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Joint Spatial Division and Multiplexing

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Joint Spatial Division and Multiplexing (JSDM) is an approach to multiuser MIMO downlink that exploits the structure of channel correlation in order to allow for a large number of antennas at the BS while requiring reduced-dimensional Channel State Information at the Transmitter (CSIT).

Channel Model

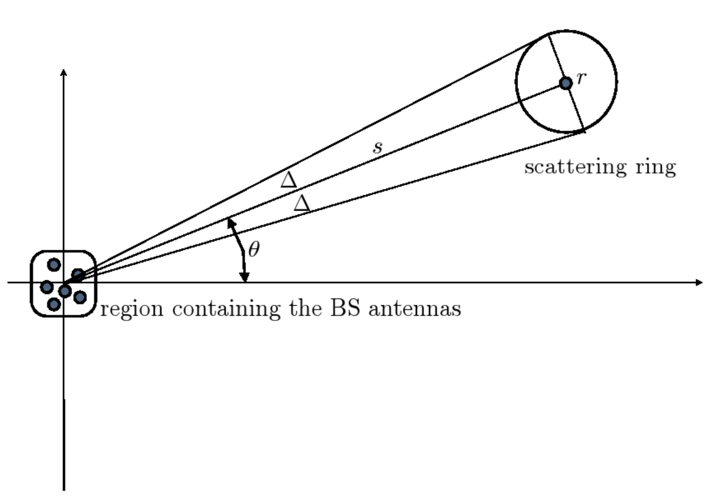

Figure: A UT at AoA θ with a scattering ring of radius r generating a two-sided AS Δ with respect to the BS

- BS has M antennas and serves K users.

$$\boldsymbol{y} = \boldsymbol{H}^{\mathsf{H}}\boldsymbol{x} + \boldsymbol{z}, \qquad (1)$$

- $\boldsymbol{H} = [\boldsymbol{h}_1 \dots \boldsymbol{h}_K]$ is the concatenation of user channels, $\boldsymbol{x} = \boldsymbol{V}\boldsymbol{d}$ is the transmit signal vector, with $\boldsymbol{V}$ the precoder and $\boldsymbol{d}$ the vector of data symbols.
- Channel of user k given by $\boldsymbol{h}_k \sim \mathcal{CN}(0, \boldsymbol{R}_k)$, $\boldsymbol{R}_k = \boldsymbol{U}_k\boldsymbol{\Lambda}_k\boldsymbol{U}_k^{\mathsf{H}}$ being the channel covariance of rank r_k.
- Equivalently, $\boldsymbol{h}_k = \boldsymbol{U}_k\boldsymbol{\Lambda}_k^{\frac{1}{2}}\boldsymbol{w}_k$, where $\boldsymbol{w}_k \in \mathbb{C}^{r_k \times 1} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$.
- A UT with AoA θ and angular spread Δ has

$$[\boldsymbol{R}_k]_{m,p} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{j\boldsymbol{k}^{\mathsf{T}}(\alpha+\theta)(\boldsymbol{u}_m - \boldsymbol{u}_p)} d\alpha$$

- $\boldsymbol{k}(\alpha) = -\frac{2\pi}{\lambda}(\cos(\alpha), \sin(\alpha))^{\mathsf{T}}$ is the wave vector for a planar wave impinging with AoA α, λ the carrier wavelength, and $\boldsymbol{u}_m, \boldsymbol{u}_p \in \mathbb{R}^2$ are vectors indicating the position of BS antennas.

JSDM Basics

- K UTs are partitioned into G different groups based on the similarity of their covariance matrices, each group containing $s = \frac{K}{G}$ users.
- We assume all users in a group g have the same covariance matrix $\boldsymbol{R}_g$ with rank r_g.
- JSDM uses a two stage precoder $\boldsymbol{V} = \boldsymbol{B}\boldsymbol{P}$.
- $\boldsymbol{B} \in \mathbb{C}^{M \times b}$, the pre beamformer, is independent of the instantaneous channel realization and $\boldsymbol{P} \in \mathbb{C}^{b \times K}$ is a precoding matrix depending on the effective channel $\mathsf{H} = \boldsymbol{B}^{\mathsf{H}}\boldsymbol{H}$ of reduced dimensions. In our work, we use two choices for $\boldsymbol{P}$, i.e., regularized-ZFBF and ZFBF.
- $\boldsymbol{B}$ can be designed jointly for all groups (joint group processing (JGP)) or separately for each group g (per group processing (PGP)).
- In PGP, the received signal by users in group g is

$$\boldsymbol{y}_g = \mathsf{H}_g^{\mathsf{H}}\boldsymbol{P}_g\boldsymbol{d}_g + \sum_{g' \neq g} \boldsymbol{H}_g^{\mathsf{H}}\boldsymbol{B}_{g'}\boldsymbol{P}_{g'}\boldsymbol{d}_{g'} + \boldsymbol{z}_g$$

where $\mathsf{H}_g = \boldsymbol{B}_g^{\mathsf{H}}\boldsymbol{H}_g$ and $\boldsymbol{B}_g \in \mathbb{C}^{M \times b_g}$ with $\boldsymbol{B} = [\boldsymbol{B}_1\boldsymbol{B}_2 \dots \boldsymbol{B}_G]$.
- In JGP, We choose $\boldsymbol{B}_g = \boldsymbol{U}_g$. This is called *eigen beamforming*.
- In PGP, the pre-beamforming process creates *virtual sectors*, similar to spatial sectorization in current cellular standards.
- Channel covariance $\boldsymbol{R}_g$ changes slowly compared to the instantaneous channel matrix. So, $\boldsymbol{R}_g$ can be estimated based on a suitable subspace estimation and tracking algorithm, exploiting the downlink training phase.

Achieving capacity with JSDM

Theorem 1: Let the channel covariances of the G groups are such that $\underline{\boldsymbol{U}} = [\boldsymbol{U}_1, \cdots, \boldsymbol{U}_G]$ is tall unitary (i.e., $\underline{\boldsymbol{U}}^{\mathsf{H}}\underline{\boldsymbol{U}} = \boldsymbol{I}$). For this scenario, JSDM achieves the same sum capacity of the corresponding MU-MIMO downlink channel (1) with full CSIT.

- Note that choosing $\boldsymbol{B} = [\boldsymbol{U}_1\boldsymbol{U}_2 \dots \boldsymbol{U}_G]$ achieves capacity, and gives a set of decoupled MU-MIMO downlink channels.
- It is beneficial to partition users into groups based on the similarity of their eigenspaces and then scheduling across groups satisfying the tall unitary condition.
- If this is not possible, block diagonalization (or approximate block diagonalization) is used to design $\boldsymbol{B}_g$ such that $\boldsymbol{H}_g^{\mathsf{H}}\boldsymbol{B}_{g'} \approx \boldsymbol{0}, \ \forall\, g' \neq g$.
- Approximate block diagonalization is important because a majority of the non-zero eigen values of $\boldsymbol{R}_g$ are very small, and hence, going for exact block diagonalization may degrade the achievable rates.

Performance Analysis

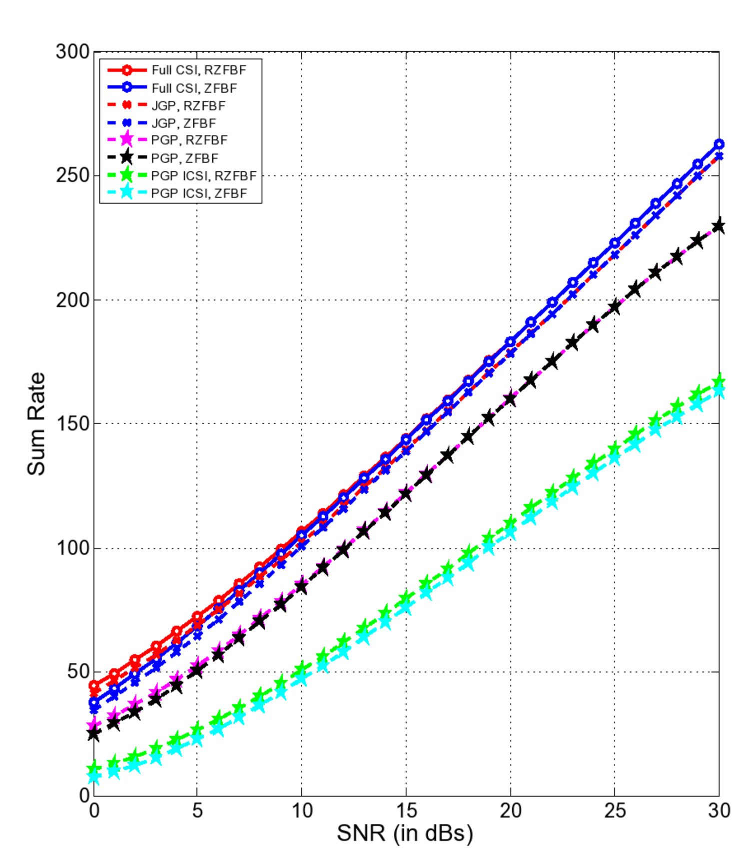

Figure: Comparison of sum spectral efficiency (bit/s/Hz) vs. SNR (dB) for JSDM.

- Plots are obtained by using an analytical tool based on random matrix theory, called *deterministic equivalents* avoiding lengthy Monte Carlo simulations.
- Uniform Circular Array with $M = 100$, $G = 6$, $K = 24$, with 4 users per group.
- JSDM with PGP performs well compared to full CSIT case.
- Imperfect CSIT (resulting from estimation of channels by users via downlink pilots and ideal feedback) reduces the achievable rates by 70 percent.

Uniform Linear Arrays

- For a uniformly linear array, the channel covariance $\boldsymbol{R}$ for a UT at AoA θ and angular spread Δ is

$$[\boldsymbol{R}_k]_{m,p} = \frac{1}{2\Delta}\int_{-\Delta+\theta}^{\Delta+\theta} e^{-j2\pi D(m-p)\sin(\alpha)} d\alpha$$

.
- For this special case, $\boldsymbol{R}$ is Toeplitz (with rank r) and can be approximated by a circulant matrix $\boldsymbol{C}$, and the approximation holds as follows.
- The set of eigenvalues $\{\lambda_m(\boldsymbol{R})\}$, $\{\lambda_m(\boldsymbol{C})\}$ are asymptotically *equally distributed*, i.e., for any continuous function $f(x)$ defined over $[\kappa_1, \kappa_2]$, we have

$$\lim_{M \to \infty} \frac{1}{M}\sum_{m=0}^{M-1} f(\lambda_m(\boldsymbol{R})) = \lim_{M \to \infty} \frac{1}{M}\sum_{m=0}^{M-1} f(\lambda_m(\boldsymbol{C}))$$

- The tall unitary matrix of the channel covariance eigenvectors, i.e., $\boldsymbol{U}$, can be approximated with a submatrix $\boldsymbol{F}_{\mathcal{S}}$ of the DFT matrix $\boldsymbol{F}$, formed by a selection of $\mathcal{S}$ columns of $\boldsymbol{F}$ in the following sense:

$$\lim_{M \to \infty} \frac{1}{M}\left\|\boldsymbol{U}\boldsymbol{U}^{\mathsf{H}} - \boldsymbol{F}_{\mathcal{S}}\boldsymbol{F}_{\mathcal{S}}^{\mathsf{H}}\right\|_F^2 = 0$$

Theorem 2: The asymptotic normalized rank $\rho = \lim_{M \longrightarrow \infty} \frac{r}{M}$ of the channel covariance matrix $\boldsymbol{R}$, with antenna separation λD, AoA θ and AS Δ, is given by $\rho = \min\{1, B(D, \theta, \Delta)\}$, where

$$B(D, \theta, \Delta) = |D\sin(-\Delta+\theta) - D\sin(\Delta+\theta)|\,.$$

A good approximation of the actual rank r for large but finite M is given by $r \approx \rho M$, where ρ is given as above.

Theorem 3: Groups g and g' with angle of arrival θ_g and $\theta_{g'}$ and common angular spread Δ have orthogonal eigenspaces if their AoA intervals $[\theta_g - \Delta, \theta_g + \Delta]$ and $[\theta_{g'} - \Delta, \theta_{g'} + \Delta]$ are disjoint.

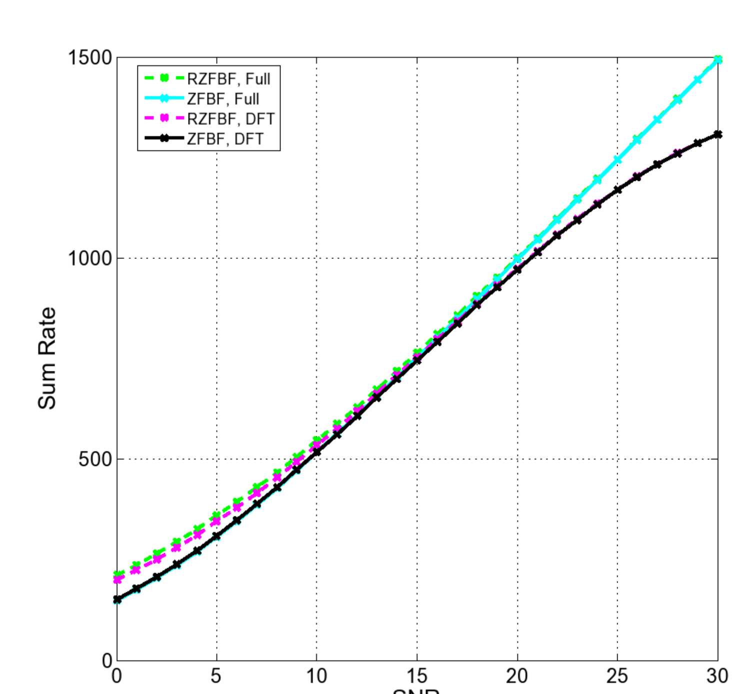

Figure: Sum spectral efficiency (bit/s/Hz) vs. SNR (dB) for JSDM for DFT pre-beamforming and PGP

DFT-prebeamforming for $G = 3$, $M = 400$, $\theta_1 = \frac{-\pi}{4}, \theta_2 = 0, \theta_3 = \frac{\pi}{4}$, $\Delta = 15$ deg. performs close to schemes with full CSIT

- When the BS has a large antenna array, an efficient way consists of selecting groups of users with almost identical AoA intervals, and scheduling G groups with non-overlapping AoAs.
- JSDM is attractive because only a coarse knowledge (AoA interval) is required, rather than an estimate of the sample covariance matrix.

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