Swarm Aggregation Algorithms for Multi-Robot Systems

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Outline

Introduction

Biological Swarming

In Nature I

“Swarm behavior, or swarming, is a collective behavior exhibited by animals of similar size which aggregate together, perhaps milling about the same spot or perhaps moving en masse or migrating in some direction.”

In Nature II

Examples of biological swarming are found in bird flocks, fish schools, insect swarms, bacteria swarms, quadruped herds

Several studies have been made to explain the social behavior which different groups of animals exhibit[1],[2].

A common understanding is that in nature there are attraction and repulsion forces between individuals that lead to swarming behavior

[1] K Warburton . and J. Lazarus. "Tendency-distance models of social cohesion in animal groups." In: *Journal of theoretical biology* 150.4 (1991), pp. 473–488.

[2] D. Grunbaum, A. Okubo, and S. A. Levin. "Modelling social animal aggregations". In: *Frontiers in theoretcial Biology: Lecture notes in biomathematics*. Springer-Verlag, 1994.

Introduction

Swarm Robotics

Swarm Robotics

From a robotic perspective, swarm behavior is the collective motion of a large number of self-propelled entities[3].

The key points of swarming robotics are:

- Focuses on a large number of simple autonomous agents,
- Achieving an aggregation through local simple interaction,
- An emergent global behavior arises from local interactions,
- Can provide high robustness and flexibility.

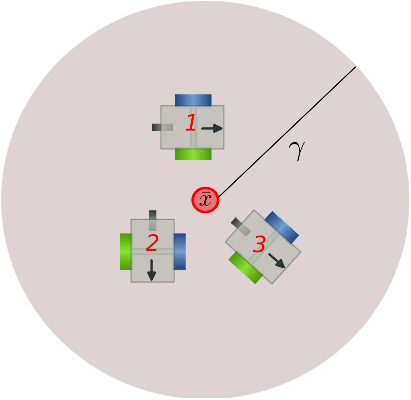
[3] O J O'Loan and M R Evans. "Alternating steady state in one-dimensional flocking". In: *Journal of Physics A: Mathematical and General* 32.8 (1999), p. L99.

Aggregation (in space)

Consider a team of n agents, the team is said to be showing an aggregative behavior if the following holds:

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| \leq 2\gamma, \quad \forall\, i, j \in 1, \ldots, n \tag{1}$$

with γ the aggregation radius.



Related Work

Several works can be found in the literature about swarm robotics:

- Veysel Gazi and Kevin M. Passino. "A class of attractions/repulsion functions for stable swarm aggregations". In: *International Journal of Control* 77.18 (2004)
- Wei Li. "Stability Analysis of Swarms With General Topology". In: *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics* 38.4 (2008), pp. 1084–1097
- D.V. Dimarogonas and K.J. Kyriakopoulos. "Connectedness Preserving Distributed Swarm Aggregation for Multiple Kinematic Robots". In: *IEEE Transactions on Robotics* 24.5 (2008), pp. 1213 –1223. ISSN: 1552-3098
- V. Gazi and K.M. Passino. *Swarm Stability and Optimization*. Springer Berlin Heidelberg, 2011. ISBN: 9783642180415

Contributions

The following major contributions have been made:

- The design of two swarm aggregation algorithms with input saturations and local interactions
- A theoretical analysis of the converge properties of the proposed interaction rules
- An experimental validation of the proposed control laws with a team of robotic platforms

Credits

This work has been done in collaboration with:

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Swarm Robotics

Reference Work

Modeling - I

Consider the following dynamics for each agent i:[4]

$$\dot{x}_i = \sum_{j=1, j\neq i}^{n} g(x_i - x_j), \quad \forall i = 1, \ldots, n \tag{2}$$

with the interaction function:

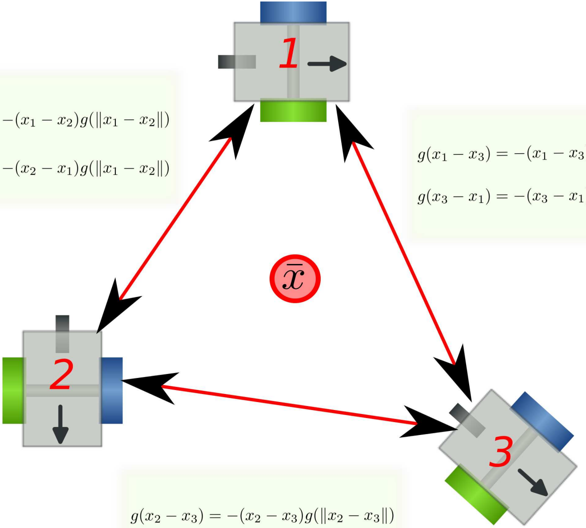
$$g(\cdot) : \mathbb{R}^d \to \mathbb{R}^d$$

Observations:

- The robot-to-robot interaction is described by a fully connected graph $\mathcal{G}(V, E)$.

[4] Veysel Gazi and Kevin M. Passino. "A class of attractions/repulsion functions for stable swarm aggregations". In: *International Journal of Control* 77.18 (2004).

Modeling - II



$$g(x_1 - x_2) = -(x_1 - x_2)g(\|x_1 - x_2\|)$$

$$g(x_2 - x_1) = -(x_2 - x_1)g(\|x_1 - x_2\|)$$

$$g(x_1 - x_3) = -(x_1 - x_3)g(\|x_1 - x_3\|)$$

$$g(x_3 - x_1) = -(x_3 - x_1)g(\|x_1 - x_3\|)$$

$$g(x_2 - x_3) = -(x_2 - x_3)g(\|x_2 - x_3\|)$$

$$g(x_3 - x_2) = -(x_3 - x_2)g(\|x_2 - x_3\|)$$

Interaction Function

The interaction function can be defined as follows:

$$g(y) = -y\left[g_a(\|y\|) - g_r(\|y\|)\right], \forall y \in \mathbb{R}^d. \tag{3}$$

where:

- The term $g_a(\|y\|)$ is the attraction component
- The term $g_r(\|y\|)$ is the the repulsion component.

Observations:

- The interaction function is odd, namely $g(y) = -g(-y)$.

Interaction Functions - Assumptions

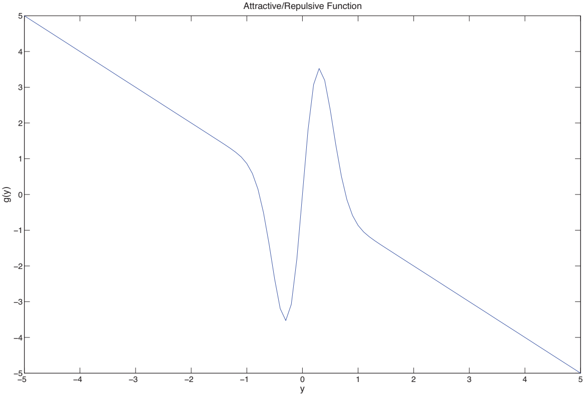
Assumption 1 (A1):There exists a unique distance δ at which $g_a(\delta) = g_r(\delta)$ and the following holds:

$$\begin{aligned} g_a(\|y\|) \geq g_r(\|y\|) \ \text{ for } \ \|y\| \geq \delta \\ g_r(\|y\|) > g_a(\|y\|) \ \text{ for } \ \|y\| < \delta. \end{aligned} \tag{4}$$

Observations:

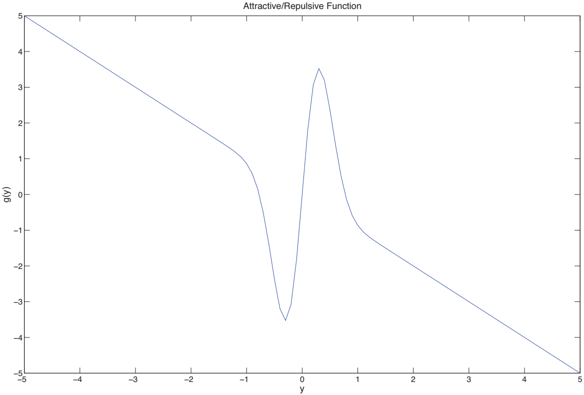
- The vector y defines the alignment,
- The term $y\, g_a(\|y\|)$ is the actual attraction,
- The term $y\, g_r(\|y\|)$ is the actual repulsion.

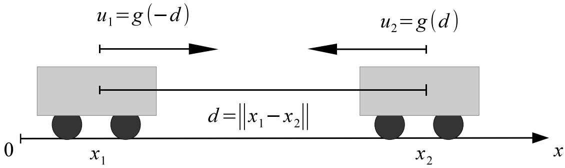
Interaction Function - An Example I



$$g(y) = -y\left[\underbrace{a}_{g_a(\|y\|)} - \underbrace{b\exp\left(-\frac{\|y\|^2}{c}\right)}_{g_r(\|y\|)}\right] \qquad \|y\| = \delta = \sqrt{c\ln\left(\frac{a}{b}\right)}$$

Interaction Function - An Example II





Swarm Aggregation Properties

The main properties of model (2) are:

P1 The barycenter $\bar{x}$ of the swarm is stationary over time.

P2 The swarm converges to an equilibrium state.

P3 The swarm converges to a bounded region.

P4 The swarm reaches the bounded region in finite time.

How to generalize this model?

The model proposed by Passino et. al. possibly represents the first mathematical characterization of the cohesive behavior observed in nature by groups of animals.

How could this model be generalized?

- Local interaction restricted by limited visibility radius,
- Time-varying interaction between pair of agents,
- Asymmetric interaction due to actuator saturations.

Swarming Algorithms

First Control Law

Aggregation - I

Let us consider the following dynamics for each mobile robot i:

$$\dot{x}_i(t) = k \frac{\sum_{j \in \mathcal{N}_i(t)} g(x_i(t) - x_j(t))}{1 + \left\| \sum_{j \in \mathcal{N}_i(t)} g(x_i(t) - x_j(t)) \right\|}, \tag{5}$$

where:

- k is the saturation gain ($k = 1$ in the sequel for simplicity),
- $g(\cdot)$ is the interaction function as in Passino et. al.,
- $\mathcal{N}_i(t)$ is the time-varying neighborhood of agent i,
- $1 + \left\| \sum_{j \in \mathcal{N}_i(t)} g(x_i(t) - x_j(t)) \right\|$ is the normalization factor.

Aggregation - II

The aggregation dynamics (5) has the following characteristics:

- The proximity graph encoding the interactions among the robots is time varying,
- Robots only interact within their visibility range,
- The control law is saturated to better comply with the actuators capabilities,
- An external connectivity maintenance control term might be required.

Properties

Compared to the Passino et. al. algorithm, the aggregation dynamics (5) exhibits the following properties:

- The barycenter of the swarm is no longer stationary over time
- The swarm converges to an equilibrium state.
- The swarm converges to a bounded region.
- The swarm approaches the bounded region in finite time.

Equilibrium State Existence

Assumption 2 (A2): There exist functions $J_a(\|y\|) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ and $J_r(\|y\|) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ such that:

$$\begin{aligned} \nabla_y J_a(\|y\|) &= y\, g_a(\|y\|) \\ \nabla_y J_r(\|y\|) &= y\, g_r(\|y\|) \end{aligned}. \tag{6}$$

Theorem: Consider a swarm of robots whose dynamics is described by (5) under **A1**, **A2**. Then the swarm reaches a steady state configuration for any given initial condition.

Equilibrium State Existence - Proof Sketch I

- Consider the following (generalized) Lyapunov candidate:

$$V(t) = \frac{1}{2} \sum_{(i,j) \in E(t)} \Big(J_a(\|y\|) - J_r(\|y\|) \Big) \tag{7}$$

- Time derivative is:

$$\dot{V}(t) = \sum_{i=1}^{n} (\nabla_{x_i} V(t))^T \dot{x}_i(t). \tag{8}$$

- By construction, we have:

$$\nabla_{x_i} V(t) = -\frac{1}{f(x_i(t))} \dot{x}_i(t), \tag{9}$$

with $f(x_i(t)) = \frac{1}{1 + \|\sum_{j \in \mathcal{N}_i(t)} g(x_i(t) - x_j(t))\|} \in (0, 1]$.

Equilibrium State Existence - Proof Sketch II

- Substituting (9) in (8), we obtain:

$$\dot{V}(t) = -\sum_{i=1}^{n} \frac{1}{f(x_i(t))} \|\dot{x}_i(t)\|^2 \leq 0. \tag{10}$$

- Using LaSalle's Invariance Principle, it follows that as $t \to \infty$ the state $\mathbf{x}(t)$ converges towards the largest invariant subset of the set where $\dot{V}(t) = 0$, that is:

$$\Omega_e = \{\mathbf{x} : \dot{\mathbf{x}}(t) = 0\}. \tag{11}$$

- Since Ω_e is made of equilibrium points, the thesis follows.

Cohesive Behavior

Assumption 3 (**A3**): The norm of the total attractive vector is bounded below by a linear function:

$$\|y\| g_a(\|y\|) \geq a\|y\|. \tag{12}$$

Assumption 4 (**A4**): The norm of the total repulsive vector is bounded:

$$\|y\| g_r(\|y\|) \leq b. \tag{13}$$

Cohesive Behavior

Assumption 5 (A5): Graph $\mathcal{G}(t)$ remains connected at all times, with

$$\lambda_2(\mathcal{L}_{f,\mathcal{G}}(t)) \geq \lambda_2^\star > 0, \quad \forall\, t > 0 \tag{14}$$

with $\mathcal{L}_{f,\mathcal{G}}(t)$ the weighted Laplacian matrix related to $f(e_i(t))$ whose elements are defined as:

$$\mathcal{L}^{ij}_{f,\mathcal{G}}(t) = \begin{cases} \sum_{j\in\mathcal{N}_i(t)} f(e_i) & \text{if } i = j, \\ -f(e_i) & \text{if } j \in \mathcal{N}_i(t), \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

The following technique can be used to ensure **A5**[5]:

- A. Leccese, A. Gasparri, L. Sabattini, and G. Ulivi. "On the Decentralized Connectivity Maintenance of Bounded Swarm Aggregation Control Laws". Submitted to the 52nd IEEE Conference on Decision and Control (CDC 2013), 2013

[5] A minor fix is required to deal with the asymmetry of the Laplacian matrix.

Cohesive Behavior

Theorem: Consider a swarm of robots whose dynamics is described by (5) under **A1**-**A5**. Then the swarm converges to the following bounded region:

$$\mathcal{B}_r = \left\{ x \in \mathbb{R}^d \;:\; \|x - \bar{x}\| \leq \frac{b}{a} \frac{(n-1)}{\lambda_2^\star} \right\}. \tag{16}$$

with $\bar{x}(t) = 1/n \sum_{i=1}^{n} x_i(t)$ the actual barycenter of the swarm.

Observations:

- The convergence radius depends on the attractive and repulsive design parameters
- The convergence radius depends on the algebraic connectivity of the network topology

Cohesive Behavior - Proof Sketch I

- Consider the following Lyapunov candidate:

$$V(t) = \frac{1}{2}\sum_{i=1}^{n} e_i(t)^T\, e_i(t), \tag{17}$$

with $e_i(t) = x_i(t) - \bar{x}(t)$ the distance between the agent i and the barycenter at time t.

- The time derivative is:

$$\dot{V}(t) = \sum_{i=1}^{n} e_i^T\, \dot{e}_i = \sum_{i=1}^{n} e_i^T\,(\dot{x}_i - \dot{\bar{x}}) = \underbrace{\sum_{i=1}^{n} e_i^T\, \dot{x}_i}_{\dot{V}_1} - \underbrace{\sum_{i=1}^{n} e_i^T\, \dot{\bar{x}}}_{\dot{V}_2}. \tag{18}$$

Cohesive Behavior - Proof Sketch II

- Let us first analyze the second term $\dot{V}_2$:

$$\dot{V}_2 = \sum_{i=1}^{n} e_i^T \dot{\bar{x}} = \left(\sum_{i=1}^{n} e_i^T \right) \dot{\bar{x}} = \mathbf{0}^T \dot{\bar{x}} = 0. \qquad (19)$$

Observations:

- The motion of the barycenter does not affect the size of the bounded region of convergence,
- The motion of the barycenter only affects the location of such a bounded region of convergence.

Cohesive Behavior - Proof Sketch III

- Let us now analyze the first term $\dot{V}_1$. Let us denote $\bar{g}(\|e_i - e_j\|) = g_a(\|e_i - e_j\|) - g_r(\|e_i - e_j\|)$.

$$\begin{aligned} \dot{V}_1 &= \sum_{i=1}^{n} e_i^T \dot{x}_i \\ &= \sum_{i=1}^{n} e_i^T \frac{\sum_{j\in\mathcal{N}_i(t)} -(x_i - x_j)\,\bar{g}(\|x_i - x_j\|)}{1 + \|\sum_{j\in\mathcal{N}_i(t)} -(x_i - x_j)\,\bar{g}(\|x_i - x_j\|)\|} \end{aligned} \tag{20}$$

- By recalling that $x_i - x_j = e_i - e_j$, we obtain:

$$\dot{V}_1 = \sum_{i=1}^{n} f(e_i)\, e_i^T \sum_{j\in\mathcal{N}_i(t)} -(e_i - e_j)\bar{g}(\|e_i - e_j\|). \tag{21}$$

Cohesive Behavior - Proof Sketch IV

- By denoting the attractive and repulsive terms, respectively as

$$\begin{aligned} h_a(\|e_i - e_j\|) &= g_a(\|e_i - e_j\|)f(e_i) \\ h_r(\|e_i - e_j\|) &= g_r(\|e_i - e_j\|)f(e_i) \end{aligned} \tag{22}$$

we have:

$$\begin{aligned} \dot{V}_1 = &\underbrace{-\sum_{i=1}^{n} e_i^T \sum_{j \in \mathcal{N}_i(t)} (e_i - e_j) h_a(\|e_i - e_j\|)}_{\dot{V}_a} \\ &\underbrace{+\sum_{i=1}^{n} e_i^T \sum_{j \in \mathcal{N}_i(t)} (e_i - e_j) h_r(\|e_i - e_j\|)}_{\dot{V}_r} \end{aligned} \tag{23}$$

Cohesive Behavior - Proof Sketch V

- For the term $\dot{V}_a$ according to **A3**, it follows:

$$\begin{aligned} \dot{V}_a &= -\sum_{i=1}^{n} e_i^T \sum_{j \in \mathcal{N}_i(t)} (e_i - e_j)\, h_a(\|e_i - e_j\|) \\ &\leq -a \sum_{i=1}^{n} e_i^T f(e_i) \sum_{j \in \mathcal{N}_i(t)} (e_i - e_j) \\ &\leq -a\, \mathbf{e}^T \mathcal{L}^d_{f,\mathcal{G}}(t) \mathbf{e} \end{aligned} \tag{24}$$

where:

$$\mathcal{L}^d_{f,\mathcal{G}}(t) = \mathcal{L}_{f,\mathcal{G}}(t) \otimes I_d \tag{25}$$

with $\otimes$ the Kronecker product and $\mathcal{L}_{f,\mathcal{G}}(t)$ the weighted Laplacian matrix related to the term $f(\cdot)$.

Cohesive Behavior - Proof Sketch VI

- It follows that:

$$\begin{aligned}
\dot{V}_a &\leq -a\,\mathbf{e}^T \mathcal{L}^d_{f,\mathcal{G}}(t)\mathbf{e} \\
&\leq -a\,\lambda_2(\mathcal{L}_{f,\mathcal{G}}(t))\|\mathbf{e}\|^2 \\
&\leq -a\,\lambda_2(\mathcal{L}_{f,\mathcal{G}}(t))\sum_{i=1}^{n}\|e_i\|^2, \\
&\leq -a\,\lambda_2^\star \sum_{i=1}^{n}\|e_i\|^2, \quad \forall\, \mathbf{e} \notin \operatorname{span}\{\mathbf{1}\otimes\xi_1, \ldots, \mathbf{1}\otimes\xi_d\}
\end{aligned} \tag{26}$$

Observation:

If $\mathbf{e} \in \operatorname{span}\{\mathbf{1}\otimes\xi_1, \ldots, \mathbf{1}\otimes\xi_d\} \Rightarrow$ then all the robots are on the same location, that is $x_1 = x_2 = \ldots = x_n$, which actually should never happen! ;-)

Cohesive Behavior - Proof Sketch VII

- For the term $\dot{V}_r$ according to **A4**, it follows:

$$\begin{aligned}\dot{V}_r &= \sum_{i=1}^{n} e_i^T \sum_{j\in\mathcal{N}_i(t)} (e_i - e_j)\, h_r(\|e_i - e_j\|) \\ &\leq \sum_{i=1}^{n} \|e_i\| \sum_{j\in\mathcal{N}_i(t)} \|e_i - e_j\|\, f(e_i)\, \frac{b}{\|e_i - e_j\|} \\ &\leq \sum_{i=1}^{n} \|e_i\|\, f(e_i)\, b\, |\mathcal{N}_i(t)| \\ &\leq \sum_{i=1}^{n} \|e_i\|\, f(e_i)\, b\,(n-1) \\ &\leq \sum_{i=1}^{n} \|e_i\|\, b\,(n-1).\end{aligned} \tag{27}$$

Cohesive Behavior - Proof Sketch VIII

- By combining the obtained results we have:

$$\begin{aligned}\dot{V}_1 &= \dot{V}_a + \dot{V}_r \\ &\leq -a\,\lambda_2^\star \sum_{i=1}^{n} \|e_i\|^2 + \sum_{i=1}^{n} \|e_i\|\, b\,(n-1) \\ &\leq -\sum_{i=1}^{n} \left[\|e_i\| \left(a\,\lambda_2^\star \|e_i\| - b\,(n-1)\right)\right]\end{aligned} \tag{28}$$

- Thus $\dot{V}_1$ is negative definite if the following holds:

$$\|e_i\| \geq \frac{b}{a}\frac{(n-1)}{\lambda_2^\star}, \quad \forall\, i \in \mathcal{V}. \tag{29}$$

Cohesive Behavior - Proof Sketch IX

- Therefore, the bound on the maximum ultimate swarm size is:

$$\|x - \bar{x}\| \leq \frac{b}{a} \frac{(n-1)}{\lambda_2^\star}, \quad x \in \mathbb{R}^d. \tag{30}$$

thus proving the statement.

Observation: If the graph is fully connected then it can be proven that $\lambda_2(\mathcal{L}_\mathcal{G}) = \lambda_2^\star = n$. The previous equation becomes:

$$\|x - \bar{x}\| \leq \frac{b}{a} \frac{(n-1)}{n} \leq \frac{b}{a}, \tag{31}$$

that is the same bound as in Passino et. al.

Time Convergence

Theorem: Let us consider a swarm of robots whose dynamics is described by eq. (5) under **A1-A5**. Then the swarm moves arbitrarily close to the bounded region $\mathcal{B}_r$ in finite-time t_f, that is:

$$t_f \leq -\frac{1}{2\,\vartheta\,a\,\lambda_2^\star} \ln\left(\frac{\xi^2}{2V(0)}\right), \tag{32}$$

with ξ defined as follows:

$$\xi = (1+\eta)\frac{b}{a}\frac{(n-1)}{\lambda_2^\star}, \tag{33}$$

where $\eta = \dfrac{\vartheta}{1-\vartheta}$ with $\vartheta \in (0,\, 1)$.

Time Convergence - Proof Sketch I

- Let us consider again the Lypaunov function:

$$V(t) = \frac{1}{2} \sum_{i=1}^{n} e_i(t)^T \, e_i(t) \tag{34}$$

- From the previous analysis, we have:

$$\begin{aligned}
\dot{V}(t) &\leq -a\,\lambda_2^\star \sum_{i=1}^{n} \|e_i\|^2 + b\,(n-1) \sum_{i=1}^{n} \|e_i\| \\
&\leq -(1-\vartheta+\vartheta)\,a\,\lambda_2^\star \sum_{i=1}^{n} \|e_i\|^2 + b\,(n-1) \sum_{i=1}^{n} \|e_i\| \\
&\leq -\vartheta\,a\,\lambda_2^\star \sum_{i=1}^{n} \|e_i\|^2 - \sum_{i=1}^{n} \|e_i\| \left[(1-\vartheta)\,a\,\lambda_2^\star \|e_i\| - b\,(n-1)\right]
\end{aligned} \tag{35}$$

with $0 < \theta < 1$.

Time Convergence - Proof Sketch II

- If the following condition holds:

$$\|e_i\| \geq \frac{1}{(1-\vartheta)} \frac{b}{a} \frac{(n-1)}{\lambda_{2,\min}} = \xi \tag{36}$$

- Then, the time derivative $\dot{V}(t)$ can be bounded as:

$$\begin{aligned} \dot{V}(t) &\leq -\vartheta \, a \, \lambda_2^\star \sum_{i=1}^{n} \|e_i\|^2 \\ &\leq -2\vartheta \, a \, \lambda_2^\star \, V(t). \end{aligned} \tag{37}$$

- Thus, the swarm moves arbitrarily close to the bounded region in finite time:

$$t_f \leq -\frac{1}{2\vartheta \, a \, \lambda_2^\star} \ln\left(\frac{\xi^2}{2V(0)}\right). \tag{38}$$

What should be next?

Compared to the Passino et. al. formulation we introduced:

- input saturations for the actuators,
- limited visibility for the robot-to-robot interaction.

What else can be done?

- ensure robot-to-robot collision-free interaction,
- consider an environment with obstacles,
- consider asymmetric input saturations,
- derive tighter bounds w.r.t. the number of agents

Swarming Algorithms

Enhanced Control Law

Aggregation Dynamics - I

Let us consider the following dynamics for each mobile robot i:

$$\dot{x}_i = \frac{\sum\limits_{j \in \mathcal{N}_i(t)} \gamma_{ij}\, g(x_i - x_j)}{\sum\limits_{j \in \mathcal{N}_i(t)} \gamma_{ij}}, \tag{39}$$

where γ_{ij} is the weighting factor between a pair of neighboring robots i and j defined as:

$$\gamma_{ij} = \frac{1}{\|x_i - x_j\|^{\alpha}}, \quad \text{with } \alpha \geq 1. \tag{40}$$

Aggregation Dynamics - II

The aggregation dynamics (39) has the following characteristics:

- The proximity graph encoding the interactions among the robots is time varying,
- Robots only interact within their visibility range,
- The control law is saturated to better comply with the actuators capabilities,
- The saturation for the feedforward and backward velocities can differ,
- The interaction is ensured to be collision-free.

Interaction Function

For the proposed dynamics, the attraction and repulsion functions are defined as follows:

$$\begin{aligned} g_a(\|y\|) &= a\,(1 - \Phi(\|y\|)), \\ g_r(\|y\|) &= b\,\Phi(\|y\|), \end{aligned} \tag{41}$$

Assumption 6 (**A6**): A generalized function $\Phi(\cdot) : \mathbb{R} \to \mathbb{R}$ satisfies:

- $\|x\| \leq \|y\| \Rightarrow \Phi(\|x\|) \leq \Phi(\|y\|)$
- $\lim_{\|y\| \to 0} \Phi(\|y\|) = 1,$
- $\lim_{\|y\| \to \infty} \Phi(\|y\|) = 0.$

Interaction Function: An Example

As an example of generalized function $\Phi(\cdot)$, let us consider:

- $\exp\left(-\frac{\|y\|^\beta}{c}\right)$ with $\beta \geq 1$ and $c > 0$,
- $\mathrm{sech}\left(-\|y\|^\beta\right)$ with $\beta \geq 1$.

Steady State Existence

Assumption 7 (A7): There exist the following functions $J_a(\|y\|) : \mathbb{R}^+ \to \mathbb{R}^+$ and $J_r(\|y\|) : \mathbb{R}^+ \to \mathbb{R}^+$ such that:

$$\begin{array}{l} \nabla_y J_a(\|y\|) = \left(\frac{1}{\|y\|}\right)^{\alpha} \frac{y}{\|y\|} g_a(y) \\ \nabla_y J_r(\|y\|) = \left(\frac{1}{\|y\|}\right)^{\alpha} \frac{y}{\|y\|} g_r(y) \end{array}. \tag{42}$$

Theorem: Consider a swarm of robots whose dynamics is described by eq. (39) under **A1**, **A6** and **A7**. Then the swarm converges to an equilibrium state for any initial condition.

Assumption

Assumption 8 (A8): The graph $\mathcal{G}(t)$ remains connected all the time with:

$$\hat{\lambda}_2(\hat{\mathcal{L}}(t)) \geq \hat{\lambda}_2^\star \tag{43}$$

where $\hat{\mathcal{L}}$ the error Laplacian matrix whose elements are defined as:

$$\hat{l}_{ij} = \begin{cases} \sum_{j \in \mathcal{N}_i(t)} \gamma_{ij} \frac{w_i}{\|e_i - e_j\|}, & j = i \\ -\gamma_{ij} \frac{w_i}{\|e_i - e_j\|}, & j \in \mathcal{N}_i(t) \\ 0 & \text{otherwise.} \end{cases} \tag{44}$$

and:

$$\gamma_i = \sum_{j \in \mathcal{N}_i(t)} \gamma_{ij} > 0. \tag{45}$$

Cohesiveness Analysis

Theorem: Let us consider a swarm of robots whose dynamics is described by eq. (39) under **A1**, **A6-A8**. Then the swarm moves towards and remains within a bounded region:

$$\mathcal{B}_r = \left\{ x \in \mathbb{R}^d : \|x - \bar{x}\| \leq \frac{1}{\hat{\lambda}_2^\star}\left(1 + \frac{b}{a}\right) \right\}. \qquad (46)$$

Observations:

- The convergence radius depends on the attractive and repulsive design parameters
- The convergence radius depends on the algebraic connectivity of the network topology

Time Convergence

Theorem: Let us consider a swarm of robots whose dynamics is described by eq. (39) under **A1**,**A6-A8**. Then the swarm moves arbitrarily close to the bounded region $\mathcal{B}_r$ in finite-time t_f, that is:

$$t_f \leq -\frac{1}{2\,\vartheta\,a\,\hat{\lambda}_2^\star} \ln\left(\frac{\xi^2}{2V(0)}\right) \tag{47}$$

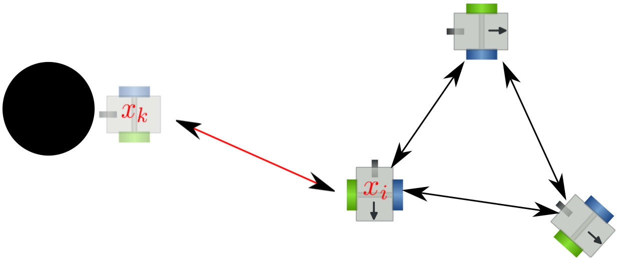
with ξ defined as follows:

$$\xi = (1+\eta)\,\frac{1}{a\,\hat{\lambda}_2^\star}\left(1+\frac{b}{a}\right), \tag{48}$$

where $\eta = \dfrac{\vartheta}{1-\vartheta}$ with $\vartheta \in (0,\,1)$.

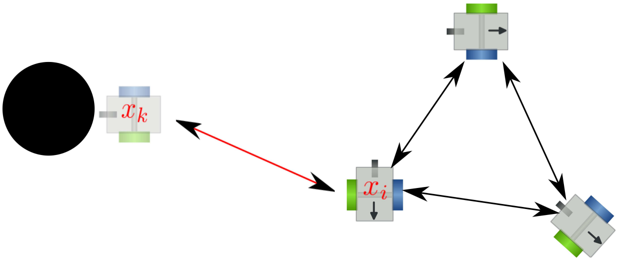
Obstacle Avoidance Integration I

- The key idea is to represent an obstacle as a virtual robot in the neighborhood of the detecting robot,
- Collisions are prevented thanks to the presence of the weighting factors γ_{ik}.



Obstacle Avoidance Integration II

- The closer the robot x_i moves to the virtual robot x_k (obstacle), the larger the term $\gamma_{ik} = \frac{1}{\|x_i - x_k\|^{\alpha}}$ becomes.
- The control term due to the interaction with the virtual robot becomes more relevant, thus preventing a collision



Algorithm Validation

Overview

To validate the proposed swarm aggregation algorithm several simulations along with experiments were carried out.

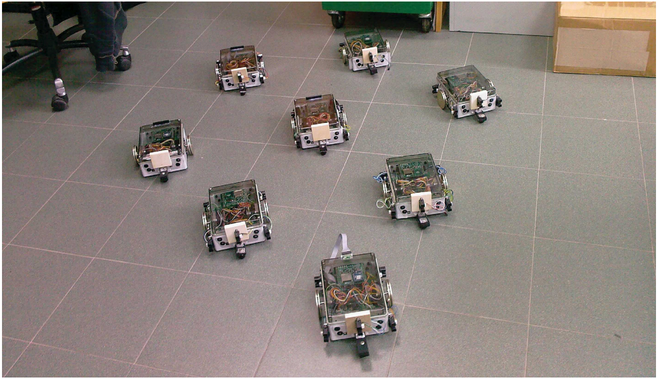
- Simulations were conducted to investigate the scalability with respect to the number of robots
- Experiments were performed to analyze the effectiveness in a real context against not-modeled factors such as noisy measurements.

Video

The following generalized function and parameters were used the control law in (39), i.e.:

$$\Phi(\|y\|) = \exp\left(-\frac{\|y\|^4}{0.02}\right) \quad \gamma_i = \frac{1}{\|x_i - x_j\|^3} \quad [-a,\ b] = [-4,\ 0.4]$$

SAETTA Team - Overview



Team of low-cost mobile robotic platforms developed at the Robotic Lab of the Engineering Department of the University "Roma Tre"

SAETTA Platform - CPUs

The SAETTA platform has a two-levels architecture

Low-level

– Microcontroller (PIC18F87J50)

– Time-constrained Tasks

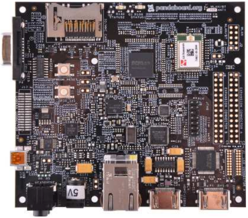
– Main Control Loop at 40Hz

Higl-level

– Pandaboard (1GHz Cortex-A9)

– High-level Tasks

– Main Control Loop at 4Hz



SAETTA Platform - Equipment

Sensors:

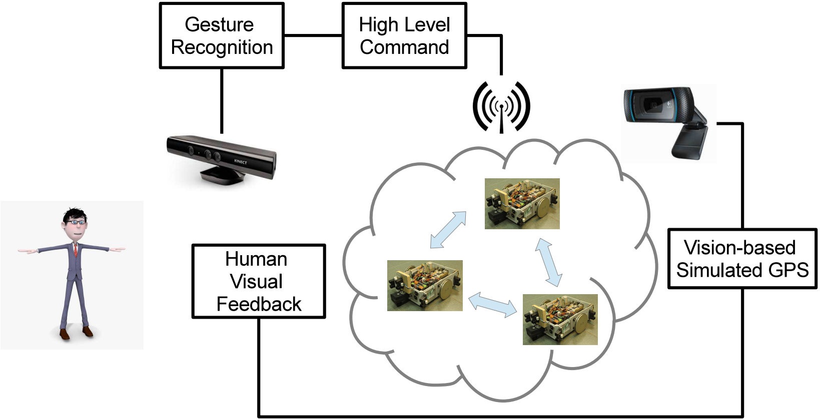
– Accelerometer – Magnetometer – Gyroscope – Infrared

Communication:

Zigbee
- Standard IEEE 802.15.4
- Baudrate 250Kbps
- Range 100/20m (out/in)
- Low consumption

Wifi
- Standard IEEE 802.11
- Baudrate up to 300Mbps
- Mainly for testing purposes
- Higher consumption

Experimental Setup



Video

Any questions?

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