

# Stochastic Resource Auctions in Smart-Grid Networks

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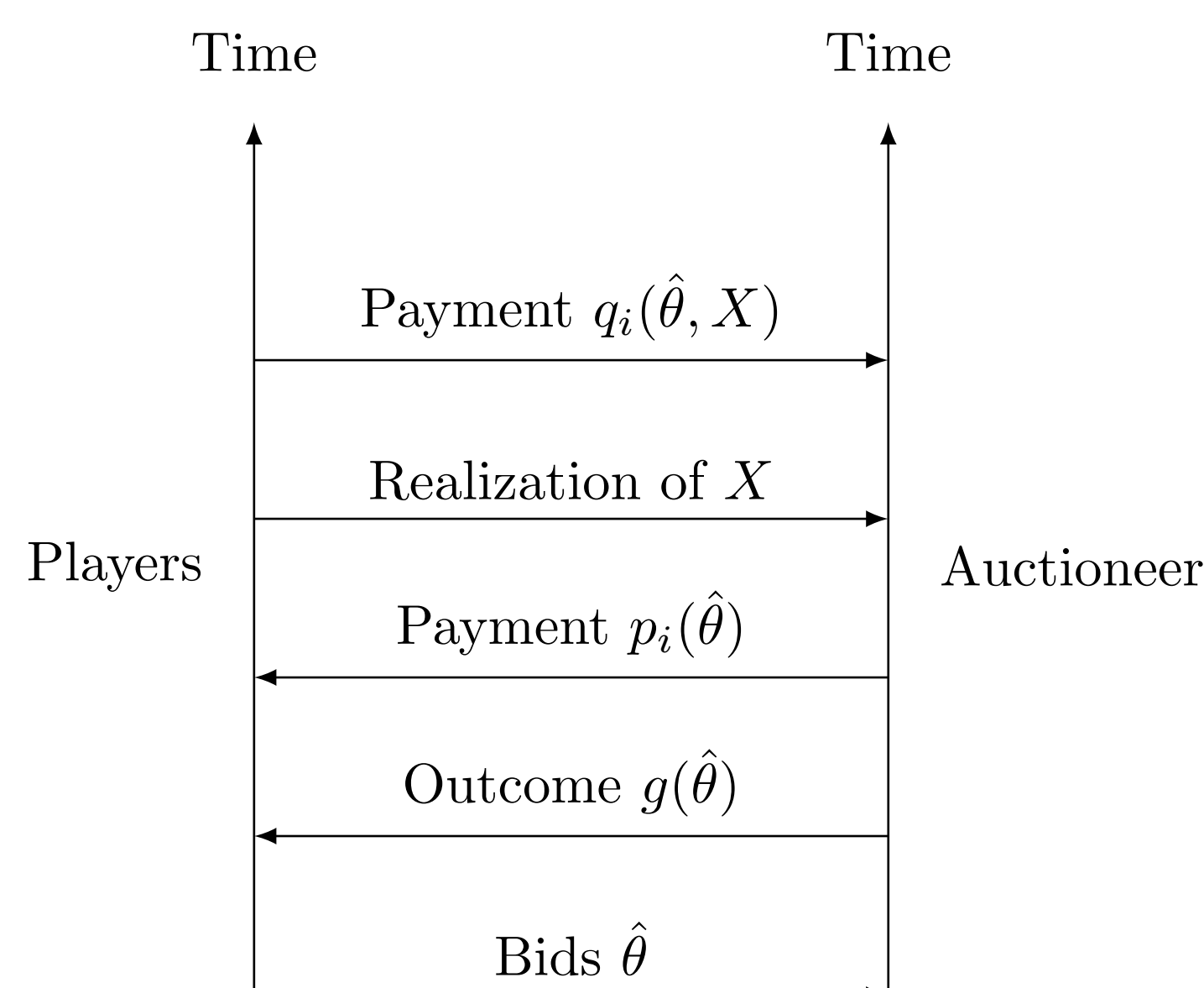
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## Motivation

- ▶ California requires one-third of the state's electricity to come from renewable energy by 2020
- ▶ Challenges of renewable energy: **variability** and **unpredictability**
- ▶ How the **electric power economy** operates
- ▶ Auctions for **stochastic** resources have not been studied
- ▶ Goal: elicit the generation **distributions** so as to achieve efficiency

## Model

- ▶ Renewable energy generators:  $i = 1, \dots, N$
- ▶ Tomorrow's generation:  $X_i$  with CDF  $F_i(\cdot) \equiv \theta_i$  (type)
- ▶ Stochastic resource auction



- ▶ Goal: contract with the one with the highest expected generation  

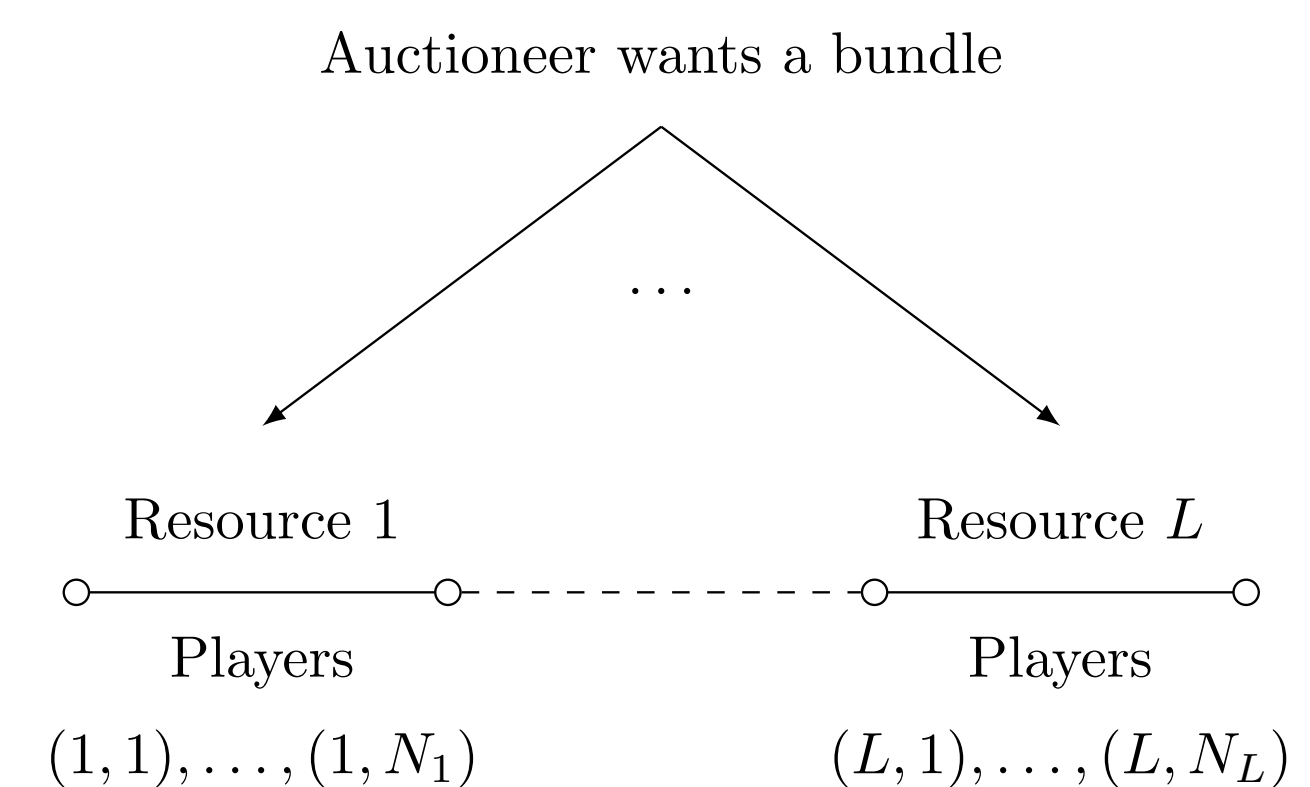
$$i' \in \arg \max_i \mathbb{E}[X_i]$$

## Two Basic Mechanisms

- ▶ Same rule to determine the winner
  - ▷ winner  $i' \in \arg \max_i \int x d\hat{F}_i(x)$
  - ▷ marginal loser  $i'' \in \arg \max_{i \neq i'} \int x d\hat{F}_i(x)$
- ▶ Stochastic VCG mechanism
  - ▷ ex ante payment  $-p_{i'} = \int x d\hat{F}_{i''}(x)$
  - ▷ ex post payment  $-q_{i'} = X_{i'}$
  - ▷ winner's payoff  $U_{i'} = X_{i'} - \int x d\hat{F}_{i''}(x)$
- ▶ Stochastic shortfall penalty mechanism
  - ▷ ex ante payment  $p_{i'} = 1$
  - ▷ ex post payment  $q_{i'} = \lambda(1 - X_{i'})$
  - ▷ penalty price  $\lambda = 1/[1 - \int x d\hat{F}_{i''}(x)]$
  - ▷ winner's payoff  $U_{i'} = 1 - \lambda(1 - X_{i'})$
- ▶ Both are **incentive compatible**
- ▶ **Quasi-duality** between the mechanisms
- ▶ Revenue comparison:  $SVCG \geq SSP$

## Generalizations

- ▶ General objective function:  $i' \in \arg \max_i \mathbb{E}[h(X_i)]$
- ▶ Bundled auction



- ▶ Multiple winners

## Generation Assignment Auction

- ▶ Generators assume risk and compete for assignment  $y$
- ▶ Generator's cost function

$$c_i(y_i) := \mathbb{E}[\lambda(y_i - X_i)^+]$$

- ▶ Aggregator's cost function

$$c(y) := \mathbb{E}[\lambda(Z - y)^+]$$

- ▶ Social welfare optimization

$$\begin{aligned} & \text{minimize}_{y_1, \dots, y_N, y} \sum_i c_i(y_i) + c(y) \\ & \text{subject to } y = \sum_i y_i, \quad 0 \leq y_i \leq 1, \quad \forall i \end{aligned}$$

- ▶ VCG mechanism for **parametric**  $c_i(\cdot)$

- ▷ allocation rule

$$\begin{aligned} & \text{minimize}_{y_1, \dots, y_N, y} \sum_i \tilde{c}_i(y_i) + c(y) \\ & \text{subject to } y = \sum_i y_i, \quad 0 \leq y_i \leq 1, \quad \forall i \end{aligned}$$

- ▷ payment scheme

$$w_i = \sum_{j \neq i} [\tilde{c}_j(y_j^{-i}) - \tilde{c}_j(y_j)] + c(y^{-i}) - c(y)$$

- ▶ i-VCG mechanism for **non-parametric**  $c_i(\cdot)$

- ▷  $b_i = (\beta_i, d_i)$ , ask price and maximum quantity offered

- ▷ allocation rule

$$\begin{aligned} & \text{minimize}_{y_1, \dots, y_N, y} \sum_i \beta_i y_i + c(y) \\ & \text{subject to } y = \sum_i y_i, \quad 0 \leq y_i \leq d_i, \quad \forall i \end{aligned}$$

- ▷ payment scheme

$$w_i = \sum_{j \neq i} \beta_j (y_j^{-i} - y_j) + c(y^{-i}) - c(y)$$