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Intra Predictive Transform Coding based on Predictive Graph Transform

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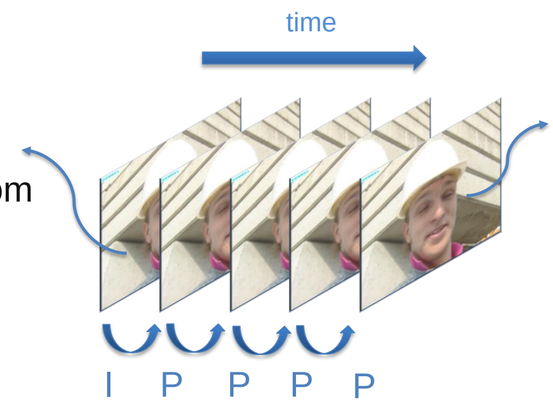
Introduction

Intra Frame Coding: prediction based on current frame.

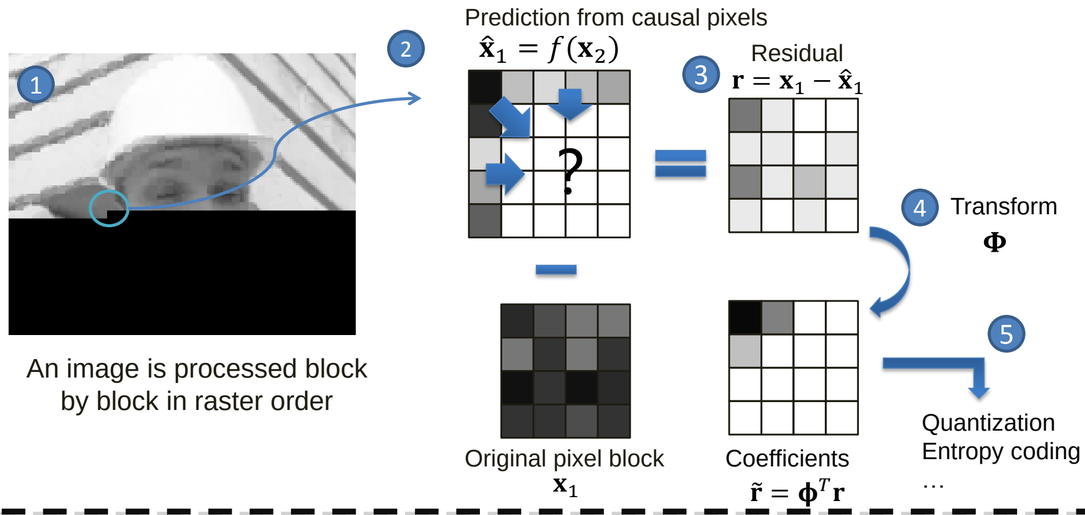
- Independent of other frames
- Allows for random access



- Dependent on other frames
- Reduce inter-frame redundancy

We will focus on the intra frame in this paper.

Predictive Transform Coding: prediction -> transform.

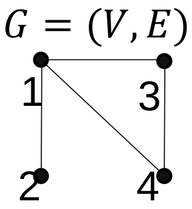


An image is processed block by block in raster order

Graph-based Transform

- A graph consists of a set of vertices/nodes *V*, and a set of links/edges *E*.
- can be represented by **adjacency matrix** *A*, or **graph Laplacian** *L*
- *L* = *D* – *A* where *D* is the **degree matrix** with $D_{ii} = \sum_j A_{ij}$

$G = (V, E)$ E.g.:

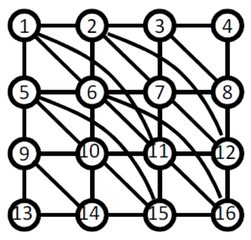


$$A = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \quad L = D - A = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

Optimal Graph Transform:

- Eigenvector matrix of *L*, **Graph Fourier Transform (GFT)**.
  - ➢Circulant graph: GFT -> DFT.
  - ➢Four neighbor regular lattice graph: GFT -> 2D DCT.

Predictive Graph Transform



Node v_i: Gaussian random variable x_i

Link e_{ij}: partial correlation between x_i and x_j

Adjacency matrix $\boldsymbol{A} = \boldsymbol{Q}$

The optimal prediction : $\hat{\mathbf{x}}_1 = -\boldsymbol{A}_{11}^{-1}\boldsymbol{A}_{12}\,\mathbf{x}_2$

The optimal transform $\boldsymbol{\Phi}$: eigenvector matrix of $\widetilde{\boldsymbol{A}} = \boldsymbol{A}_{11}$

Shortcoming: No practical design was provided.

Question: Is the model-driven the only approach?

The Equivalent PGT on covariance graph where $A' = A^{-1}$:

The optimal prediction: $\hat{\mathbf{x}}_1 = \boldsymbol{A'}_{12}\boldsymbol{A'}_{22}^{-1}\mathbf{x}_2$

The optimal transform: eigenvector matrix of $\widetilde{\boldsymbol{A'}} = \boldsymbol{A'}_{11} - \boldsymbol{A'}_{12}\boldsymbol{A'}_{22}^{-1}\boldsymbol{A'}_{21}$

Full covariance leads to Wiener predictor + KLT:

Optimal for predictive transform, however too many parameters.

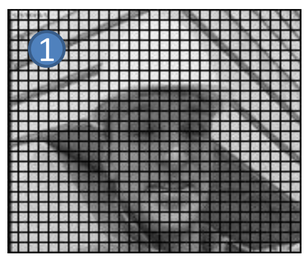
Solution 1: Covariance can be estimated at the decoder side: bandwidth vs. complexity.

Solution 2: **graph sparsification**, sparsify the graph connections and update the weights.

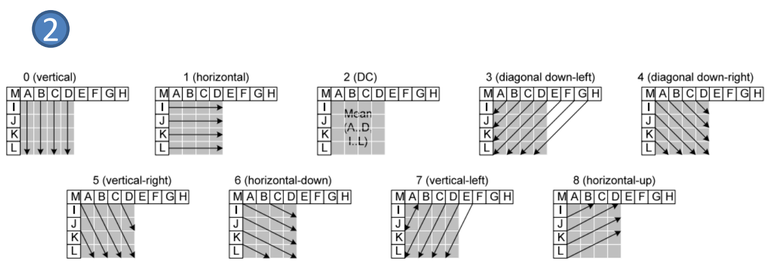
1. Graphical lasso
2. Graph regression model

Application to Intra Coding

Mode-Adaptive PGT:



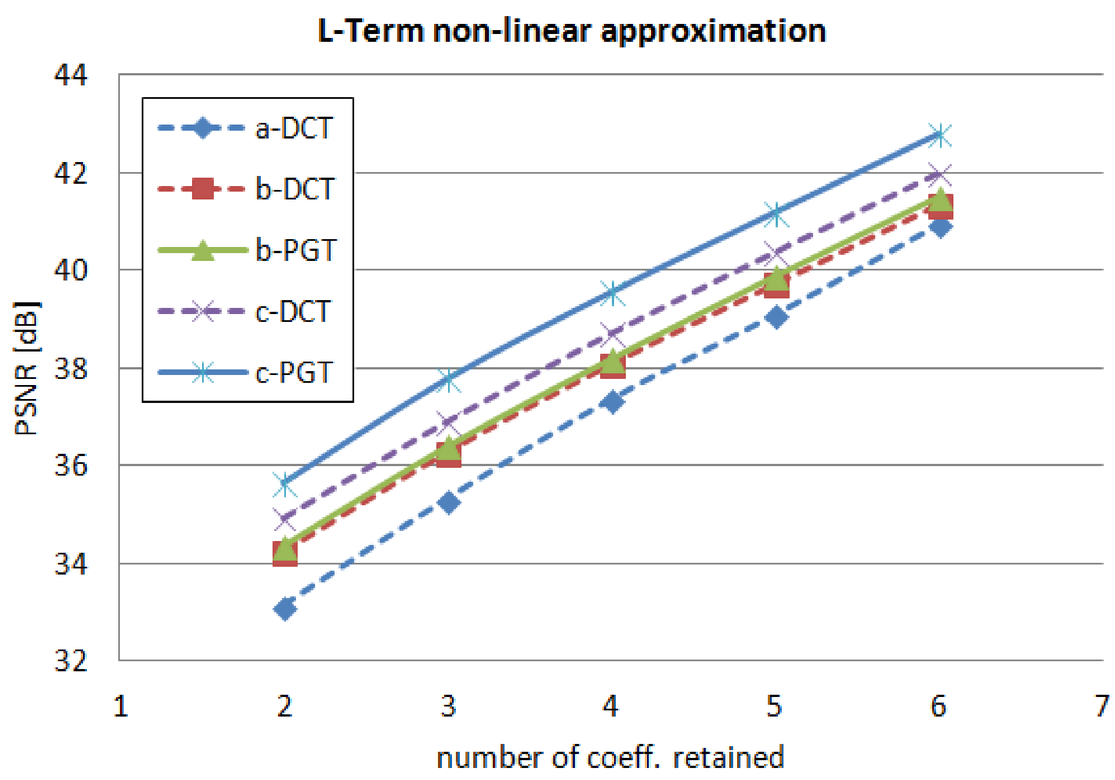
An image is divided into N non-overlapping blocks.



Blocks are classified into 9 classes, corresponding to the 9 modes in H.264/AVC.

3. A sample covariance matrix $\mathbf{S}_{mode}(mode \in [1,9])$ is extracted for each class. (9 per scene).
4. An optimal Prediction and Transform can be calculated for each class.
5. Replace all nine modes in H.264/AVC intra prediction.

Results



More in our ICIP'13 paper

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