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Kalman-like state tracking and control in POMDPs with applications to body sensing networks

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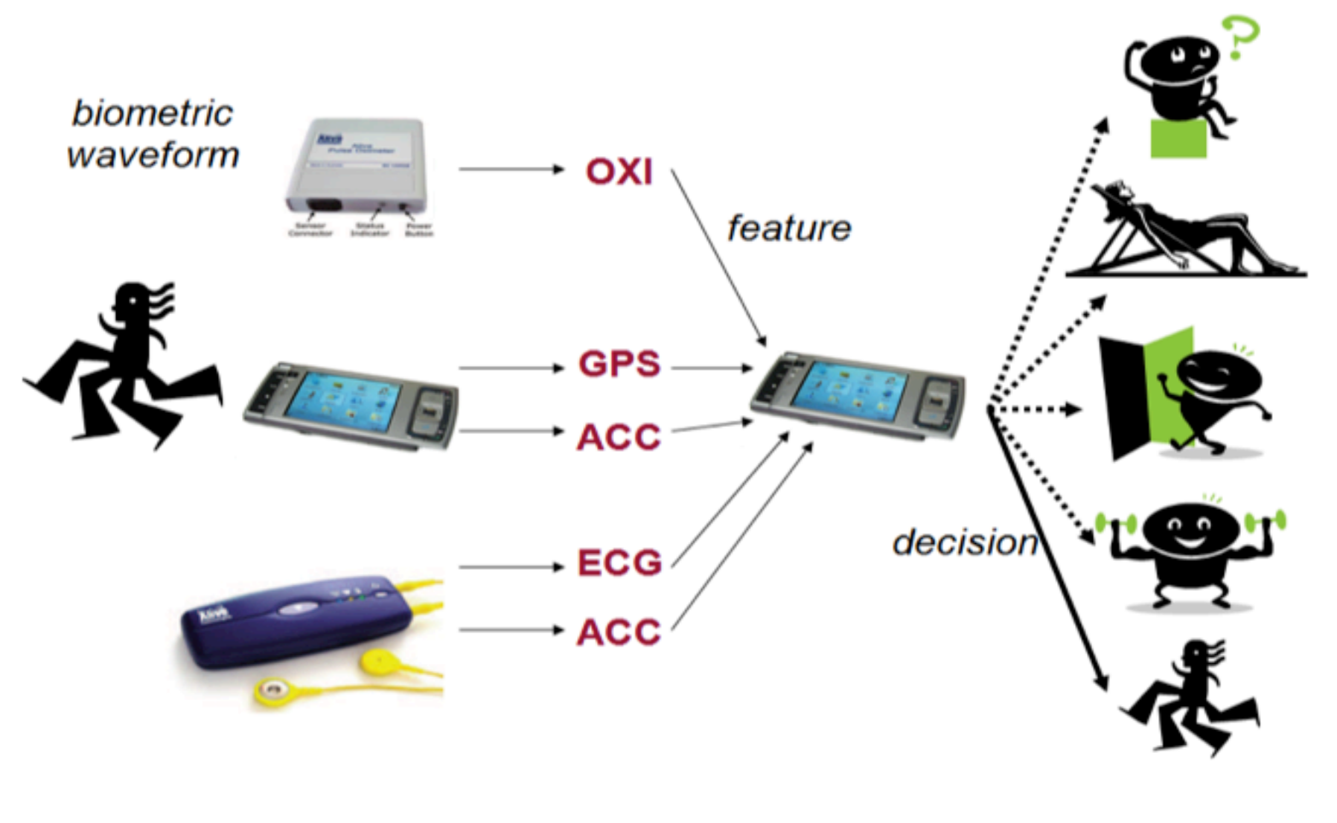
Motivation



Body Sensing Networks

Intelligent Home

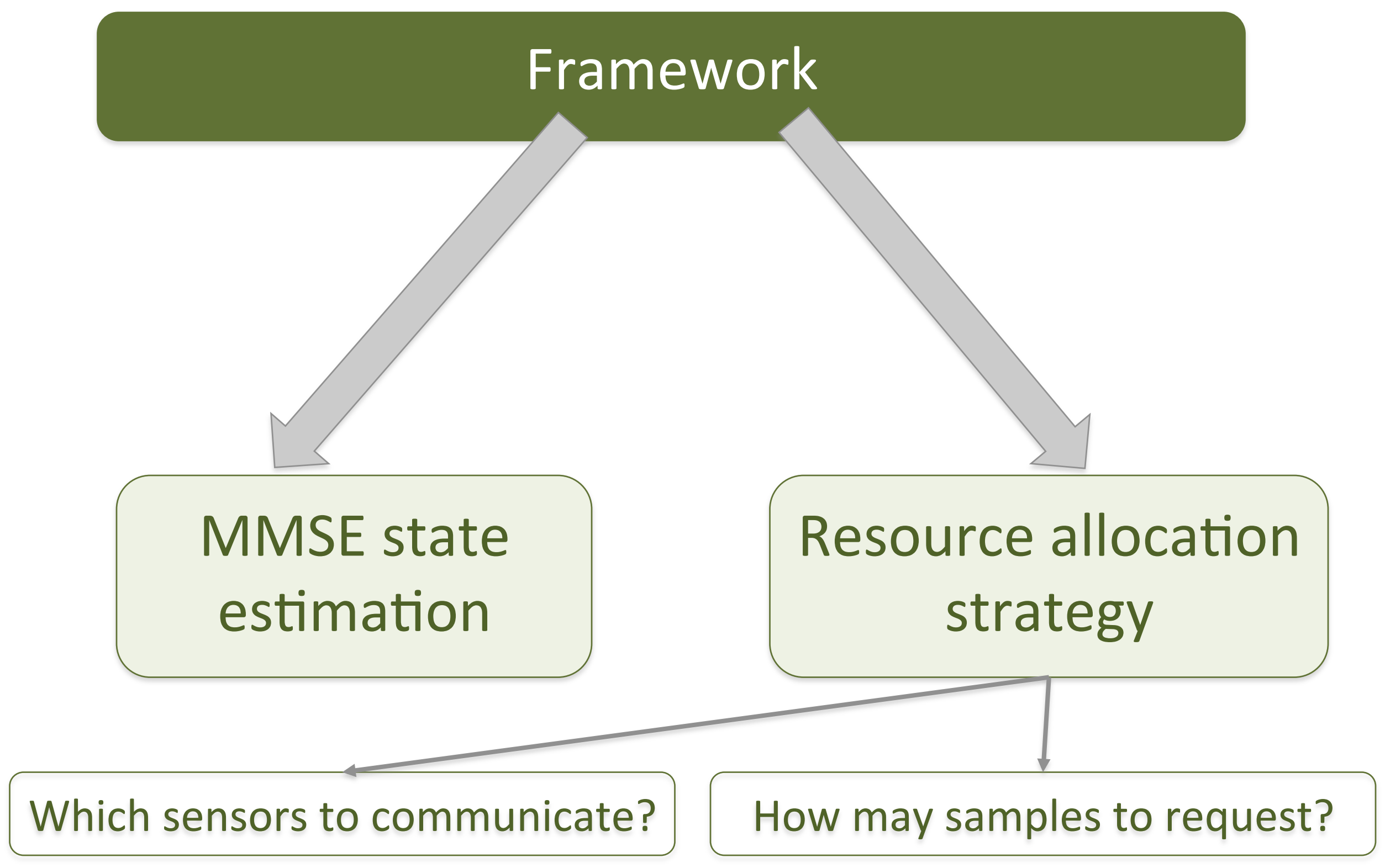
Wildlife Conservation

State estimation from noisy observations

Markov chain state evolution

Heterogeneous sensors

Goal



Framework

MMSE state estimation

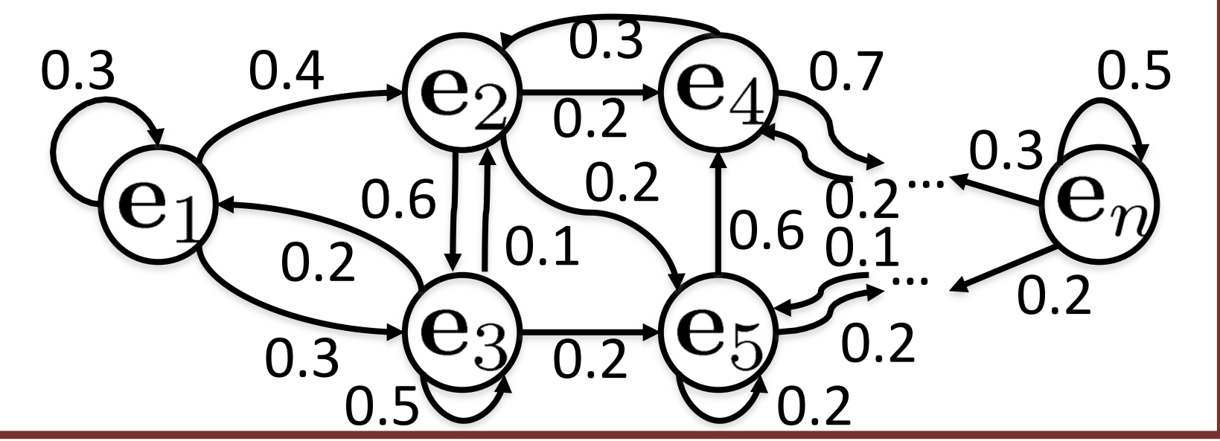
Resource allocation strategy

Which sensors to communicate?

How may samples to request?

Proposed System

Markov chain system state



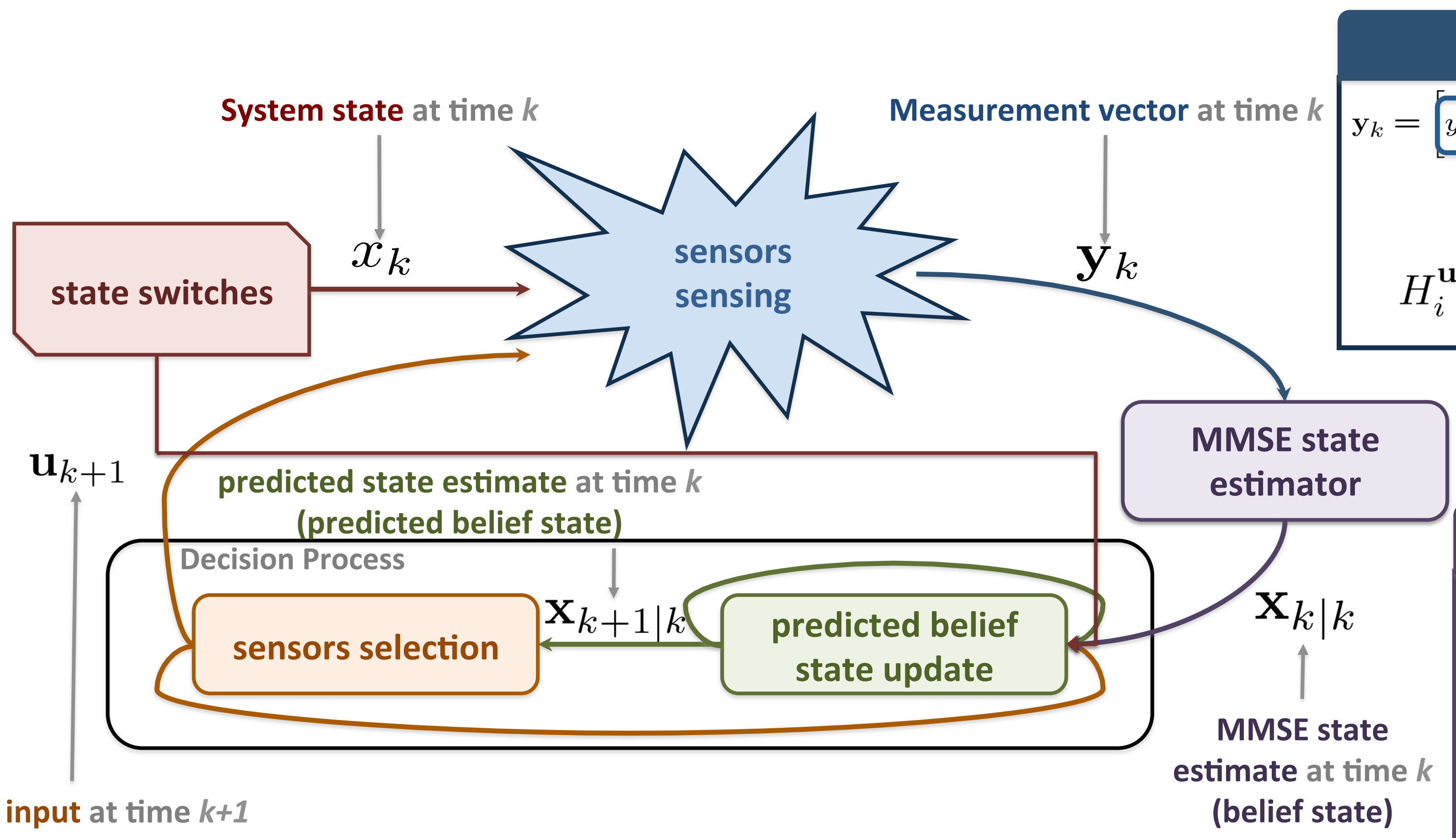
Control Input

sensors in the network

K – tuple $[N_1^{\mathbf{u}_k}, N_2^{\mathbf{u}_k}, \ldots, N_l^{\mathbf{u}_k}, \ldots, N_K^{\mathbf{u}_k}]^T$

Accelerometer | GPS sensor | Gyroscope | Camera

constraint: $\sum_{l=1}^{K} N_l^{\mathbf{u}_k} \leqslant N$



Signal Model

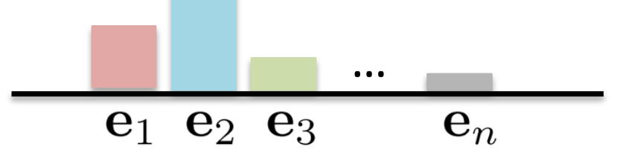
$$\mathbf{y}_k = [\underbrace{y_k^1, \ldots\ldots}_{N_1}\underbrace{\ldots\ldots}_{N_2}\underbrace{\ldots\ldots\ldots\ldots}_{N_3}\ldots\ldots\underbrace{\ldots\ldots\ldots\ldots\ldots\ldots}_{N_{K-1}}\underbrace{\ldots\ldots, y_k^N}_{N_K}]$$

Multivariate Gaussian model:

$$H_i^{\mathbf{u}_{k-1}} : \mathbf{y}_k | \mathbf{e}_i, \mathbf{u}_{k-1} \backsim \mathcal{N}(\mathbf{m}_i^{\mathbf{u}_{k-1}}, \mathbf{Q}_i^{\mathbf{u}_{k-1}}), \; \forall \mathbf{e}_i \in \mathcal{X}$$

MMSE state estimator

- **Conditional pmf over system states**



- KALMAN-LIKE **recursive structure**

Control Policy Design

Optimization Problem

Trace of filtering error covariance matrix

Cost function

$$J_\gamma = \underset{\mathbf{y}_0, \mathbf{y}_1, \ldots, \mathbf{y}_L}{\mathbb{E}} \left\{ \sum_{k=1}^{L} \overbrace{\operatorname{tr}\left(\mathbf{\Sigma}_{k|k}(\mathbf{y}_k, \mathbf{u}_{k-1})\right)} \right\}$$

- **partially observable stochastic control problem:**

$$\min_{\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{L-1}} J_\gamma$$

Optimal Solution

Nonlinear, statistics and *gain dependent*

- **Stochastic dynamic programming:**

Current cost

$$\underbrace{\overline{J}_k(\mathbf{p}_{k|k-1})}_{\text{Optimal cost-to-go}} = \min_{\mathbf{u}_{k-1} \in \mathcal{U}} \Bigg[\overbrace{\mathbf{p}_{k|k-1}^T \mathbf{h}(\mathbf{p}_{k|k-1}, \mathbf{u}_{k-1})} + \underbrace{\int \zeta(\mathbf{p}_{k|k-1}, \mathbf{y}, \mathbf{u}_{k-1}) \overline{J}_{k+1}(\mathbf{p}_{k+1|k}) d\mathbf{y}}_{\text{Expected future cost}} \Bigg]$$

MMSE Estimator

Theorem

(Proof by innovations theory)

The MMSE Markov chain system estimate at time step k is recursively defined as

$$\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1} + \mathbf{G}_k[\mathbf{y}_k - \mathbf{y}_{k|k-1}], \; k \geqslant 0$$

with

$$\mathbf{x}_{k|k-1} = \mathbf{P}\mathbf{x}_{k-1|k-1}, \quad \mathbf{x}_{0|-1} = \pi$$

transition probability matrix

initial distribution over states

conditional covariance matrix of prediction error

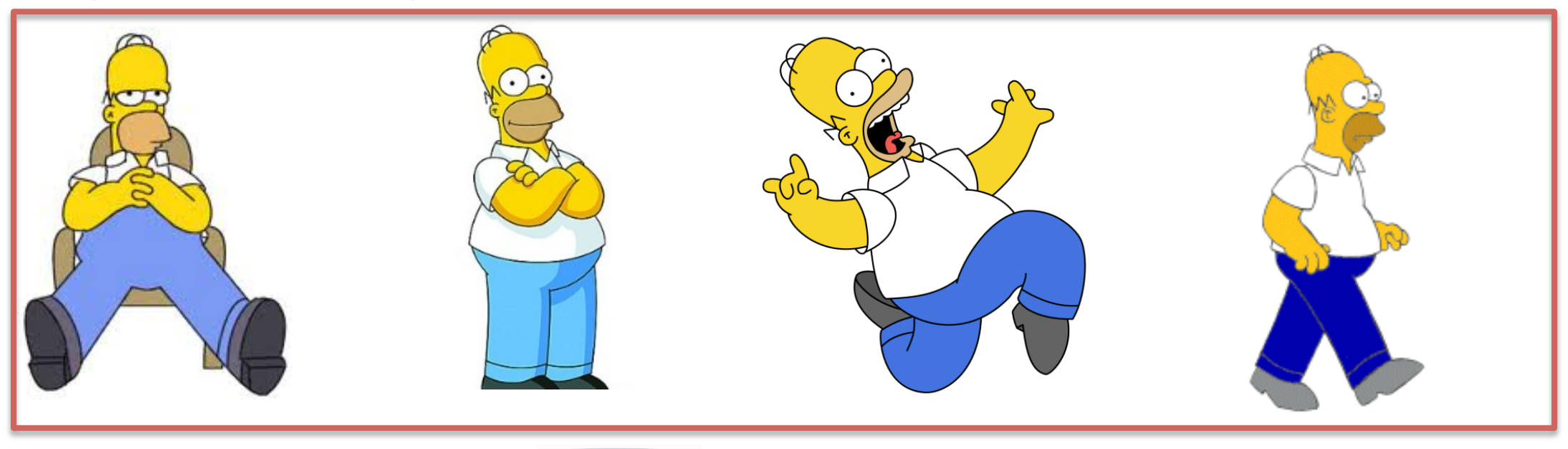
$$\mathbf{y}_{k|k-1} = \mathcal{M}(\mathbf{u}_{k-1})\mathbf{x}_{k|k-1}, \quad \mathcal{M}(\mathbf{u}_{k-1}) = [\mathbf{m}_1^{\mathbf{u}_{k-1}}, \ldots, \mathbf{m}_n^{\mathbf{u}_{k-1}}]$$

$$\mathbf{G}_k = \mathbf{\Sigma}_{k|k-1}\mathcal{M}^T(\mathbf{u}_{k-1})(\mathcal{M}(\mathbf{u}_{k-1})\mathbf{\Sigma}_{k|k-1}\mathcal{M}^T(\mathbf{u}_{k-1}) + \mathbf{R})^{-1}$$

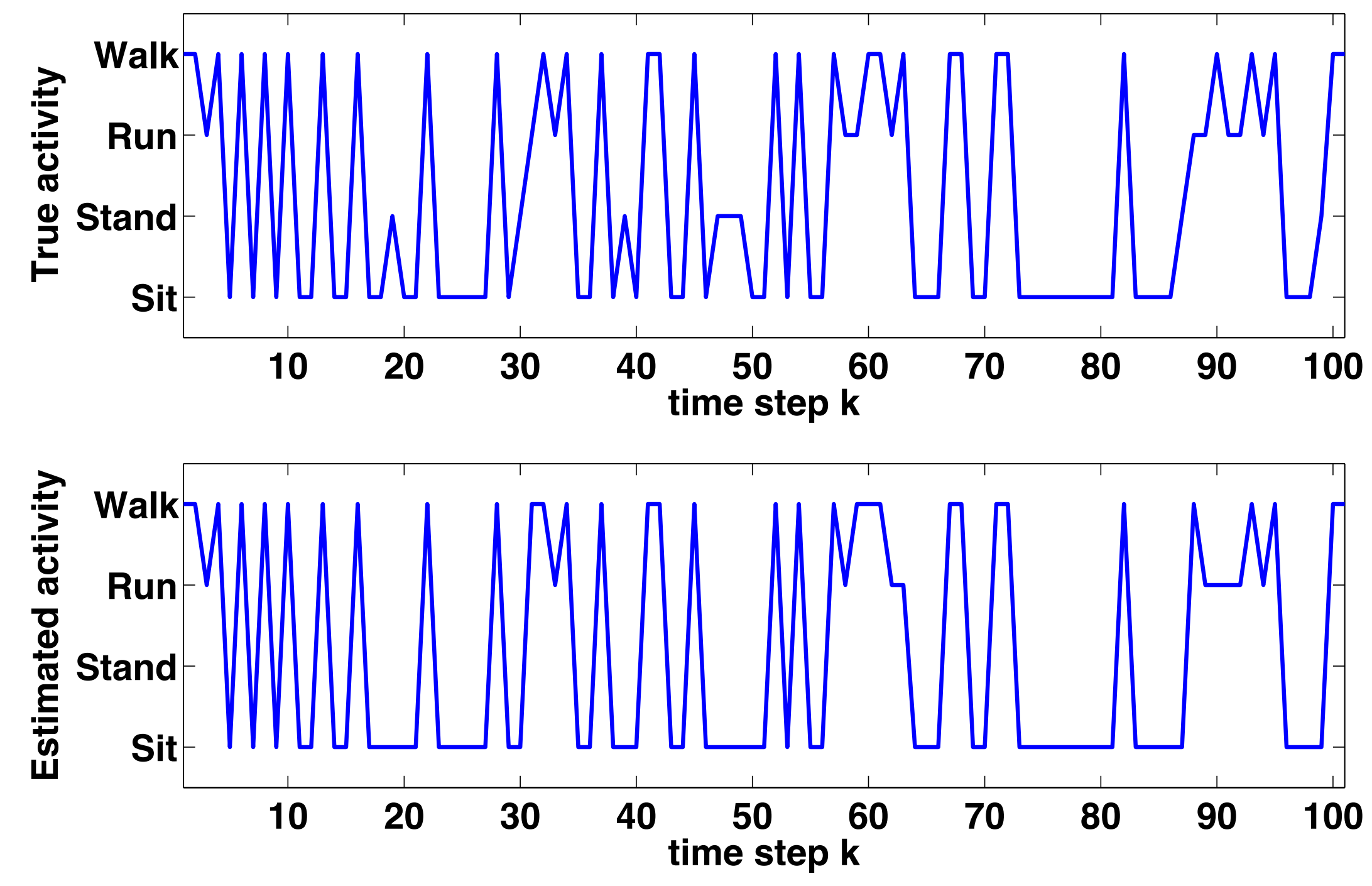
$$\mathbf{R} = \sum_{i=1}^{n} x_{k|k-1}^i \mathbf{Q}_i^{\mathbf{u}_{k-1}}$$

Application

Physical Activity Detection



Body sensing network =  + accelerometers + heart-rate monitor



Control Policy	Detection Accuracy
ACC 1 only	74%
ACC 2 only	77%
HRM only	40%
Optimal (adaptive)	87%

- Very good tracking performance
- Adaptive resource allocation is beneficial

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This research has been funded in part by the following grants and organizations: ONR N00014-09-1-0700, NSF CNS-0832186, NSF CCF-0917343, AFOSR FA9550-12-1-0215, the National Center on Minority Health and Health Disparities (NCMHD) (supplement to P60 MD002254), Nokia and Qualcomm.

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