1,3),...,(|V|,|V|)\}$$

- **Interested concepts:**
  - **PET (Inter Pairwise Encounter Time)**
  - **IAET (Inter Any Encounter Time)**
- $P(x,y,t)$ is the probability given that the walker 1 initially stays at vertex x, walker 2 initially stays at vertex y, they can first meet after t time steps:

$$P(x,x,0) = 1, \forall x \in A$$
$$P(x,y,0) = 0, \forall x,y \in A, x \neq y$$
$$P(x,y,t) = \sum_{\substack{(x,y)\in B \\ (x',y')\in B \\ x'\neq y'}} P(x',y',t-1)\cdot M_{(x',y')(x,y)}$$

- Inter-pairwise encounter time probability:

$$P_{PET}(t) = \sum_{z\in A} P(z,z,t)\cdot \widetilde{\pi}_z$$

- The probability given that the walker 1 initially stays at vertex x, walker 2 initially stays at vertex y, the pair hasn't met for t time slots:

$$\overline{P}(x,y,t) = \overline{P}(x,y,t-1) - P(x,y,t), x \in A, y \in A$$

- Considering all N walkers:

$$\overline{P}(L_z,t) = \prod_{i=2}^{N} \overline{P}(z,l_i,t)$$

- The probability that the particular walker meet one of remaining walkers at location z and hasn't met any other walkers since then up to time slot t:

$$\hat{P}(z,t) = \frac{\sum_{L_z} \overline{P}(L_z,t)}{|V|^{N-2}}$$

- Inter any encounter time probability:

$$\overline{P}_{IAET}(t) = \sum_{z\in A} \hat{P}(z,t)\cdot \pi_z$$

$$P_{IAET}(t) = \overline{P}_{IAET}(t-1) - \overline{P}_{IAET}(t)$$

Illustrative example

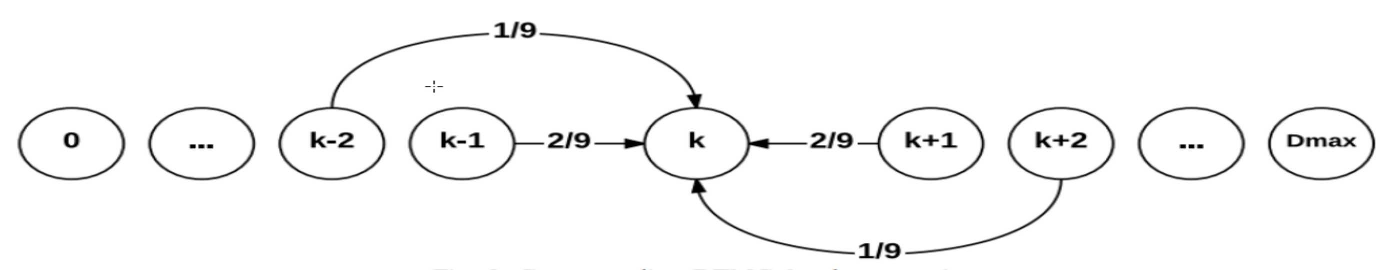


Fig. 2: Corresponding DTMC for the example

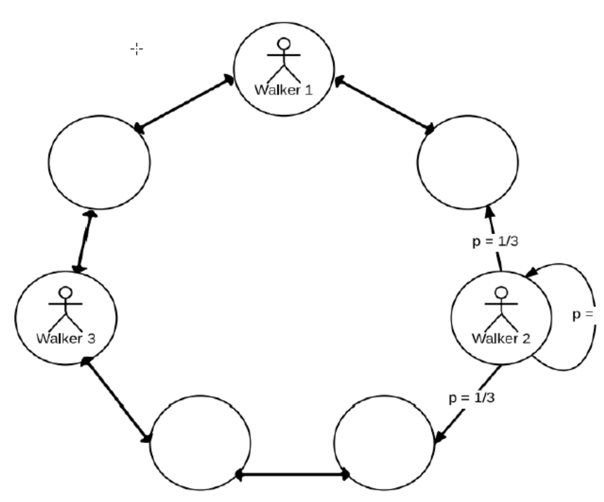
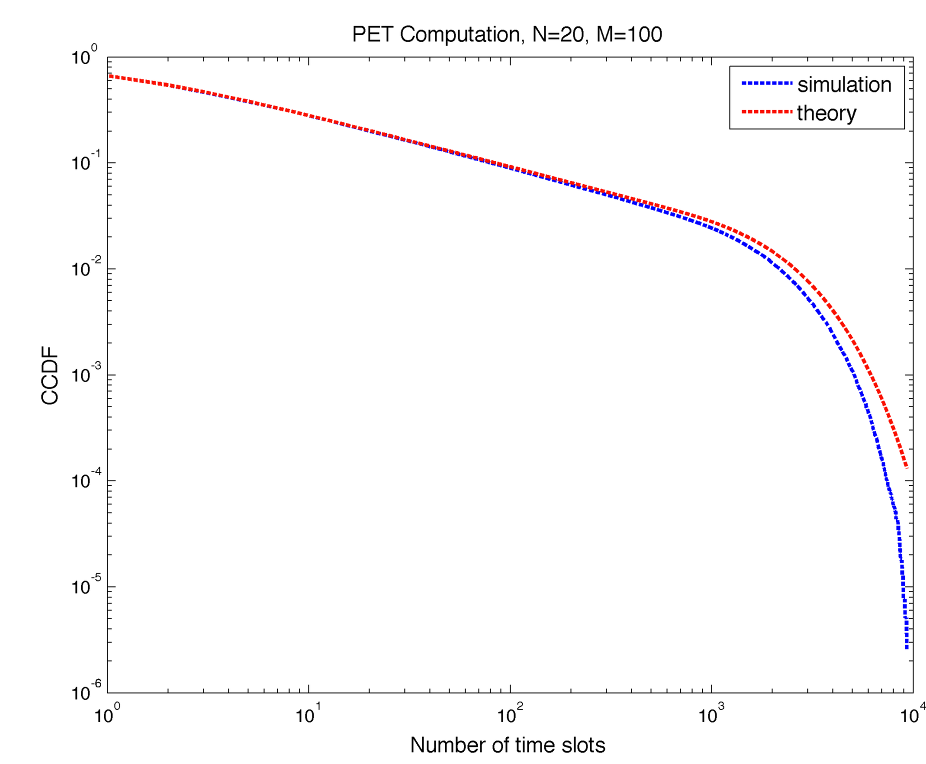
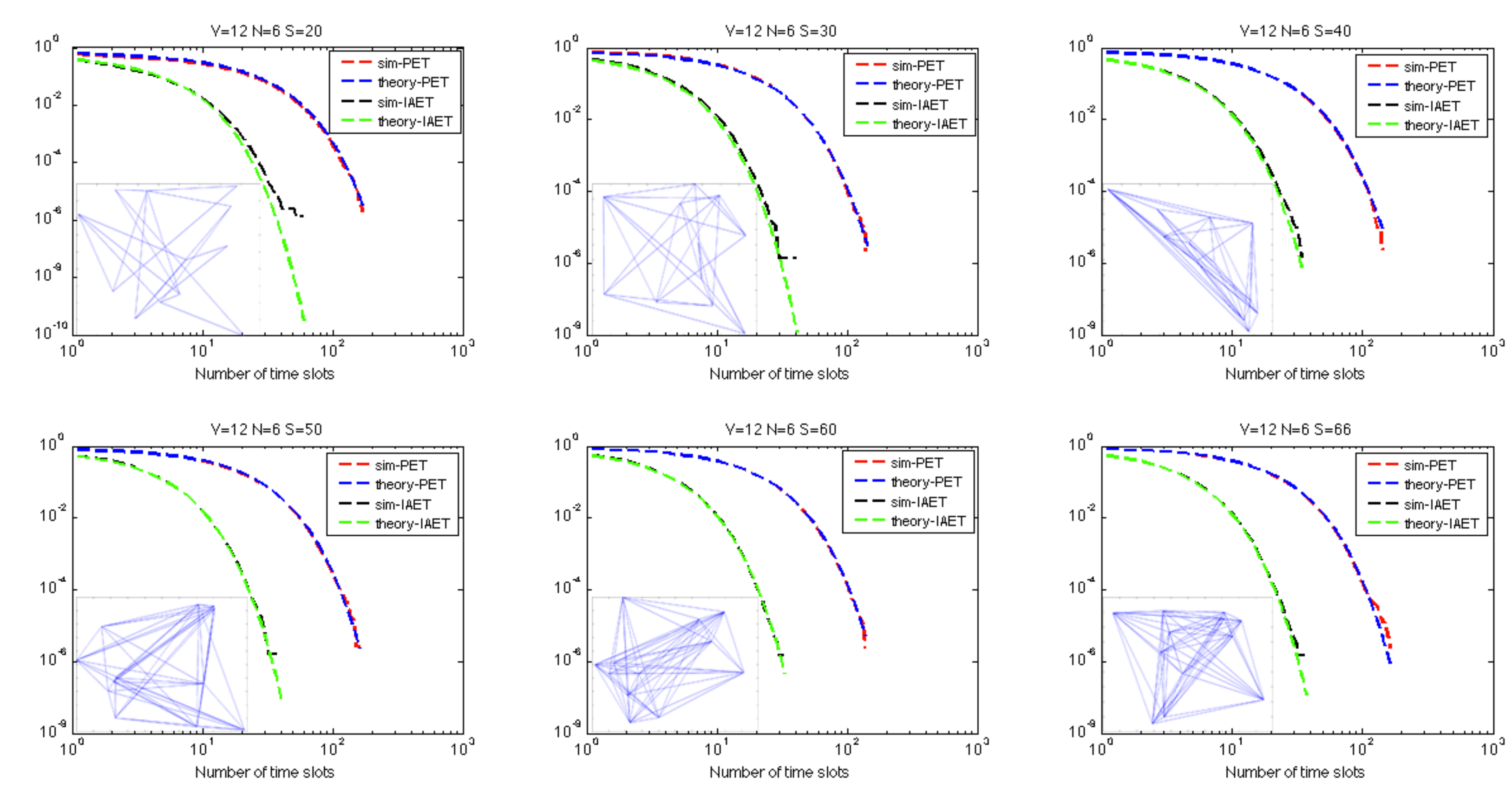


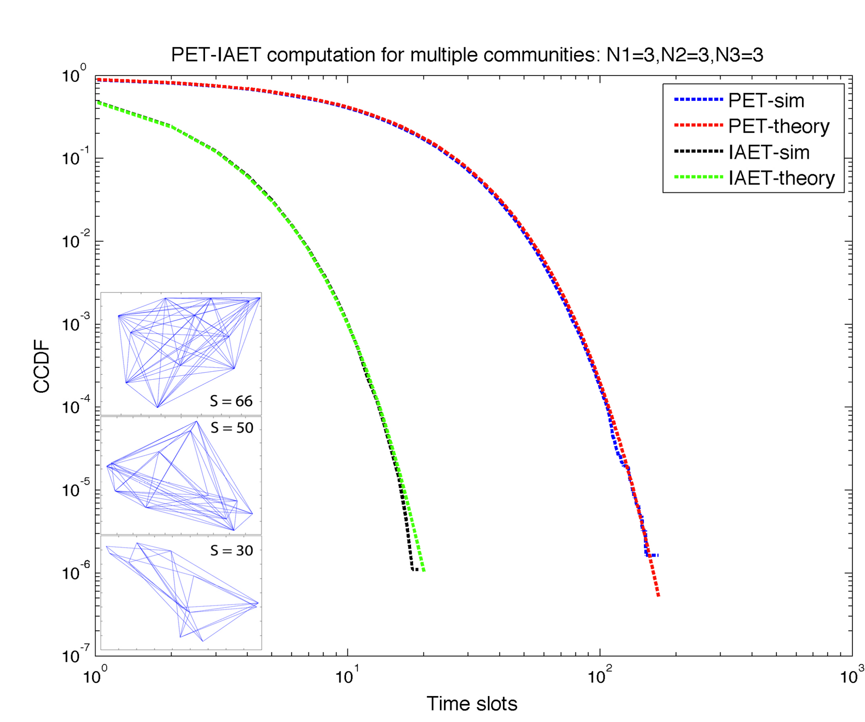
Fig. 1: Circular Random Walk example

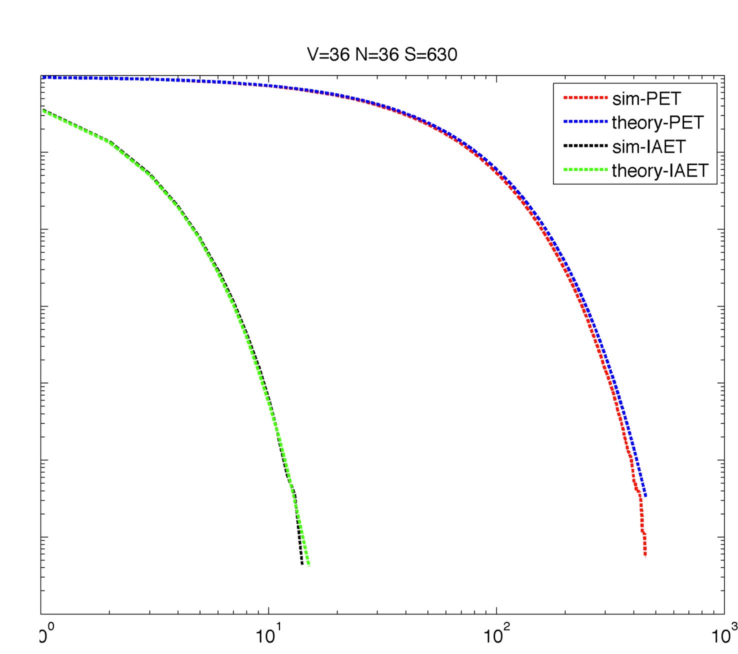


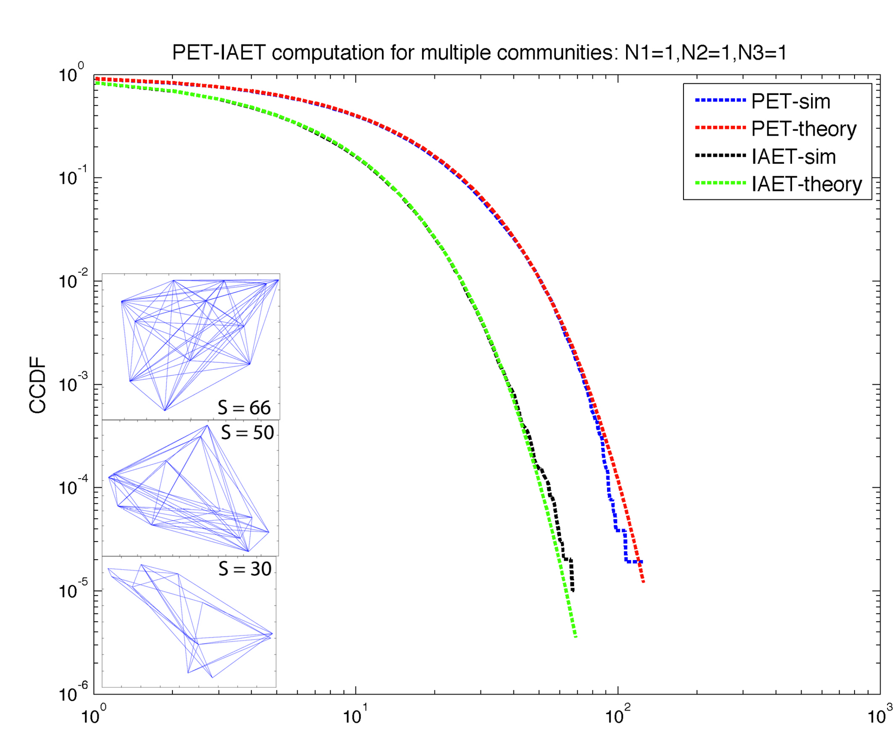
Simulation results



Extension to multi-communities case







- PET computation: $O(T|V|^2)$
- IAET computation: $O(T|V|^2)$
- Community case: $O(TN^2|V|^2)$

Conclusions

Recursive polynomial-time computation yielding of Pairwise inter-encounter time and approximate computation of inter-any encounter time.

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