The Spread Spectrum Concept

ROBERT A. SCHOLTZ, SENIOR MEMBER, IEEE

Abstract—**This paper describes an idealized spread-spectrum communication system. The processing gain concept is developed as a measure of a well-designed system's robust performance against independent wide-sense stationary interference. Multipath and repeater jammer rejection, partial correlation problems, and security requirements are related to spread-spectrum code properties.**

1. INTRODUCTION

THOUGH well known for more than twenty years, the subject of spread spectrum (SS) communications has not enjoyed the limelight until recently.[1-3] Undoubtedly the low profile of SS techniques has been due to its distinct military (classified) advantages which are based on a robust immunity to interference and jamming. The recent upsurge in open discussions of SS techniques has probably been caused in part by applications of the concept to multiple-user communication situations where large amounts of interference are encountered, and in part by a rapidly advancing technology which is making more intricate signal processing feasible.

The objective of this tutorial paper is to explain how immunity to interference is achieved through use of SS techniques and to present a reasonably rigorous spectral analysis of the SS system. In the process, several major technical problems confronting the system engineer will be revealed.

2. SPREAD SPECTRUM SIGNALS

Roughly speaking, a spread spectrum signal is generated by modulating a data signal onto a wideband carrier so that the resultant transmitted signal has bandwidth which is much larger than the data signal bandwidth and which is relatively insensitive to the data signal. A "mathematical" block diagram of the transmitter is shown in Figure 1. In the notation used here, u(e.g., $d(u, t)$) simply indicates that the quantity involved should be viewed as being random in some way. Double lines in block diagrams indicate in-phase and quadrature channel signals with the in-phase channel carrying the real part of the indicated complex signal, and the quadrature channel carrying the imaginary part. Thus the transmitted signal $s(u, t)$ is viewed as the real part of the product of three complex random signals:

$$s(u, t) = \text{Re}\ [d(u, t)c(u, t)e^{j(\omega_0 t+\varphi(u))}] \ . \tag{1}$$

The actual mechanization of the transmitter may differ considerably from that indicated in Figure 1 (e.g., single channel processing at an i.f. or r.f. frequency may be used to replace two channel processing at baseband), but the mathematical model of the signal given in (1) generally will be applicable.

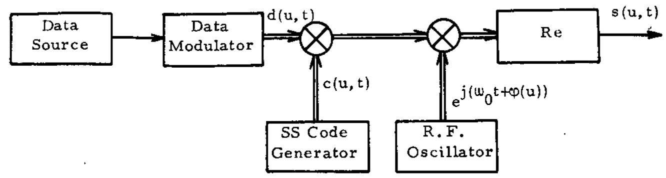

Figure 1. A Spread Spectrum Transmitter.

The expression for the SS transmitted signal $s(u, t)$ indicated in (1) would be the same as for non-SS modulation schemes if it were not for the extra factor $c(u, t)$ which we will refer to as the *spread spectrum code* signal. To simplify the demodulation process, SS codes usually are frequency or phase modulated, and not amplitude modulated, i.e.,

$$|\, c(u, t) \,| = 1, \tag{2}$$

the one obvious exception to this being time-hopping SS codes in which $|\, c(u, t) \,|$ takes on the values 0 or 1, turning the transmitter off and on in an irregular manner. By far the two most widely discussed code signals are the following.

a) *Direct sequence (DS) Signal with chip time T_c:*

$$c(u, t) = \sum_n a_n \boxed{T_c}\, (t - nT_c) \tag{3}$$

where $|\, a_n \,| = 1$ for all n. This signal contains no random parameters.

b) *Noncoherent Frequency Hopping (FH) Signal with hop-time T_h:*

$$c(u, t) = \sum_n e^{j(\omega_n t+\varphi_n(u))} \boxed{T_h}\, (t - nT_h) \tag{4}$$

where $\varphi_n(u)$ is a sequence of independent random phase variables, uniform on $(-\pi, \pi)$.

The notation $\boxed{T}(t)$ is used here to denote a square T-second pulse of unit amplitude, centered at the time origin, and hence it is easily verified that both the DS and FH signals satisfy the constant power condition (2).

The complex sequence $\{a_n\}$ in the DS case, or the frequency sequence $\{\omega_n\}$, in the FH case, must be agreed upon in advance by transmitter and receiver, and in fact have a status similar to that of a key in a cryptographic system. That is, with knowledge of the appropriate sequence, demodulation is possible and without knowledge of that sequence, demodulation is extremely difficult. From a cryptographic viewpoint it would be nice to make the SS code sequences purely random with no mathematical structure. However since all

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The author is with the Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90007, and with the Missile Systems Group, Hughes Aircraft Company, Canoga Park, CA 91304.

systems have a finite memory constraint, all practical SS code sequences have some periodic structure, i.e.,

$$a_n = a_{n+N} \text{ or } \omega_n = \omega_{n+N} \tag{5}$$

for all n where N will be used to denote the period of the appropriate sequence.

3. THE RECEIVER

Neglecting interference and receiver noise, the receiver ideally is presented with a waveform $r(u, t)$ from which the data modulation $d(u, t)$ must be extracted. We assume that

$$r(u, t) = \text{Re}\,\{c(u, t - \tau(u))d(u, t - \tau(u)) \cdot e^{j((\omega_0+\omega_d(u))t+\theta(u))}\}, \tag{6}$$

i.e., the channel inserts a random delay and Doppler shift. This simple model is sufficient to illustrate the demodulation difficulties which the receiver encounters.

A mathematical block diagram of an SS receiver is shown in Figure 2. Again in-phase and quadrature baseband signaling has been chosen for tutorial simplicity. The indicated multiplications (mixing operations) are the receiver's attempt to first reduce the received signal to baseband and then strip the SS code from the data signal. The baseband filter can be considered to be the basic data detection filter (e.g., a matched filter in the digital signal case), possessing a bandwidth comparable to the bandwidth of the data modulation. Assuming that the RF filter passes $r(u, t)$ without distortion, the output of the baseband filter is

$$v(u, t) = \int_{-\infty}^{\infty} 2h(t - \alpha)r(u, \alpha)c_r^*(u, \alpha - \hat{\tau}) \cdot e^{-j((\omega_0+\hat{\omega}_d)\alpha+\hat{\theta})}\, d\alpha \tag{7}$$

where $h(t)$ is the impulse response of the baseband filter, $c_r(u, t)$ is a receiver generated replica of the transmitter's SS code, and $(\ )^*$ denotes conjugation. Assuming that second harmonics of the carrier frequency are eliminated and ideal mixing takes place, $v(u, t)$ in (7) can be evaluated for the signal (6) to give

$$v(u, t) = \int_{-\infty}^{\infty} h(t - \alpha)c(u, \alpha - \tau(u))d(u, \alpha - \tau(u)) \cdot c_r^*(u, \alpha - \hat{\tau})e^{j((\omega_d(u)-\hat{\omega}_d)\alpha+\theta(u)-\hat{\theta})}\, d\alpha. \tag{8}$$

Considerable analytical effort is required to determine the effects of the nonideal RF filters and the nonideal mixers in real systems.

The values $\hat{\omega}_d$, $\hat{\theta}$, $\hat{\tau}$ are provided by synchronization tracking loops (note shown in Figure 2) in an attempt to align the receiver VCO and SS code generator with the corresponding received signal components. With regard to terminology:

$$\hat{\tau} \approx \tau(u) \Rightarrow \text{SS code sync} \tag{9a}$$

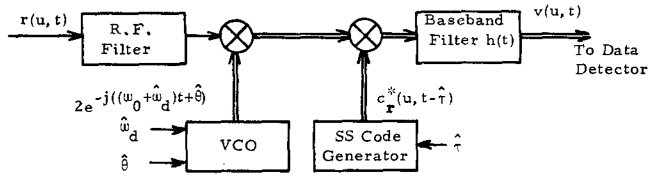

Figure 2. A Spread Spectrum Receiver.

$$\hat{\omega}_d \approx \omega_d(u) \Rightarrow \text{frequency lock} \tag{9b}$$

$$\hat{\theta} \approx \theta(u) \Rightarrow \text{phase lock.} \tag{9c}$$

When the receiver is perfectly locked (exact equalities in (9)) and the receiver SS code signal $c_r(u, t)$ is an exact replica of a constant power (2) transmitted SS code, the output of the baseband filter is

$$v(u, t) = \int_{-\infty}^{\infty} h(t - \alpha)d(u, \alpha - \hat{\tau})d\alpha \quad \text{(perfect lock).} \tag{10}$$

Obviously in this "perfect" case, an ideal equivalent complex baseband channel from the data modulator in the transmitter to the baseband filter in the receiver has been created.

In many cases the receiver cannot reproduce a perfect replica of the transmitted SS code $c(u, t)$ due to random parameters in the code. For example the noncoherent FH code contains random phase jumps inserted by the transmitter when frequency changes occur. Not knowing these phase jumps *a priori* and not making an attempt to learn them, the receiver will output the signal

$$v(u, t) = \int_{-\infty}^{\infty} h(t - \alpha)d(u, \alpha - \hat{\tau})\sum_n e^{j\varphi_n'(u)} \cdot \boxed{T_h}\,(\alpha - nT_h - \hat{\tau})\,d\alpha \quad \text{(phase incoherent)} \tag{11}$$

where $\{\varphi_n'(u)\}$ is a sequence of independent random phases. In effect here we have assumed ideal SS code sync and frequency lock, but no phase lock. Hence the baseband equivalent channel from data modulator to baseband filter now contains random phase jumps every T_h seconds. These jumps must be considered part of the data signal [as in (11)] and have the effect of increasing the bandwidth of the baseband filter $h(t)$ to the order of $1/T_h$ and requiring that envelope detection and possibly postdetection integration techniques be employed in data demodulation.

4. PRIVACY VS. FLEXIBILITY

If the SS system is built so that the SS generator-modulator in the transmitter is operated independently of the data modulator, then it may be possible to use various modulation formats with the same SS code system and thereby build flexibility into the system. This is possible provided that the code synchronizer (which provides τ in the receiver) is compatible with a variety of data modulation formats, and provided that the data demodulators can all be built to cope with any uncertainties (e.g., phase jumps) left in the signal by the SS code modulator-demodulator system.

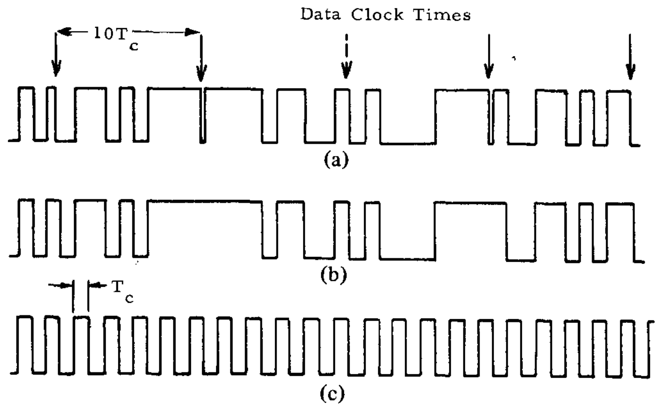

Figure 3. SS Transmitted Modulation with (a) "independent" and (b) "coincident" data and SS code clocks. (c) SS Code Clocks.

One problem with making the data modulator and SS code generator independent is that it may be possible for anyone to read the data directly from a clean copy of the received signal. For example, suppose the data and a DS-SS code are both biphase modulated onto the carrier, with the SS code bit rate at about 10 times the data bit rate, as shown in Figure 3. (The code rate used in this illustration is extremely low. Code bit rates are normally on the order of 1000 or more times higher than the data rate.) If one first estimates the SS code clock [Figure 3 (c)], then it is an easy matter to determine unscheduled phase shifts [solid arrows in Figure 3 (a)] in the transmitted modulation which must be due to phase modulation of the data. It is then possible in Figure 3 (a) to determine the data clock pulses (solid and dashed arrows) and determine the sequence of data bit changes. However if the data clock is divided down from the SS clock so that possible phase change times in the data modulation line up with phase change times in the SS code modulation, no unscheduled phase shifts occur [see Figure 3(b)]. Hence making the SS code and data clocks coincident means that the data cannot be read unless the SS code is known by the receiver.

Systems which have coincident data and SS code clocks are often said to have a data *privacy* feature. In fact systems with privacy features are also simpler to build, with much of the modulation and demodulation equipment shared by the SS code and data signal. Typical privacy system transmitted modulations are the following.

a) *DD-SS System with PSK Data Modulation:*

$$d(u,t)c(u,t) = \sum_n a_n d_{[n/M]}(u)\,\boxed{T_c}\,(t - nT_c) \tag{12}$$

where $\{a_n\}$ is the SS code, $\{d_m(u)\}$ is the data sequence, and $[n/M]$ denotes the integer part of n/M. Hence the data clock rate is $1/M^{\text{th}}$ of the SS code clock rate. This difference in clock rates (M is large) is necessary to produce spread spectrum effects.

b) *Noncoherent FH-SS System with FSK Data Modulation:*

$$d(u,t)c(u,t) = \sum_n e^{j((\omega_n + d_{[n/M]}(u))t + \varphi_n(u))}\,\boxed{T_h}\,(t - nT_n) \tag{13}$$

where $\{\omega_n\}$ is the SS code and $\{d_n(u)\}$ is the data sequence. Again the data clock rate is $1/M^{\text{th}}$ of the SS clock rate. However spectral spreading can be achieved by use of a variety of frequencies $\{\omega_n\}$, and M is sometimes 1 in this system. Since one data symbol time MT_h contains $M-1$ internal phase jumps, the data demodulator will have to employ coherent detection within T_h second internals, followed by postdetection integration over M hop times.

Both of these privacy systems have combined the SS code and data modulators into single units.

5. WIDE-SENSE STATIONARY INDEPENDENT INTERFERENCE

Assuming that all tracking loops are operating in a stable fashion with $\hat{\tau}$, $\hat{\omega}_d$, and $\hat{\theta}$ relatively constant, we can ignore feedback effects through these loops and treat the signal processing from receiver input to baseband filter output as linear processing. Hence it is possible to separately analyze the effects of various components of the input signal on the output. Here we consider any wide-sense stationary random process $N(u, t)$ appearing at the output of the RF filter

$$N(u,t) = \text{Re}\,\{n(u,t)e^{j\omega_0 t}\} \tag{14}$$

where the spectral relationship between the real valued RF signal $N(u, t)$ and the complex valued baseband signal $n(u, t)$ is given by

$$S_N(f) = \tfrac{1}{4}[S_n(f - f_0) + S_n(-f - f_0)] \tag{15}$$

with $f_0 = \omega_0/2\pi$. Hence the baseband power spectral density $S_n(f)$ cannot have bandwidth exceeding B_{RF}, the bandwidth of the RF filter which determines the maximum spectral width of $S_N(f)$. We further assume that $n(u, t)$ is independent of all other receiver inputs and random parameters within the receiver. Certainly the usual receiver noise, some types of jamming, and unrelated interference can be modeled in this fashion. Since the processing is linear, we can normalize the signal so that $n(u, t)$ has unit average power.

$$E\{|n(u,t)|^2\} = 1 = \int_{-\infty}^{\infty} S_n(f)\,df. \tag{16}$$

We shall now determine the amount of power in the baseband filter output $v_n(u, t)$ caused by $n(u, t)$.

Following the processing indicated in Figure 2, the baseband filter output is related to $n(u, t)$ by

$$v_n(u,t) = \int_{-\infty}^{\infty} h(t-\alpha)n(u,\alpha)e^{-j(\hat{\omega}_d\alpha + \hat{\theta})}c_r{}^*(u, \alpha - \hat{\tau})\,d\alpha. \tag{17}$$

After a change of variables, the expected squared value of this output under the stationarity and independence assumptions is easily shown to be:

$$E\{|v_n(u,t)|^2\} = \iint_{-\infty}^{\infty} h(\alpha)h^*(\beta)R_n(\beta-\alpha) \cdot e^{j\hat{\omega}_d(\alpha-\beta)}E\{c_r^*(u,t-\hat{\tau}-\alpha) \cdot c_r(u,t-\hat{\tau}-\beta)\}\, d\alpha\, d\beta \tag{18}$$

where $R_n(\tau)$ is the ensemble autocorrelation of $n(u, t)$, i.e. the Fourier transform of $S_n(f)$. The result of this computation depends on $t - \hat{\tau}$, and hence the resultant output interference power is a periodic function of t, synchronized with the period of the SS receiver code.

By settling for a time-averaged value of the output power (18), it is possible to significantly simplify both the computation and the interpretation of results. Denoting time average by $\langle\,\rangle$, time averaging (18) gives

$$\sigma_{v_n}^2 \triangleq \langle E\{|v_n(u,t)|^2\}\rangle = \iint_{-\infty}^{\infty} h(\alpha)h^*(\beta)R_n(\beta-\alpha) \cdot e^{j\hat{\omega}_d(\beta-\alpha)}\mathcal{R}_{c_r}(\alpha-\beta)\, d\alpha\, d\beta \tag{19}$$

where

$$\mathcal{R}_{c_r}(\tau) \triangleq \langle E\{c_r(u,t+\tau)c_r^*(u,t)\}\rangle \tag{20}$$

is no longer a function of t. The Fourier transform of $\mathcal{R}_{c_r}(\tau)$ is the power spectral density $S_{c_r}(f)$ of the SS code signal replica. In terms of Fourier transforms (19) reduces to

$$\sigma_{v_n}^2 = \int_{-\infty}^{\infty} |H(f)|^2 [S_{c_r}(f) * S_n(f-\hat{f}_d)]\, df \tag{21}$$

where ( )*( ) denotes convolution, $H(f)$ is the system function of the baseband filter, and $\hat{f}_d = \hat{\omega}_d/2\pi$. Hence the *interference power spectral density $S_n(f-\hat{f}_d)$ is spread by the convolution with the code power spectral density $S_{c_r}(f)$ and then reduced by the baseband filter.*

A typical sequence of power spectral densities for the processing of a SS signal and narrowband interference is shown in Figure 4. The key operation is obviously the mixing process with the SS code which compresses the desired signal into the bandwidth of the baseband filter and simultaneously spreads the interference power.

By interchanging the order of convolution and integration over f in (21), it is possible to perform a worst case analysis:

$$\sigma_{v_n}^2 = \int_{-\infty}^{\infty} S_n(\alpha-\hat{f}_d)g(\alpha)\, d\alpha \tag{22a}$$

where

$$g(\alpha) \triangleq \int_{-\infty}^{\infty} |H(f)|^2 S_{c_r}(f-\alpha)\, df. \tag{22b}$$

Since $n(u, t)$ has unit power [see (16)], the highest possible

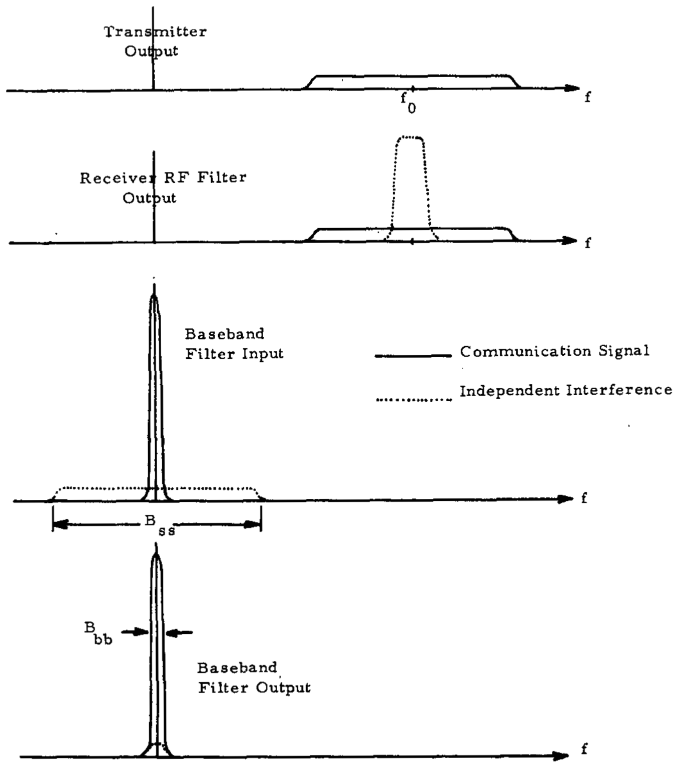

Figure 4. Power Spectral Density Plots for Signal and Interference; Signal-to-Interference Ratio = 1 at RF filter output; Processing Gain = 16.

value of $\sigma_{v_n}^2$ occurs when $S_n(\alpha - \hat{f}_d)$ is concentrated at the maximum of $g(\alpha)$. Hence

$$\sigma_{v_n}^2 \leq \max_{\alpha} \int_{-\infty}^{\infty} |H(f)|^2 S_{c_r}(f-\alpha)\, df. \tag{23}$$

To minimize this bound by good SS code selection, one must make the power spectral density $S_{c_r}(f)$ as flat as possible over the bandwidth B_{ss} of the SS code, i.e., the SS bandwidth of the system. In other words, since $c_r(u, t)$ has unit power in bandwidth B_{ss}, $\max_\alpha g(\alpha)$ can be underbounded by

$$\max_{\alpha} g(\alpha) \geq \frac{1}{B_{ss}} \int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{B_{bb}}{B_{ss}} \tag{24}$$

with equality possible for ideal SS code design. The last equality in (24) is made by assuming that the baseband filter has a maximum gain of unity, and using B_{bb} for the noise bandwidth of the baseband filter. Furthermore *for the ideal flat SS code spectrum, $g(\alpha)$ is approximately B_{bb}/B_{ss} for all α of interest and the baseband filter power output is approximately independent of the shape of the interference's spectral composition.*

$$\sigma_{v_n}^2 \approx \frac{B_{bb}}{B_{ss}} \quad \text{(for ideal SS code).} \tag{25}$$

The synchronous receiver, as described in this section and the previous section recovers the data modulation in the baseband filter without attenuation, i.e., with unit gain. On the other hand, wide-sense stationary independent interference modulation is reduced on the average by the factor in (25). Hence the receiver provides a

$$\text{Processing Gain} \triangleq \frac{B_{ss}}{B_{bb}} \tag{26}$$

advantage to the desired data signal.

One byproduct of SS system design with high processing gain is the *inherent nonobservability of the transmitted signal.* Suppose for example that an SS system with a 30 dB processing gain is operating with a 10 dB signal-to-noise energy ratio at the output of the baseband filter. This implies that the signal-to-noise energy ratio in the RF portion of the receiver is –20 dB. Another receiver, with an identical antenna and RF section but not containing the SS code multiplier, would have an extremely difficult time determining the presence of the RF signal at –20 dB SNR. Somehow the surreptitious listener would have to develop a 20 dB advantage over the SS receiver by reducing the length of the propagation path, using a higher gain antenna, a cooler receiver, etc. Even if this were possible and the listener could detect the SS signal, he could not demodulate data in the privacy format without first knowing the SS code.

6. CODE-RELATED INTERFERENCE

Many types of interference cannot be modeled by wide-sense stationary independent random processes. Two examples of such interference are multipath and repeater jamming. In both cases the interfering signal is modulated with the proper SS code but is received with a greater delay than the direct path and possibly at a different carrier frequency. Hence an adequate model for this interference is

$$m(u, t) = \mathrm{Re}\,\{c(u, t - \tau(u) - \tau_M)d(u, t - \tau(u) - \tau_M) \cdot e^{j((\omega_0 + \omega_d(u) + \omega_M)t + \theta_M(u))}\} \tag{27}$$

where as in Section 4, $\tau(u)$ and $\omega_d(u)$ are the direct path parameters, and now τ_M and ω_M are differential parameters indicating the added time delay and frequency shift of the multipath or jamming relative to the direct path. The modulation $d(u, t)$ can be viewed as data modulation, jammer modulation, or both as the case may be.

Assuming that the receiver is properly synchronized in frequency and time to the direct path parameters, the component of the output signal due to $m(u, t)$ is given by

$$v_m(u, t) = \int_{-\infty}^{\infty} h(t - \alpha)c(u, \alpha - \hat{\tau} - \tau_M)c_r^*(u, \alpha - \hat{\tau}) \cdot d(u, \alpha - \hat{\tau} - \tau_M)e^{j(\omega_M \alpha + \theta_M(u))}\, d\alpha. \tag{28}$$

There are two approaches to output power computations depending on the relation between the "data" modulation $d(u, t)$ and the SS code modulation. In either case, ensemble averages will indicate that $E\{|v_m(u, t)|^2\}$ is a function, not only of τ_M and f_M, but also of $t - \hat{\tau}$. Again as in the case of non-code related interference, the expected power in the output interference is a periodic function of t, relative to the epoch of the SS receiver code. Hence we will compute appropriate time averaged values of the expected output power.

(a) Case I: Wide-Sense Stationary Independent Modulation

Assume $d(u, t)$ is a wide-sense stationary random process, independent of $c(u, t)$ and $c_r(u, t)$. Then

$$\sigma_m^2(\tau_M, \omega_M) \triangleq \langle E\{|v_m(u, t)|^2\}\rangle = \iint_{-\infty}^{\infty} h(\alpha)h^*(\beta)R_{cc_r^*(\tau_M)}(\beta - \alpha) \cdot R_d(\beta - \alpha)e^{-j\omega_M(\beta - \alpha)}\, d\alpha\, d\beta \tag{29}$$

where $R_d(\tau)$ is the correlation function of $d(u, t)$ and

$$R_{cc_r^*}(\tau) = \langle E\{c(u, t + \tau - \tau_M)c_r^*(u, t + \tau)c^*(u, t - \tau_M) \cdot c_r(u, t)\}\rangle \tag{30}$$

is the correlation function of the SS code product $c(u, t - \tau_M)c_r^*(u, t)$. Defining $S_{cc_r^*(\tau_M)}(f)$ and $S_d(f)$ to be the spectral densities of the corresponding correlation functions, (29) can be simplified to:

$$\sigma_m^2(\tau_M, \omega_M) = \int_{-\infty}^{\infty} |H(f)|^2\, [S_{cc_r^*(\tau_M)}(f) * S_d(f + \hat{f}_M)]\, df. \tag{31}$$

Notice that equations (31) and (21) are formally the same and thus the analysis techniques of the previous section are now applicable. Hence by analogy a processing gain advantage of K/B_{bb} against a properly SS coded signal with relative delay τ_M can be guaranteed if the power spectral density of the code product signal $c(u, t - \tau_M)c_r^*(u, t)$ (a unit power signal) is no higher than $1/K$. Even with a flat power spectral density this requires that the code product signal have bandwidth of at least K.

(b) Case II: Modulation in the Privacy Format

Analyses involving privacy formats can become notationally complicated if done in great generality. Here we shall analyze a DS-SS system carefully, indicating the types of manipulations which lead to simplified results in both DS and FH cases.

The output (28) of a DS-SS privacy system with multipath interference at the input is:

$$v_m(u, t) = \int_{-\infty}^{\infty} h(t - \alpha)\left[\sum_n a_n d_{[n/M]}(u)\, \boxed{T_c}\,(\alpha - \hat{\tau} - \tau_M - nT_c)\right]\left[\sum_m a_m\, \boxed{T_c}\,(\alpha - \hat{\tau} - mT_c)\right]^* \cdot e^{j(\omega_M \alpha + \theta_M(u))}\, d\alpha. \tag{32}$$

We assume that the baseband filter-sampler is actually mechanized as an integrate and dump circuit with integration time equal to the duration of a data bit, namely MT_c. (In a noncoherent FH system the integration time is limited by phase jumps to T_h.) Hence we assume in (32) that when $v_m(u, t)$ is sampled at $t = \hat{\tau} + (iM - \frac{1}{2})T_c$, then the baseband filter integrates over the $m = (i-1)_M, (i-1)M + 1, \cdots, iM - 1$ pulses in the SS code pulse train. After inserting a scale factor to give the filter a maximum gain of unity, we have

$$h(t) = \frac{1}{MT_c} \boxed{MT_c} \left(t - \frac{MT_c}{2} \right). \tag{33}$$

Another simplification comes from the fact that in (32), each pulse in the replica code $c_r(u, \alpha - \hat{\tau})$ is overlapped by at most two pulses in the multipath pulse train. The overlapping pulses are easily determined mathematically by first computing

$$\tau_M = K_M T_c + \tau_0, \qquad 0 \leqslant \tau_0 < T_c \tag{34}$$

which indicates that τ_M is a delay of K_M (integer) chip times plus an additional delay τ_0 of less than a chip time. The result of these simplifications to (32) is:

$$\begin{aligned} v_i(u) &\triangleq v_m(u, \hat{\tau} + (iM - \tfrac{1}{2})T_c) \\ &= e^{j(\theta_M(u) + \omega_M \hat{\tau})}[\psi(i, M, K_M, \omega_M T_c)\chi(\tau_0, \omega_M) \\ &\quad + \psi(i, M, K_M + 1, \omega_M T_c)\chi(\tau_0 - T_c, \omega_M)] \end{aligned} \tag{35}$$

where

$$\psi(i, M, K, \gamma) = \frac{1}{M} \sum_{m=(i-1)M}^{iM-1} d_{\lfloor \frac{m-K}{M} \rfloor}(u) a_{m-K} a_m^* e^{jm\gamma} \tag{36}$$

$$\chi(\tau, \omega) = \begin{cases} e^{j\omega\tau/2}\left(1 - \dfrac{|\tau|}{T_c}\right) \dfrac{\sin(\omega(T_c - |\tau|)/2)}{\omega(T_c - |\tau|)/2}, & \tau \leqslant T_c \\ 0, & \tau > T_c. \end{cases} \tag{37}$$

The well-versed reader will recognize $\chi(\tau, \omega)$ as the radar ambiguity function of a T_c second square pulse, and, since

$$|\chi(\tau, \omega)| \leqslant \chi(0, 0) = 1 \tag{38}$$

it is a simple matter to work out relatively tight bounds on $|v_i(u)|$ in terms of the $\psi(i, M, K, \gamma)$ sums by themselves.

Though the precise computations are straightforward, let us further reduce the amount of bookkeeping by specializing to the case $\tau_0 = 0$ and using the bound

$$\begin{aligned} |v_i(u)| &\leqslant |\psi(i, M, K_M, \omega_M T_c)| \\ &= \Bigg| d_{i-2-A}(u) \frac{1}{M} \sum_{m=(i-1)M}^{(i-1)M+B-1} a_{m-K_M} a_m^* \\ &\quad \cdot e^{jm\omega_M T_c} + d_{i-1-A}(u) \frac{1}{M} \\ &\quad \cdot \sum_{m=(i-1)M+B}^{iM-1} a_{m-K_M} a_m^* e^{jm\omega_M T_c} \Bigg|. \end{aligned} \tag{39}$$

Here A and B are integers determined using the equation

$$K_M = AM + B, \qquad 0 \leqslant B < M. \tag{40}$$

Since the data variables are outside the sums in (39), it is now a simple matter to compute moments of the data filter output.

If data bits are uncorrelated with unit variance and if M and N are relatively prime, then the time (i) and ensemble (u) averaged, code-related interference level is given by

$$\begin{aligned} \langle E\{|v_i(u)|^2\}\rangle = \frac{1}{N} \sum_{n=0}^{N-1} [&|f(n, B, K_M, \omega_M T_c)|^2 \\ &+ |f(n, M-B, K_M, \omega_M T_c)|^2] \end{aligned} \tag{41}$$

where

$$f(n, B, K_M, \omega_M T_c) = \frac{1}{M} \sum_{m=0}^{B-1} a_{m+n-K_M} a_{m+n}^* e^{jm\omega_M T_c}. \tag{42}$$

It generally is very difficult to evaluate (42) without resorting to a computer, and in cases where the code period is large, even complete computer results may be impossible to obtain.

When $\{a_n\}$ is a maximum length shift register (MLSR) sequence[4] and $K_M \neq 0$, then $\{a_{n-K_M} a_n^*\}$ is another MLSR sequence. Further assuming that $\omega_M = 0$, (41) can be evaluated using the results of Lindholm[12] to give

$$\begin{aligned} \langle E\{|v_i(u)|^2\}\rangle = \frac{1}{M}\Bigg[&\frac{B}{M}\left(1 - \frac{B-1}{N}\right) \\ &+ \left(1 - \frac{B}{M}\right)\left(1 - \frac{M-B-1}{N}\right) \Bigg] \end{aligned} \tag{43}$$

which is on the order of $1/M$. Hence in this case with $K_M \neq 0 \bmod N$, a processing gain of M is possible. This again illustrates the fact that code related interference rejection requires that $\{a_{n+K_M} a_n^*\}$ also have the properties of SS code.

7. DIRECT SEQUENCE SPECTRAL COMPUTATIONS

Evaluation of processing gain against various types of interference requires that certain SS code power spectral density computations be performed. In the DS case the transmitted code $c(u, t)$ and the receiver's replica $c_r(u, t)$ are identical periodic coherent waveforms having no random parameters. Hence the SS code correlation function required in (19) is given by

$$\begin{aligned} R_{c_r}(\tau) &= \frac{1}{NT_c} \int_0^{NT_c} c_r(u, t+\tau) c_r^*(u, t)\, dt \\ &= \left(1 - \frac{\tau_0}{T_c}\right) R_a(K) + \frac{\tau_0}{T_c} R_a(K+1) \end{aligned} \tag{44a}$$

where

$$\tau = KT_c + \tau_0, \qquad 0 \leqslant \tau_0 < T_c \tag{44b}$$

and

$$R_a(K) = \frac{1}{N}\sum_{m=0}^{N-1} a_{m+K}a_m^*. \tag{44c}$$

It is apparent from (44) that $R_{c_r}(\tau)$ is a periodic correlation function with period NT_c in the variable τ, since $\{a_n\}$ is a periodic sequence.

The periodic form of $R_{c_r}(\tau)$ implies that the power spectral density of $c_r(u, t)$ must be a line spectral density with

$$S_{c_r}(f) = \sum_n P_n \delta\left(f - \frac{n}{NT_c}\right) \tag{45}$$

where

$$P_n = \frac{1}{NT_c}\int_0^{NT_c} R_{c_r}(\tau)e^{-j2\pi n(\tau/NT_c)}\,d\tau$$

$$= S_a(n)\left(\frac{\sin(\pi n/N)}{\pi n/N}\right)^2 \tag{46}$$

Here $S_a(n)$ is the discrete Fourier transform of one period of the SS code sequence:

$$S_a(n) = \frac{1}{N}\sum_{K=0}^{N-1} R_a(K)e^{j2\pi n(K/N)}. \tag{47}$$

The spectral density $S_{c_r c_r^*(\tau_M)}(f)$ required to evaluate the effects of code related interference can be evaluated by applying these same techniques to the new sequence $\{a_{n-K_M}a_n^*\}$ when $\tau_M = K_M T_c$.

Since $S_{c_r}(f)$ is a line spectral density, i.e., max $S_{c_r}(f) = \infty$, it would seem possible that the transmitted signal is in fact observable by a surreptitious listener and also that independent wide-sense-stationary interference may cause significantly more trouble than is predicted in Section 5. Such is not the case if the following precautions are observed: (a) *Make sure that the data modulation bandwidth is much larger than* $1/NT_c$ (the reciprocal of the SS code modulation's period). This insures that the width of $|H(f)|^2$ is much greater than the line-spacing in $S_{c_r}(f)$ and hence that $g(\alpha)$ in (22b) is a smooth function of α. Hence independent interference cannot cause unexpected difficulties. In this case data modulation will also spread the lines in the SS code's spectral density to yield a smooth transmitted signal power spectral density, devoid of lines. (b) If it is impossible to guarantee data modulation of sufficient bandwidth, then *make sure that* NT_c *is very large,* implying very close line-spacing in $S_{c_r}(f)$. Several periods of the SS code will be required to surreptitiously detect the SS transmitted signal using a filter narrow enough to isolate a spectral line and gain an advantageous signal-to-noise ratio for detection. If NT_c is large enough, this type of detection will be impossible due to oscillator drifts, Doppler shift variations, etc.

8. NONCOHERENT FREQUENCY HOPPING SPECTRAL COMPUTATIONS

The independent uniformly distributed random phase jumps which occur in noncoherent FH signals lead to significant mathematical simplifications and eliminate the possibility of a line structure to the spectral density. Computation of the ensemble autocorrelation gives

$$E\{c_r(u, t+\tau)c_r^*(u, t)\}$$
$$= \sum_n e^{j\omega_n\tau}\,\boxed{T_h - |\tau|}\,(t - nT_h + \tau/2). \tag{48}$$

Here it is understood that $\boxed{X}\,(t)$ is identically zero when X is negative. The function in (48) is periodic in t since the hopping sequence is periodic, but it is not periodic in τ. Let us denote the set of frequencies present in the FH sequence $\{\omega_n\}$ by Ω and let $k(\omega)$ be the number of times that the frequency ω occurs in one period of the FH sequence. Then the time average of (48) is easily computed:

$$R_{c_r}(\tau) = \begin{cases}\left(1 - \frac{|\tau|}{T_h}\right)\sum_{\omega'\epsilon\Omega}\frac{k(\omega')}{N}e^{j\omega'\tau}, & |\tau| < T_h\\ 0, & |\tau| \geqslant T_h.\end{cases} \tag{49}$$

Fourier transforming gives the spectral density:

$$S_{c_f}(f) = T_h\sum_{\omega'\epsilon\Omega}\frac{k(\omega')}{N}\left|\frac{\sin(\pi(f-f')T_h)}{\pi(f-f')T_h}\right|^2 \tag{50}$$

where $f' = \omega'/2\pi$. Notice that this power spectral density does not depend on the order of the frequencies in the FH sequence, but only on the relative usage $k(\omega')/N$ of frequencies $\omega'\epsilon\Omega$.

As in the case of DS-SS codes, the product of a FH-SS code signal with a $K_M T_h$ shifted version of itself is a new FH-SS signal. When $\tau_M = K_M T_h$, then the spectral density $S_{c_r c_r^*(\tau_M)}(f)$ required in the study of code-related interference can be determined by applying the above computational techniques to the sequence $\{\omega_{n-K_M} - \omega_n\}$. One conclusion about FH code design which can be drawn at this point is that codes with ω_n increasing or decreasing linearly with n are to be avoided if code-related interference is expected.

9. CLOSING COMMENTS

Even in the simple SS system models discussed here, there are two major classes of theoretical problems which we have not yet touched: (1) the synchronization problems involving phase, frequency and SS code acquisition and tracking, and (2) the problems of SS code design. The following are just a few of the problems related specifically to the SS concept.

In an SS system, the signal energy in any portion of the RF spectrum is dominated by the noise energy. This fact makes it extremely difficult to first lock up the RF carrier tracking loop which supplies $\hat{\omega}_d$ and possibly $\hat{\theta}$ in the receiver, even using sophisticated loops.(13) It seems that *SS code sync must be established first* to concentrate the received energy in a narrow band and allow suppressed carrier tracking to be established. The current literature(13-15) contains several techniques at various levels of sophistication for handling special cases of this problem.

Since an SS system uses bandwidth resources extravagantly, the frequency spectrum generally must be shared with other systems, leading to a (SS) code division multiple access (CDMA) mode of operation. A well known CDMA design for DS-SS codes is the Gold codes(5, 6) whose cross-correlation properties are nearly optimal.(7) Good FH-SS code sets have been proposed by Lempel and Greenberger(8) and by Solomon.(9) Unfortunately these code sets and the simple MLSR sequences for single DS-SS systems *are not cryptographically secure.* The structure of a complete code sequence can be determined from an observation of a small segment of the sequence.(11) It is conceivable that an intelligent listener with an operating gain advantage can use this fact either to read the data or jam the system.

Another CDMA problem is the *near-far problem,* which occurs when the power levels of the undesired signals of the other users are very large compared to the power level of the desired signal. When the processing gain of the system is not large enough to counteract this power imbalance, then time hopping modes of operation may be required to eliminate all but occasional interference. One possible time hopping code is discussed by Cohen *et al.*(10)

While this article has not touched on hardware design problems, it already should be obvious that significant problems must be solved in any system design which hopes to achieve the significant advantages of the SS concept.

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Robert A. Scholtz (S'56-M'59-SM'73) was born in Lebanon, Ohio on January 26, 1936. He received the B.S. degree from the University of Cincinnati in 1958, the M.S. degree from the University of Southern California in 1960, and the Ph.D. degree from Stanford University in 1964.

He has been with Hughes Aircraft Company since 1958, first as a Hughes Master's and Doctoral Fellow, and now as Senior Staff Engineer in the Missile Systems Group. In 1963 he joined the faculty of the University of Southern California where he is now a Professor of Electrical Engineering. He has published numerous papers in the areas of communication theory and coding.

Dr. Scholtz is a past chairman of the Los Angeles Chapter of the IEEE Professional Group on Information Theory, and is an active member of the Communication Theory Committee of the IEEE Communications Society. He was general chairman of the Third Annual Communication Theory Workshop and is presently serving as Financial Chairman for NTC '77.

Dr. Scholtz is a member of Sigma Xi, Tau Beta Pi, Eta Kappa Nu, and Phi Eta Sigma.