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Pricing Mechanisms in the Wholesale Electricity Market

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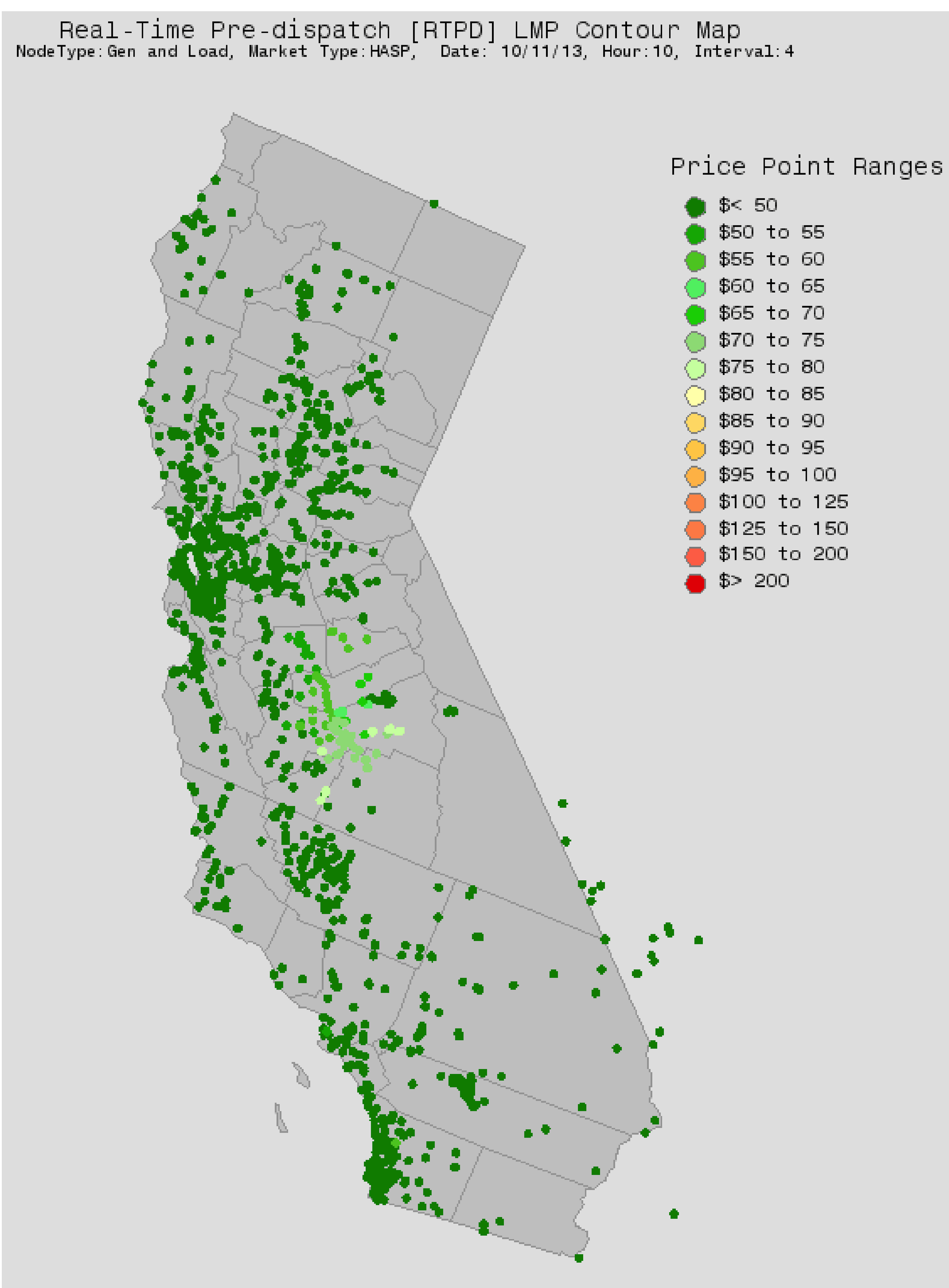
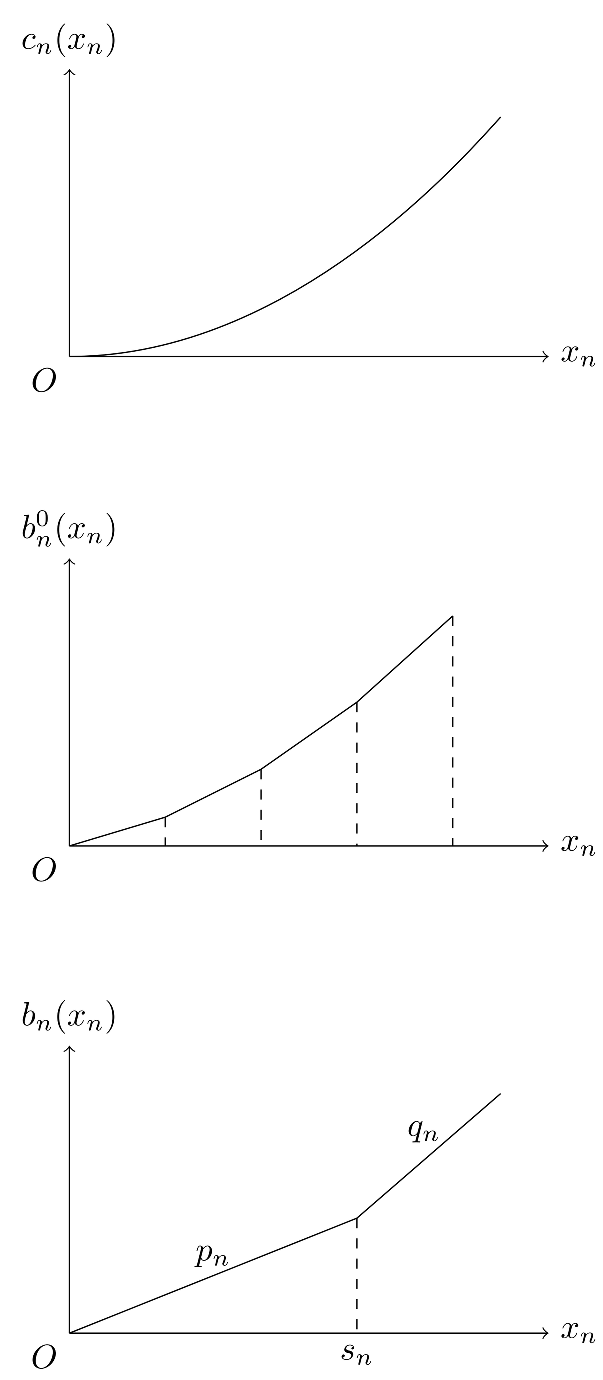
Motivation





- Locational marginal pricing is widely employed
- The underlying assumption is a competitive environment
- But the truth is that LMP is subject to market manipulation
- We use game theory to investigate the pros and cons of LMP
- We also propose the power network second price mechanism

Model

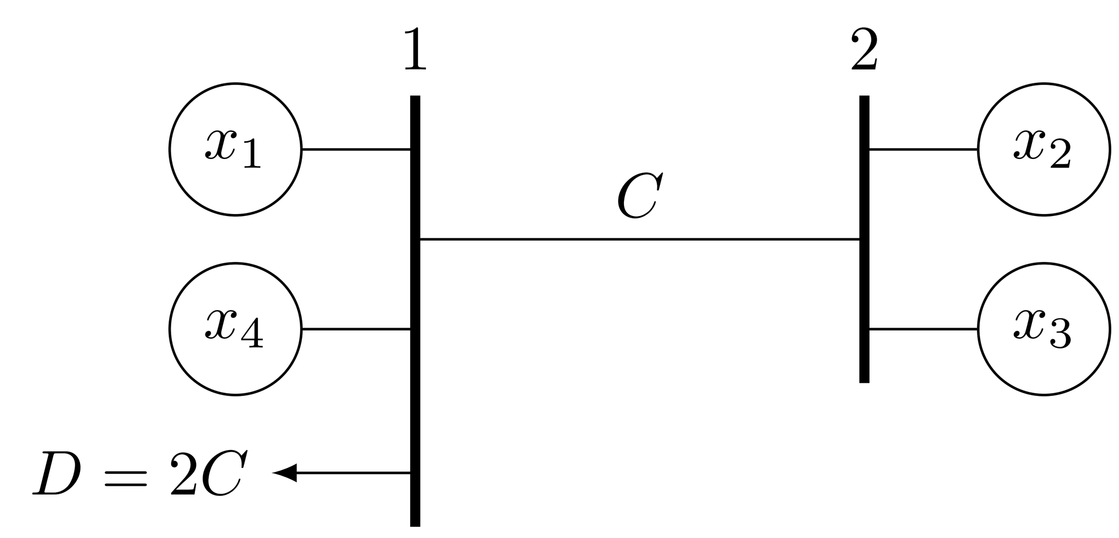
- I nodes and N generators, with N_i the generator set at node i
- Y_{ij} and C_{ij}: admittance and capacity limit of line i-j
- θ_i and D_i: phase angle and inelastic demand at node i
- $c_n(x_n)$: cost of generator n as a function of its generation x_n
- Economic dispatch problem

$$\begin{aligned}
\min_{x_n,\theta_i} \quad & \sum_n c_n(x_n) & \\
\text{s.t.} \quad & \sum_{n\in N_i} x_n - D_i = \sum_j Y_{ij}(\theta_i-\theta_j),\ \forall i & [\pi_i] \\
& Y_{ij}(\theta_i-\theta_j) \le C_{ij},\ \forall (i,j) & [\mu_{ij}] \\
& x_n \ge 0,\ \forall n &
\end{aligned}$$

- π_i: LMP at node i
- Payoff of generator $n \in N_i$: $u_n = \pi_i x_n - c_n(x_n)$
- Economic dispatch game
  - Generators may not reveal their cost functions truthfully
  - bid: reported cost function
  - $b_n^0(x_n)$: multi-segment bid in practice
  - $b_n(x_n)$: two-segment bid in our model
  - Replace the objective function by $\sum_n b_n(x_n)$

Main Results

- LMP does not always work
  - A Nash equilibrium may not exist
  - Even when a Nash equilibrium exists, the price of anarchy may be arbitrarily large



  Suppose $c_1(x) = x$, $c_2(x) = c_3(x) = kx$, $c_4(x) = 2kx$. The economic dispatch is $x^* = (2C, 0, 0, 0)$ with social cost $c_1(2C) = 2C$. One Nash equilibrium is $b_1(x) = b_4(x) = 2kx$, $b_2(x) = b_3(x) = kx$. The resulting dispatch is $x = (C, C, 0, 0)$ with social cost $c_1(C) + c_2(C) = C + kC$. The PoA is bounded below by $(C + kC)/2C = (k+1)/2 \to \infty$
- LMP works well in most cases
  - Under either of the following two conditions, not only a Nash equilibrium but also an efficient one exists
  - Congestion-free condition: no line flow constraint is binding in the economic dispatch problem ($\mu_{ij} \equiv 0$)
  - Monopoly-free condition: there are at least two generators at each node ($|N_i| \ge 2$ for all i)
- Our findings coincide with the policy proposed in *The California Electricity Crisis*: ensure competition in wholesale markets

PNSP Mechanism

- The same bid format and dispatch rule as LMP
- The payment rule is different
  - $(x_1^{-n_0}, \dots, x_N^{-n_0})$: dispatch when generator n_0 is excluded
  - Payment made to generator n_0 (positive externality):

$$w_{n_0} = \sum_{n\neq n_0} b_n(x_n^{-n_0}) - \sum_{n\neq n_0} b_n(x_n),$$

  - Payoff of generator n_0: $u_{n_0} = w_{n_0} - c_{n_0}(x_{n_0})$
- The PNSP mechanism always induces an efficient Nash equilibrium
  - Consider the bid profile: $p_n = c_n'(x_n^*)$, $s_n = x_n^*$, $q_n > p_n$
  - It induces the economic dispatch x^*
  - It can be shown to be a Nash equilibrium (using convexity)
- Comparison with LMP
  - PNSP specifies the total payment to each generator, while generators at the same node get the same unit price in LMP
  - Both may have undesirable Nash equilibria so that new designs of pricing mechanisms are needed

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