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Game-Theoretic Study of Pricing Mechanisms in Power Networks

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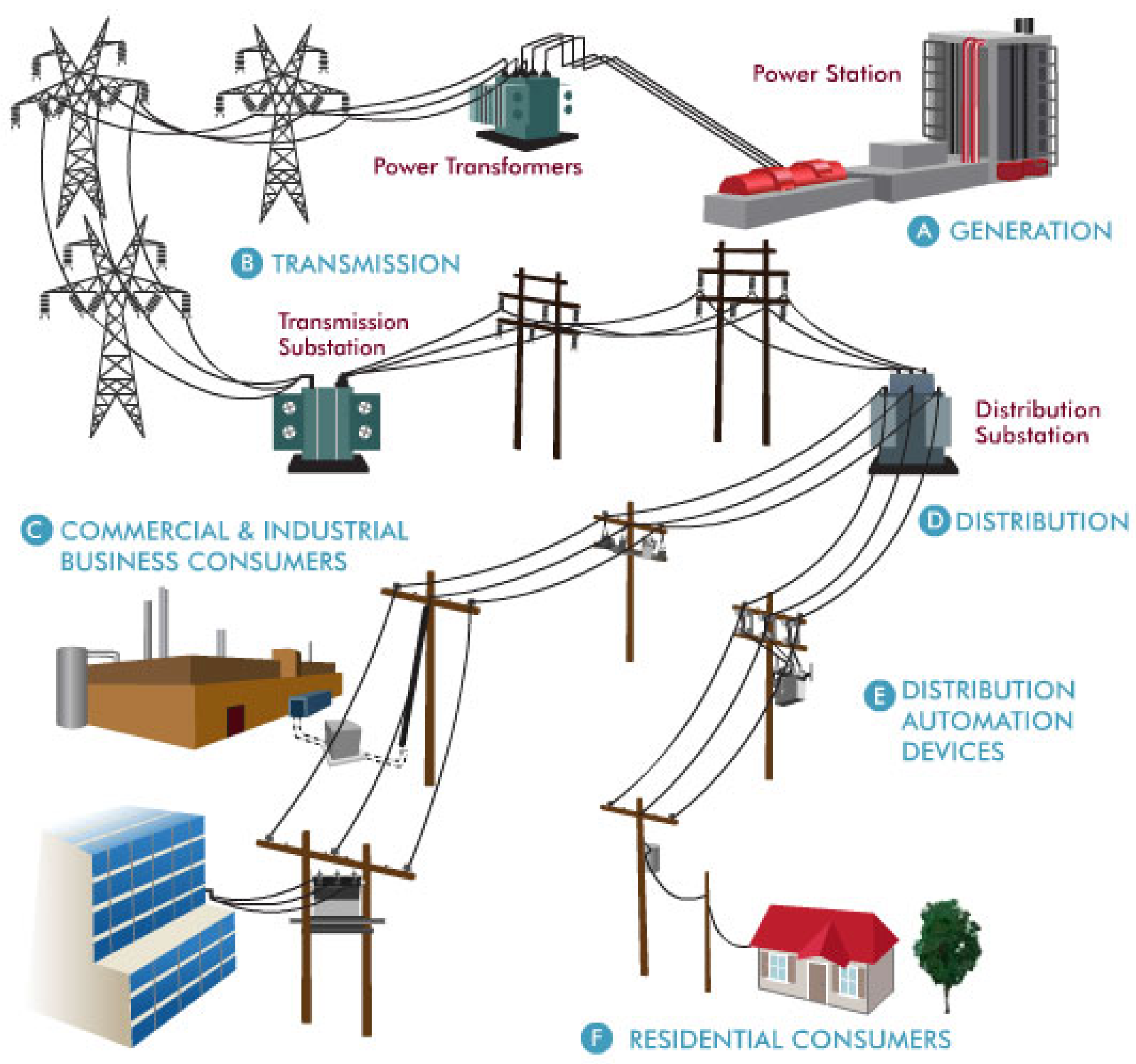
Motivation



- Nodal pricing is a widely adopted transmission pricing scheme
- Nodal price = locational marginal price
- Underlying assumption: competitive economic environment
- How come if market power exists?
- Game theory: a new perspective on pricing in power networks

Model

- N nodes: $i = 1, \ldots, N$
- Admittance of line (i,j): $Y_{ij} = Y_{ji}$
- Real power flow from i to j: $q_{ij} = -q_{ji}$
- Cost-benefit function: $c_i(q_i)$ (increasing and convex on $\mathbb{R}$)
- Approximate model: AC flow $\rightarrow$ DC flow
- Economic dispatch problem

$$\begin{aligned} \underset{q_i,\theta_i}{\text{minimize}} \quad & \sum_i c_i(q_i) \\ \text{subject to} \quad & q_i = \sum_j Y_{ij}(\theta_i - \theta_j), \quad \forall i \qquad [p_i] \\ & Y_{ij}(\theta_i - \theta_j) \le C_{ij}, \quad \forall (i,j) \qquad [\mu_{ij}] \end{aligned}$$

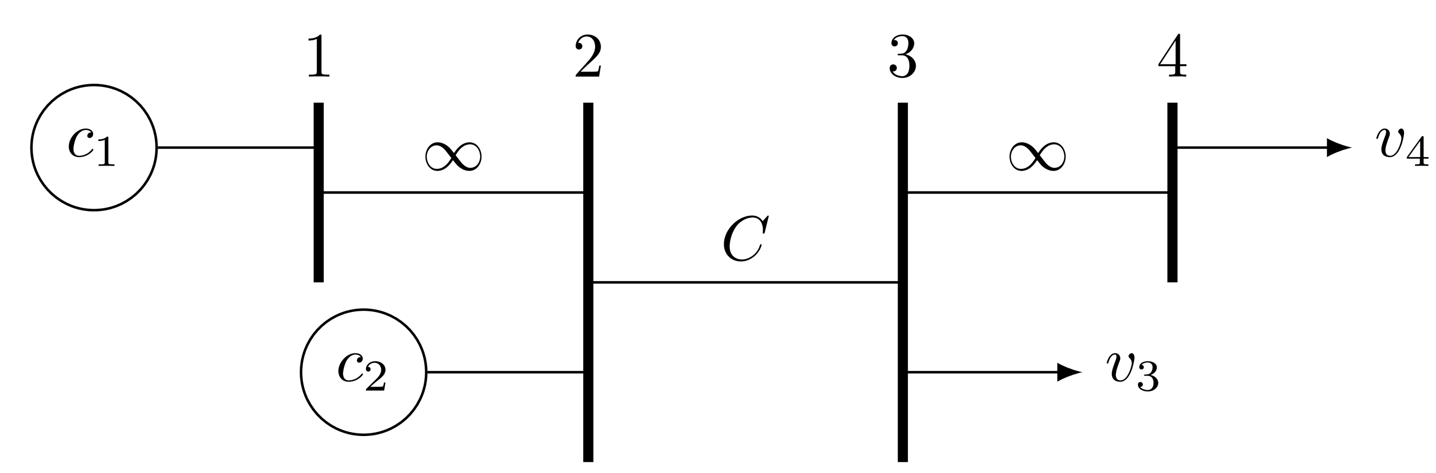
- A convex program: KKT conditions

$$\begin{aligned} c_i'(q_i) - p_i &= 0, \quad \forall i \\ \sum_j Y_{ij}(p_i - p_j + \mu_{ij} - \mu_{ji}) &= 0, \quad \forall i \\ q_i - \sum_j Y_{ij}(\theta_i - \theta_j) &= 0, \quad \forall i \\ \mu_{ij}[Y_{ij}(\theta_i - \theta_j) - C_{ij}] &= 0, \quad \forall (i,j) \\ Y_{ij}(\theta_i - \theta_j) - C_{ij} &\le 0, \quad \forall (i,j) \end{aligned}$$

- Nodal price = p_i (shadow price)

From General Equilibrium to Nash

- Real world: oligopoly or monopoly may exist
- Generators/consumers have no incentive to reveal their private information (marginal cost/benefit) truthfully
- Convert the original model to a game
  - Bid space: piecewise linear function
  - Transformation of the dispatch and pricing scheme
- Focus: Nash equilibrium
- Main result: nodal pricing is subject to manipulation
- Counterexample
  - Assumption: line (2,3) is binding in the efficient dispatch
  - Claim: Nash equilibrium does not even exist!



- Relationship to the supply function equilibrium?

VCG-Type Mechanism

- Standard VCG mechanism applies
- However, the type space cannot be parameterized
- We seek a Nash-implementation mechanism
- Two-dimensional bid $b_i = (\beta_i, d_i)$
  β_i: bid price
  d_i: maximum quantity
- Generalized mechanism
  - $\tilde{x} = (\tilde{x}_1, \ldots, \tilde{x}_N)$ as a solution of the following

$$\begin{aligned} \underset{x_i,y_i}{\text{maximize}} \quad & \sum_i \beta_i x_i \\ \text{subject to} \quad & g_k(x,y) \le 0, \quad k = 1, \ldots, m \\ & h_l(x,y) = 0, \quad l = 1, \ldots, p \\ & x_i \le d_i, \quad i = 1, \ldots, n \end{aligned}$$

  - $\tilde{x}^{-i}$ as a solution with $d_i = 0$ (when i is not present)
  - Payment made by player i (externality imposed on others)

$$w_i = \sum_{j \ne i} \beta_j \tilde{x}_j^{-i} - \sum_{j \ne i} \beta_j \tilde{x}_j$$

  - Player i's payoff

$$u_i = v_i(\tilde{x}_i) - w_i$$

- (Theorem) There exists an efficient Nash equilibrium in the proposed mechanism
- Apply the generalized mechanism to the power network
- VCG pricing works, while nodal pricing does not

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