Learning to Communicate

Robert Calderbank

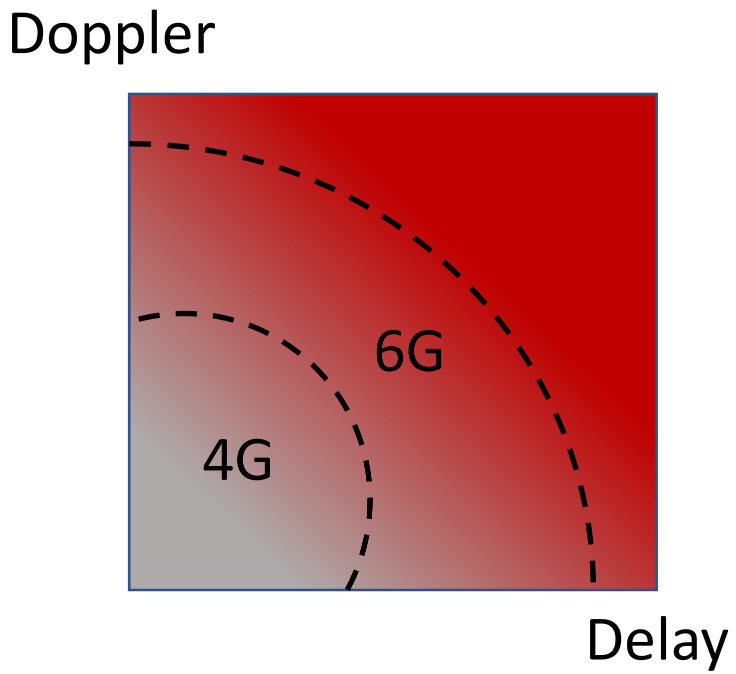
Duke University

Abstract: We describe how pulsones interpolate between TDM and FDM, and when it is possible to learn input-output relations without learning the channel, opening the door to machine learning.

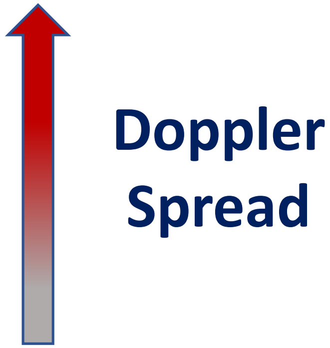
Learn More - IEEE BITS Magazine: *A Mathematical Foundation for Communications and Sensing in the Delay-Doppler Domain, Parts I and II* – in collaboration with Saif Khan Mohammed, Ronny Hadani, and Ananthanarayanan Chockalingam

Disclosure: Advisor to Cohere Technologies

So Many Channels, So Little Time



Leo-Satellite Channel
UAV/Aeronautical Channel
mmWave Mobile Channel
Terrestrial Mobile Channel
Terrestrial Pedestrian Channel

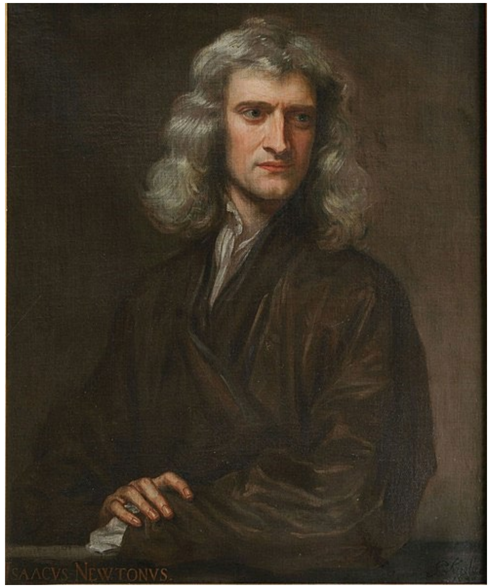


Today we design wireless systems using mathematical models

This approach is losing ground as wireless channels become more complex and Doppler becomes more significant

Might it be possible to operate model-free

We have Asked This Question Before



Newton's Laws of Motion

Model-based approach that develops understanding at the most fundamental level.

Kepler's Laws of Planetary Motion

Model-free approach that uses data to make predictions

Why Ask It Now?

Machine learning has revolutionized image and natural language processing

Data-driven discovery has revolutionized bioinformatics

Machine learning (ML) is about approximating functions – broad impact comes from the fact that it is particularly effective in high dimensions

Classically we measure complexity of functions by smoothness – how many times the function can be differentiated

ML measures complexity by how well the function can be approximated by a particular neural network model – reproducing kernel Hilbert spaces, for example

**Weinan E*, The Dawning of a New Era in Applied Mathematics, Notices of the American Mathematical Society, May 2021

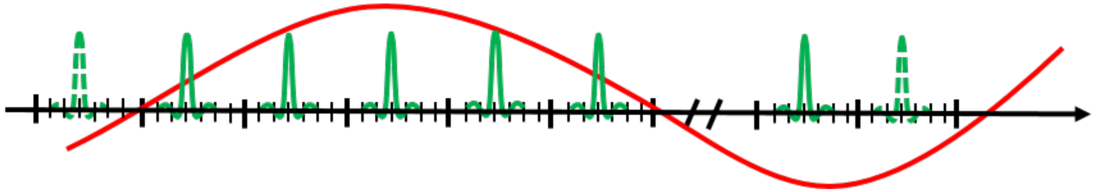
Localization in Delay and Doppler

Radar as a game of 20 questions with an operator

P.M. Woodward: *Probability and Information Theory, with Applications to Radar,* Pergamon Press, 1953

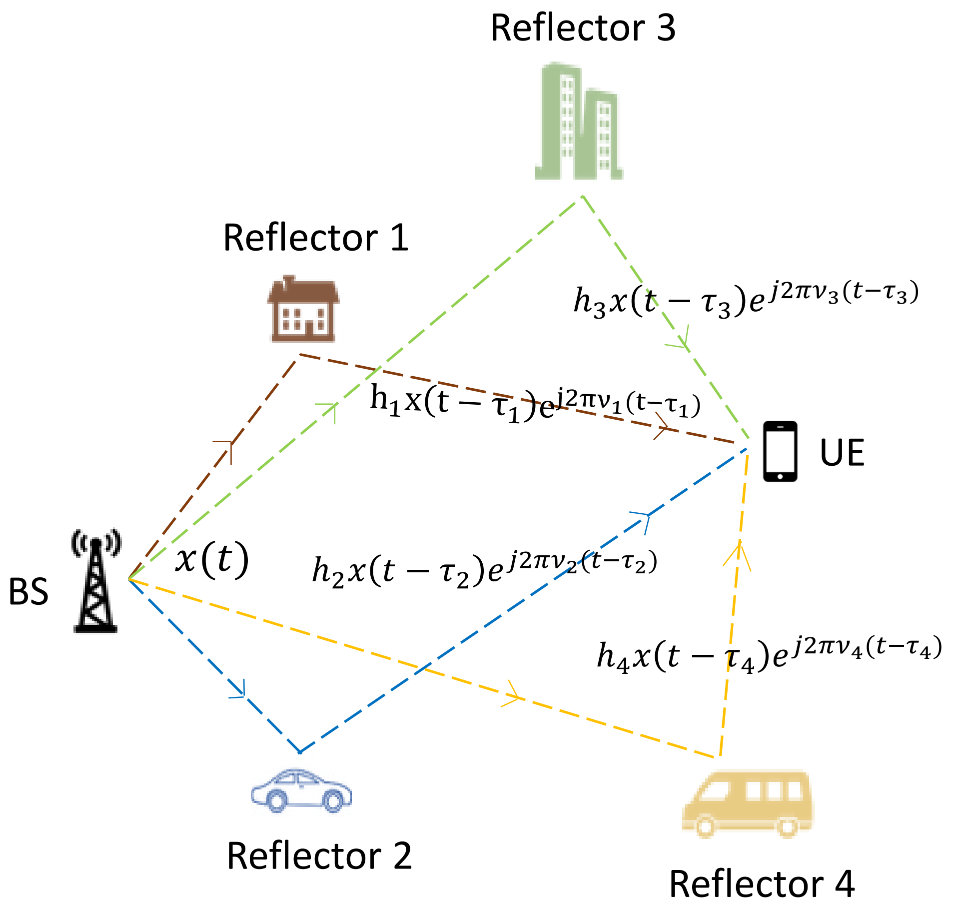
He viewed the problem of localizing a scatterer in delay and Doppler as using a waveform to ask questions of the operator defined by the radar scene

How to Design a Question:

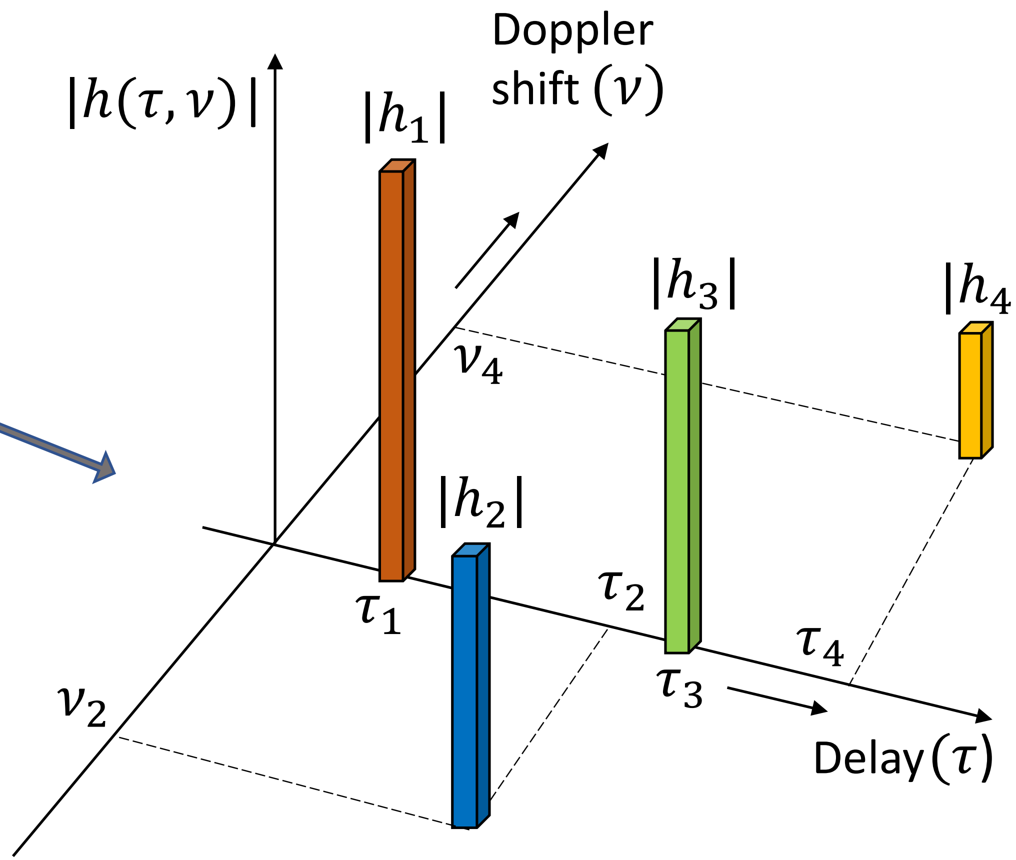


Prediction as a game of 20 questions with a doubly spread channel

Representing Doubly Spread Channels



Taps in Delay and Doppler

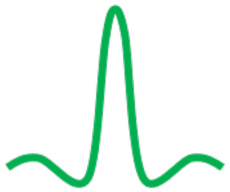


What is the right question?

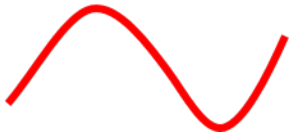
What Constitutes a Good Question?

Doubly Spread Channel: A sum of operators $D(\tau_i, \nu_j)$ introducing path delay τ_i and Doppler shift ν_j

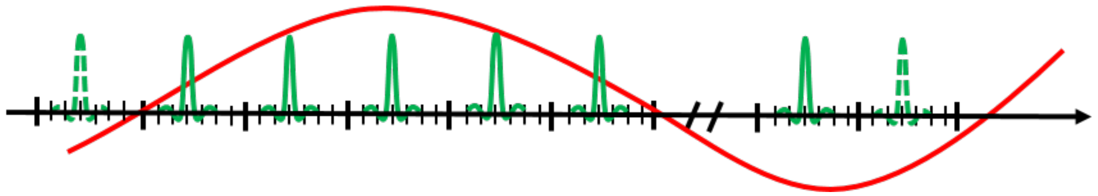
Waveforms are questions, returns are answers, objective is prediction



Time Domain (TD) Pulse: Good question for pure delay channels

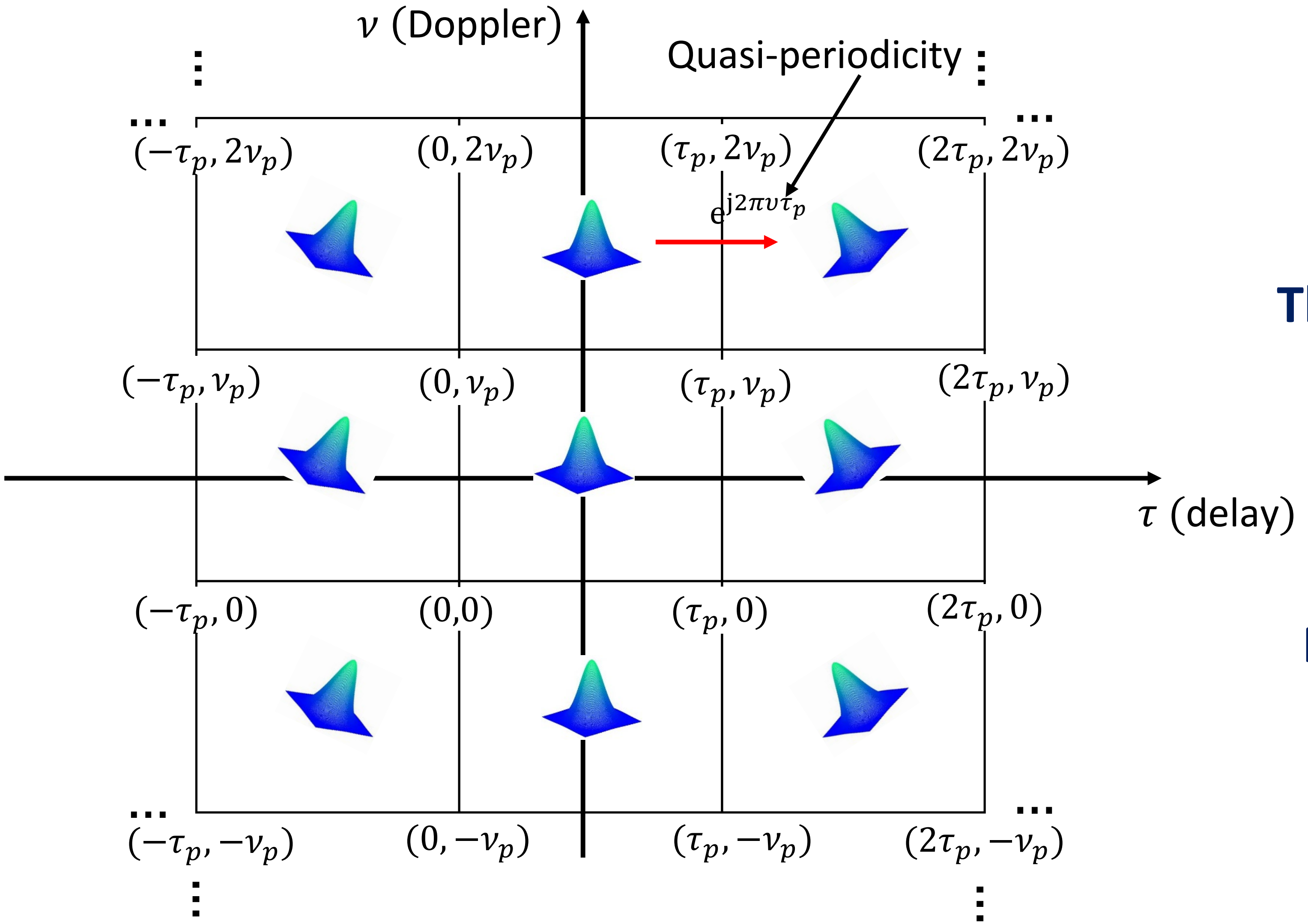


Frequency Domain (FD) Pulse: Good question for pure Doppler channels



Delay-Doppler (DD) Domain Pulse:
Good question for doubly spread channels

A Pulse in the Delay-Doppler Domain

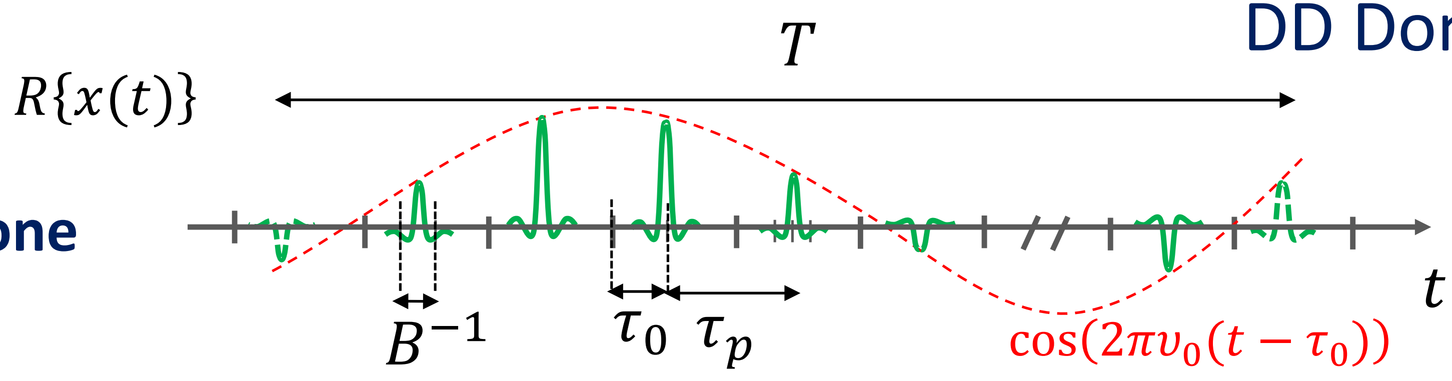


The DD realization of a TD signal is a quasi-periodic function

Fundamental Domain defined by the delay period τ_p and the Doppler period ν_p

TD Pulsone from a Quasi-Periodic DD Domain Pulse

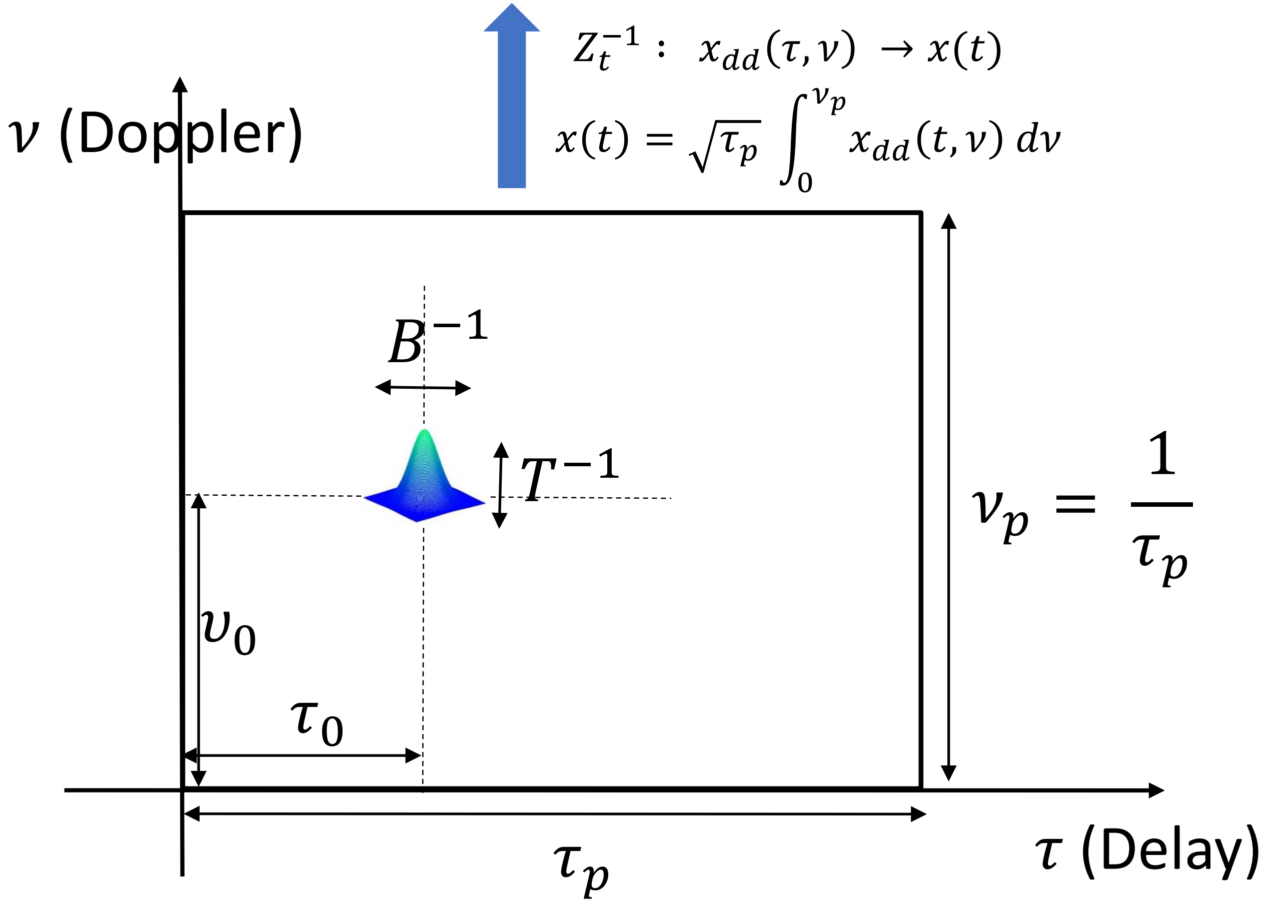
TD Pulsone



TDM is a limiting case
FDM is a limiting case

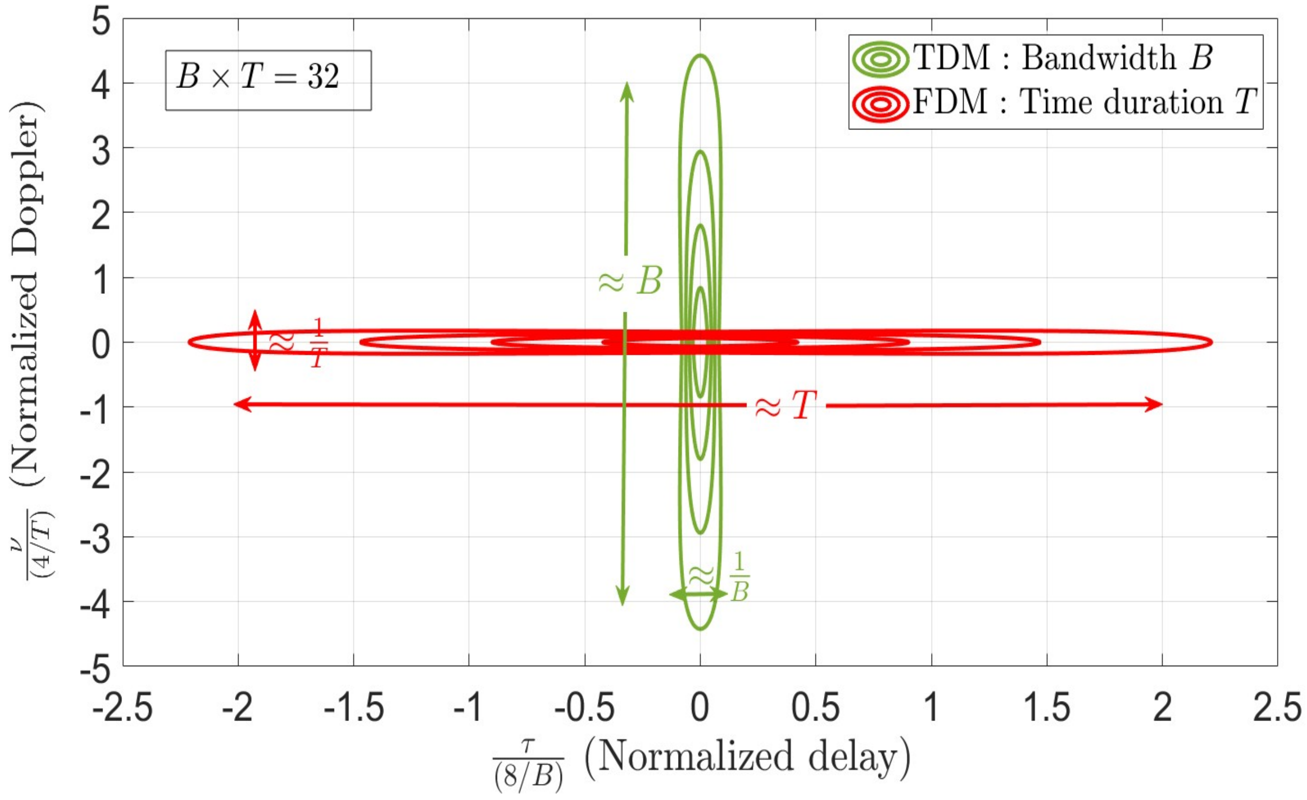
Pulsones parametrized by τ_p interpolate between TDM and FDM

When does a TD Pulsone question have predictive value?



What Constitutes an Ambiguous Answer?

Ambiguity Function $|A_{s,s}(\tau, \nu)|^2$ We transmit a waveform s to illuminate a radar scene, then correlate the result with the transmitted waveform



$\text{sinc}((B - |\nu|)\tau)$

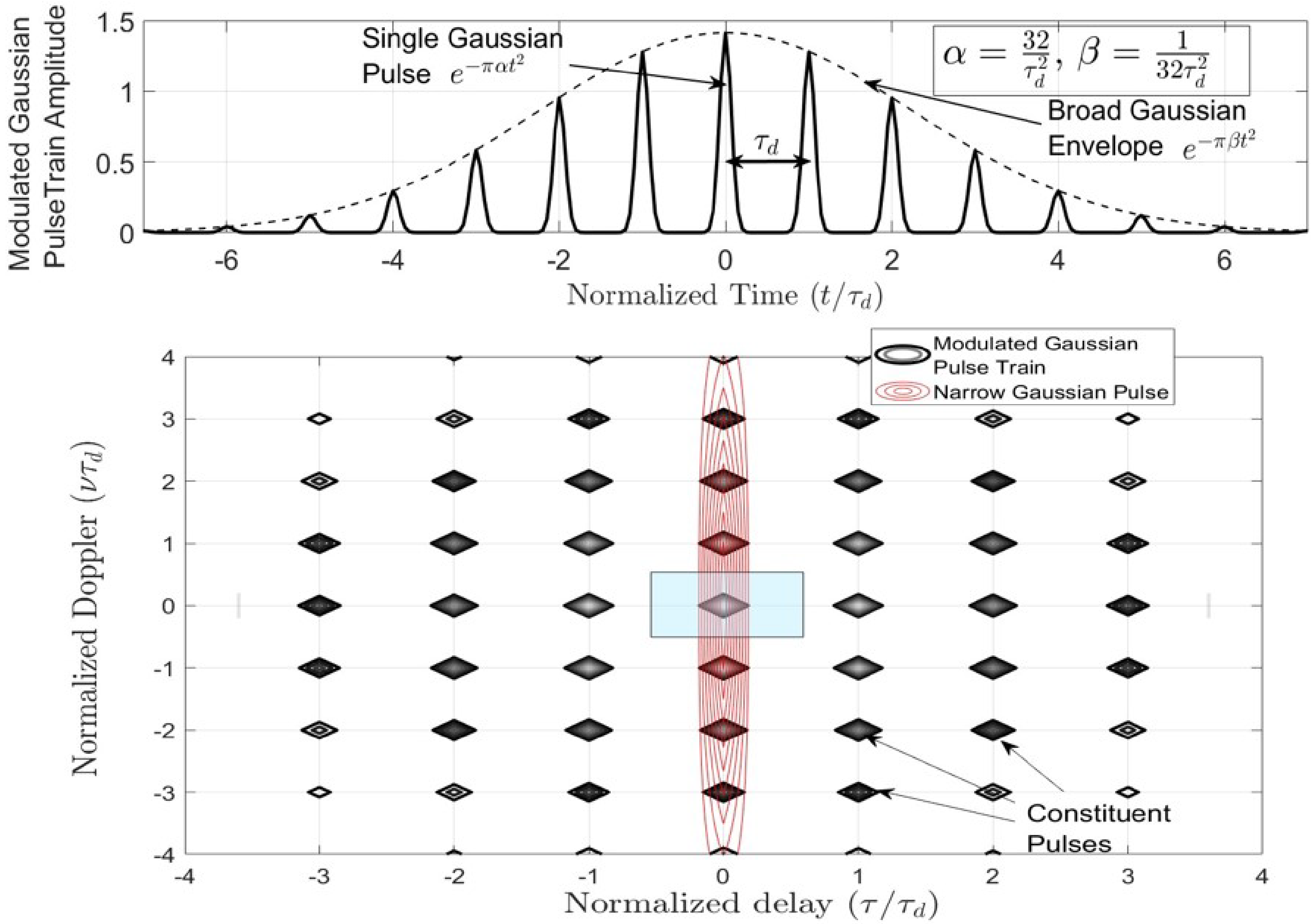
TDM: Not able to separate targets in Doppler

$\text{sinc}((T - |\tau|)\nu)$

FDM: Not able to separate targets in delay

Woodward, 1954

Volume under $|A_{s,s}(\tau, \nu)|^2$ fixed by Moyal's Identity but can be redistributed to enable resolution of radar targets



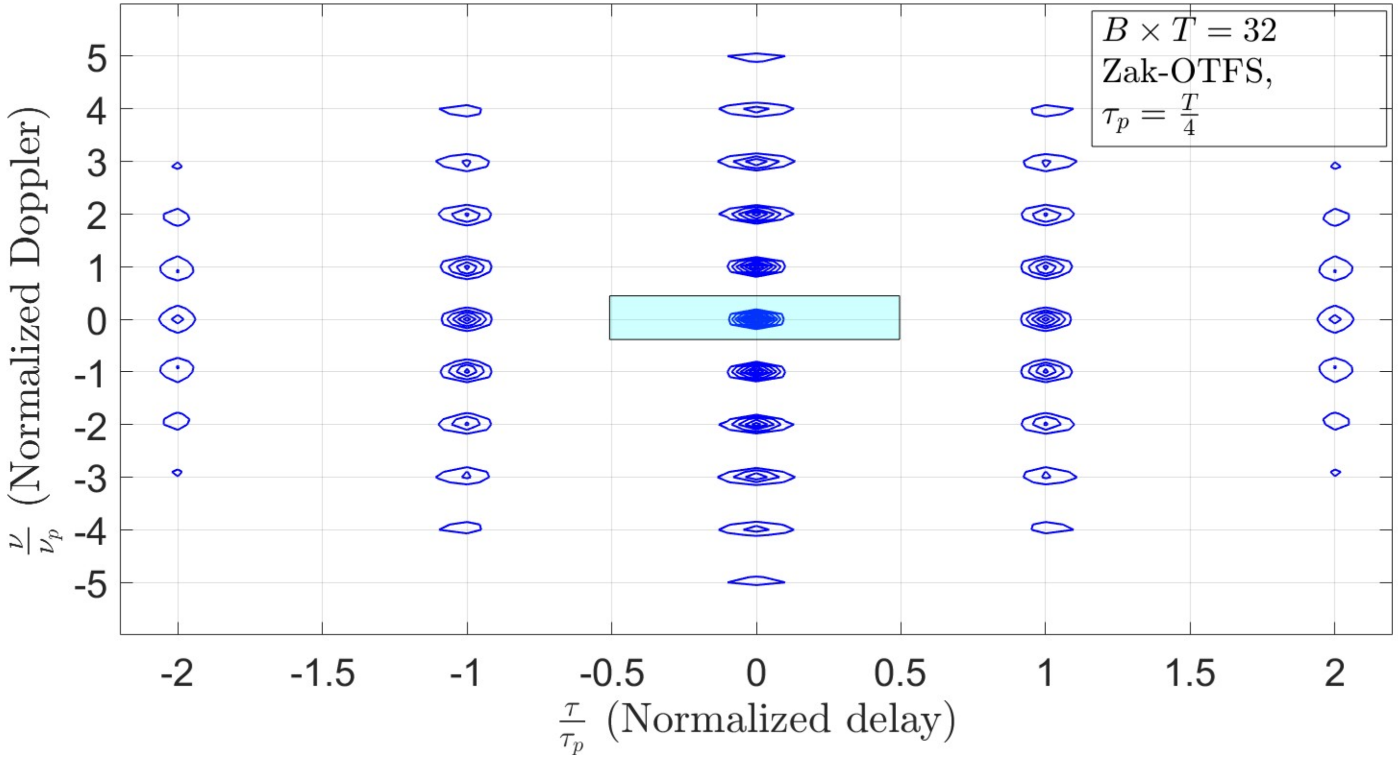
Red Ellipse: Ambiguity function of narrow Gaussian pulse

Modulate a train of narrow TD Gaussian pulses with a broad Gaussian envelope

Black Ellipses

Pulsones, 2023

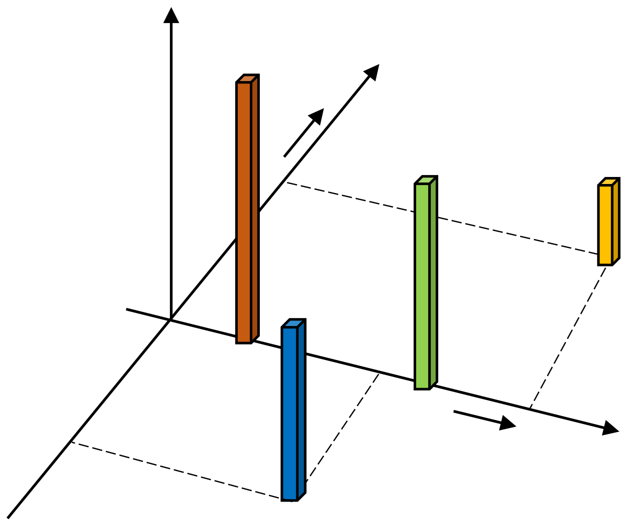
Ambiguity Function $|A_{s,s}(\tau,\nu)|^2$ - pulsone carrier waveform

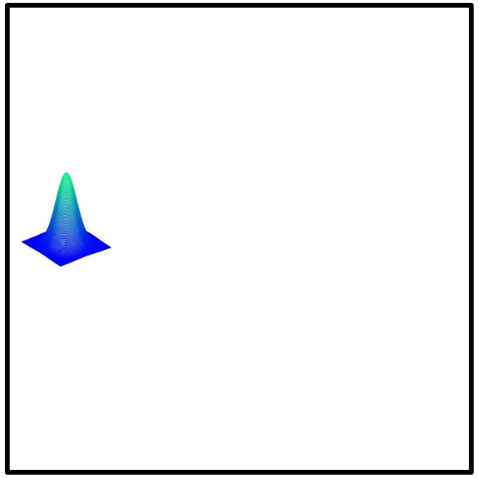


Narrow DD domain impulses separated by τ_p along the delay axis and ν_p along the Doppler axis

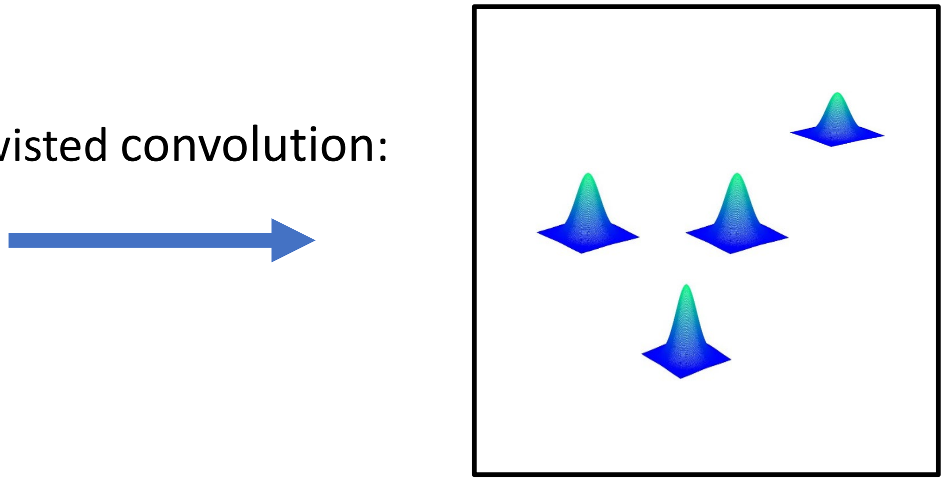
Each impulse has a spread of 1/B along the delay axis and 1/T along the Doppler axis

Doubly Spread Channels Acting on Pulsones





Twisted convolution:

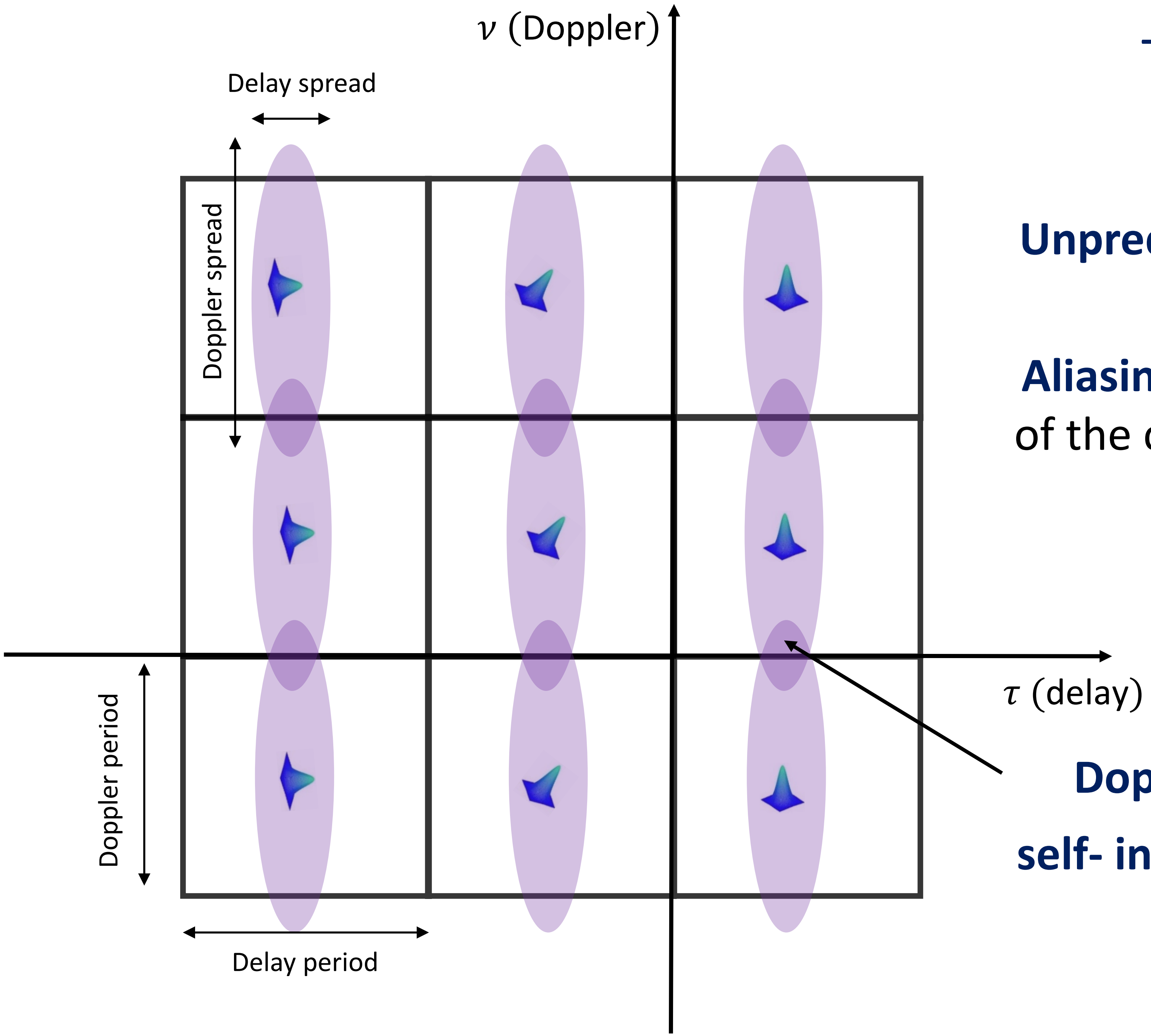


Crystalline Regime: The delay domain period τ_p is greater than the channel path delay spread, and the Doppler domain period ν_p is greater than the path Doppler spread:

$$\tau_p > \text{delay spread} \quad \text{and} \quad \nu_p > \text{Doppler spread}$$

The interaction of a doubly spread channel with a TD pulsone is predictable and geometric

The Origins of Unpredictability



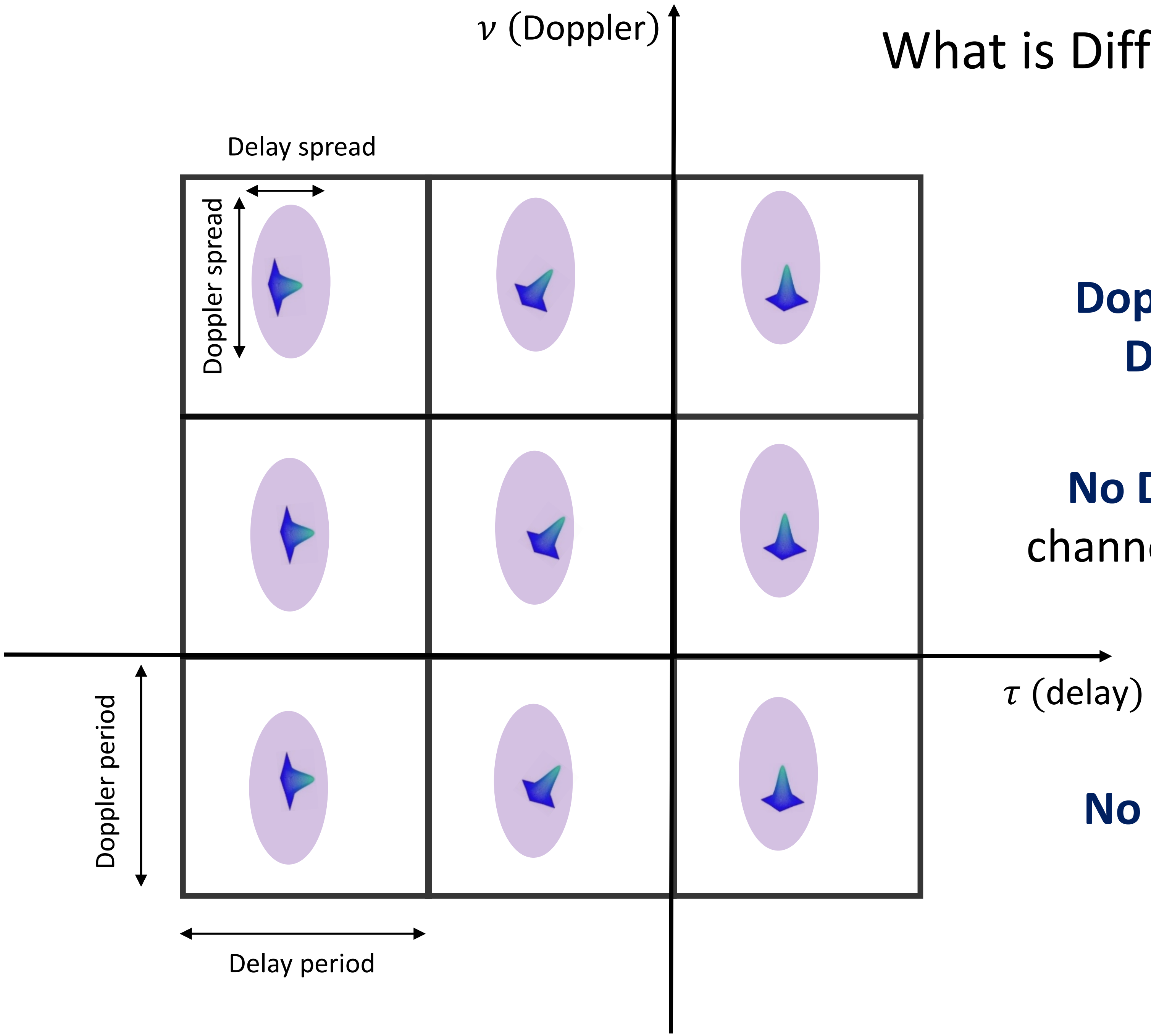
Unpredictability results from aliasing in the DD domain

Aliasing occurs when the DD spreads of the channel are bigger than the DD periods of the pulsone

Doppler spread > Doppler period

self- interaction leads to unpredictability

What is Different in the Crystalline Regime?



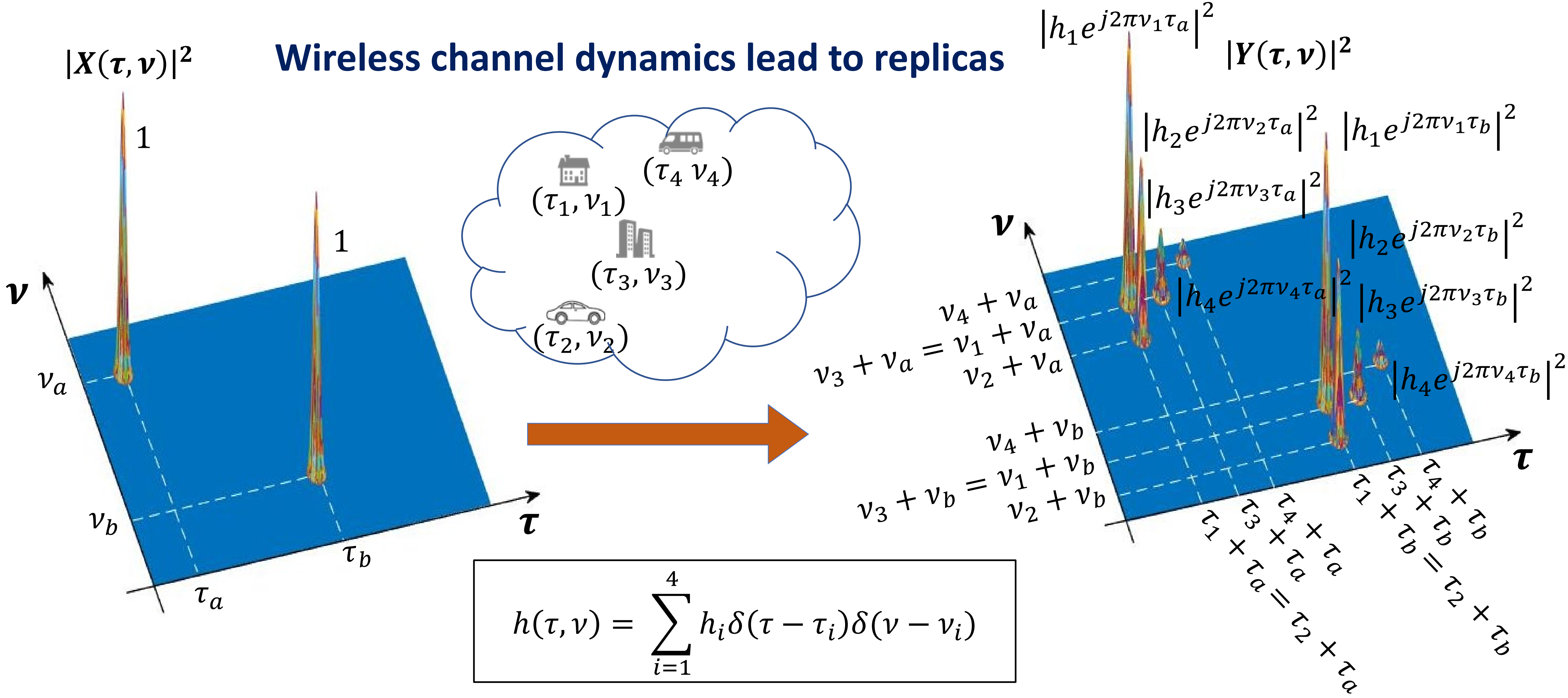
Doppler spread < Doppler period
Delay spread < Delay period

No DD Aliasing - the DD spreads of the channel are smaller than the DD periods of the pulsone.

No Self-Interaction means the channel response is predictable

Picturing Predictability

Wireless channel dynamics lead to replicas



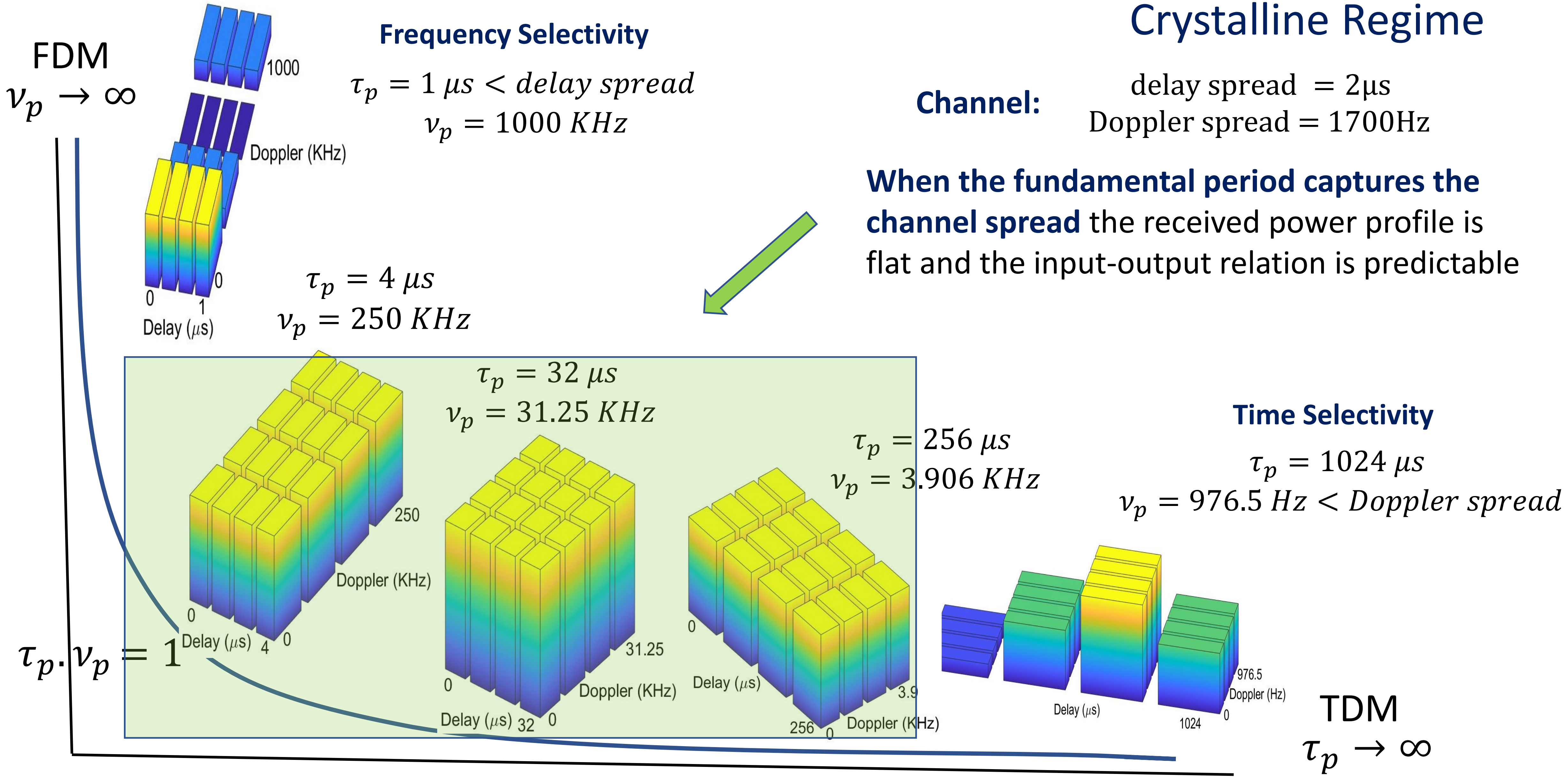
$$h(\tau,\nu)=\sum_{i=1}^{4} h_i\delta(\tau-\tau_i)\delta(\nu-\nu_i)$$

Twisted convolution: $Y(\tau,\nu)=h(\tau,\nu)*_\sigma X(\tau,\nu)$

Crystalline Regime

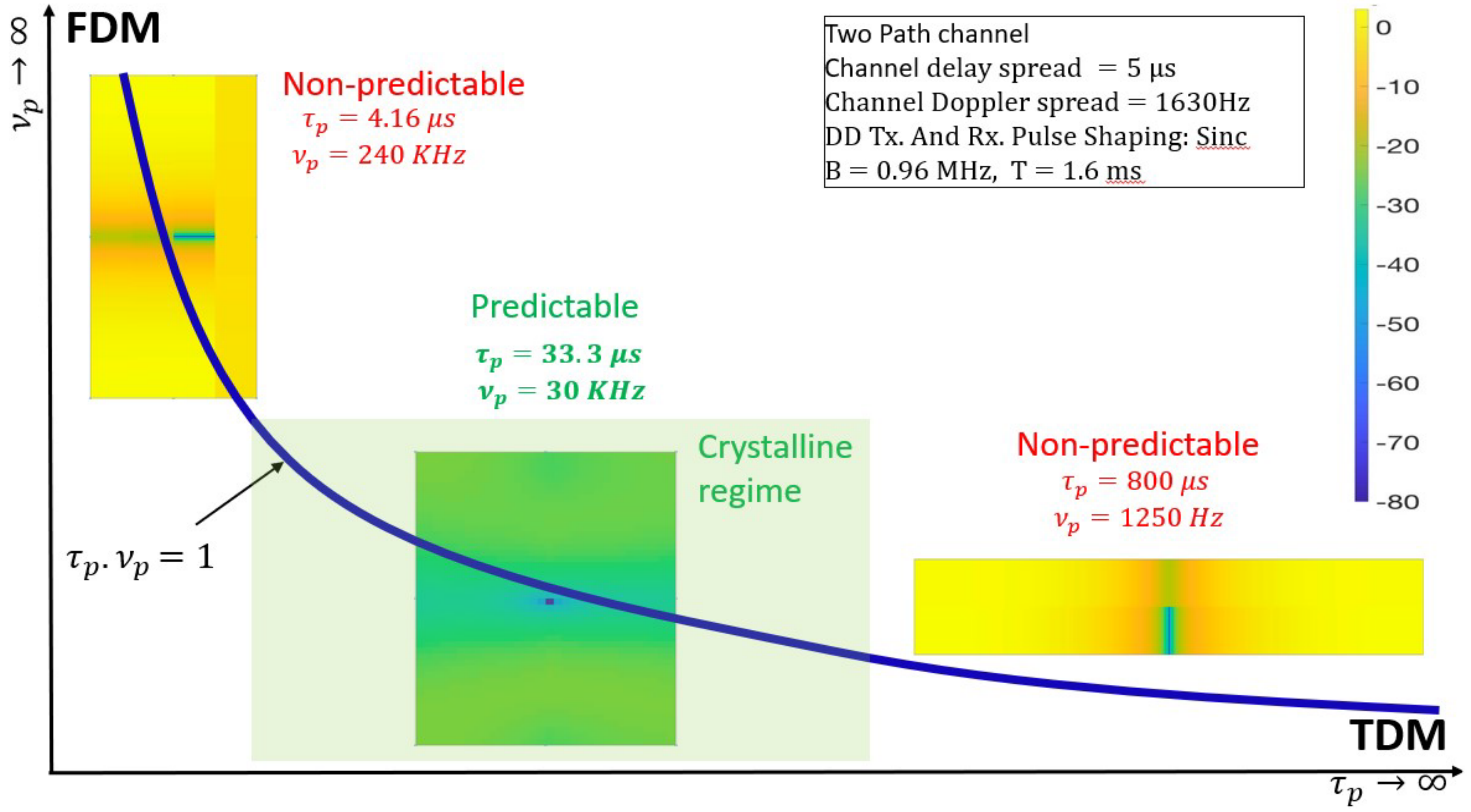
Channel: delay spread $= 2\mu s$, Doppler spread $= 1700 Hz$

When the fundamental period captures the channel spread the received power profile is flat and the input-output relation is predictable



Predictability of the I/O Relation

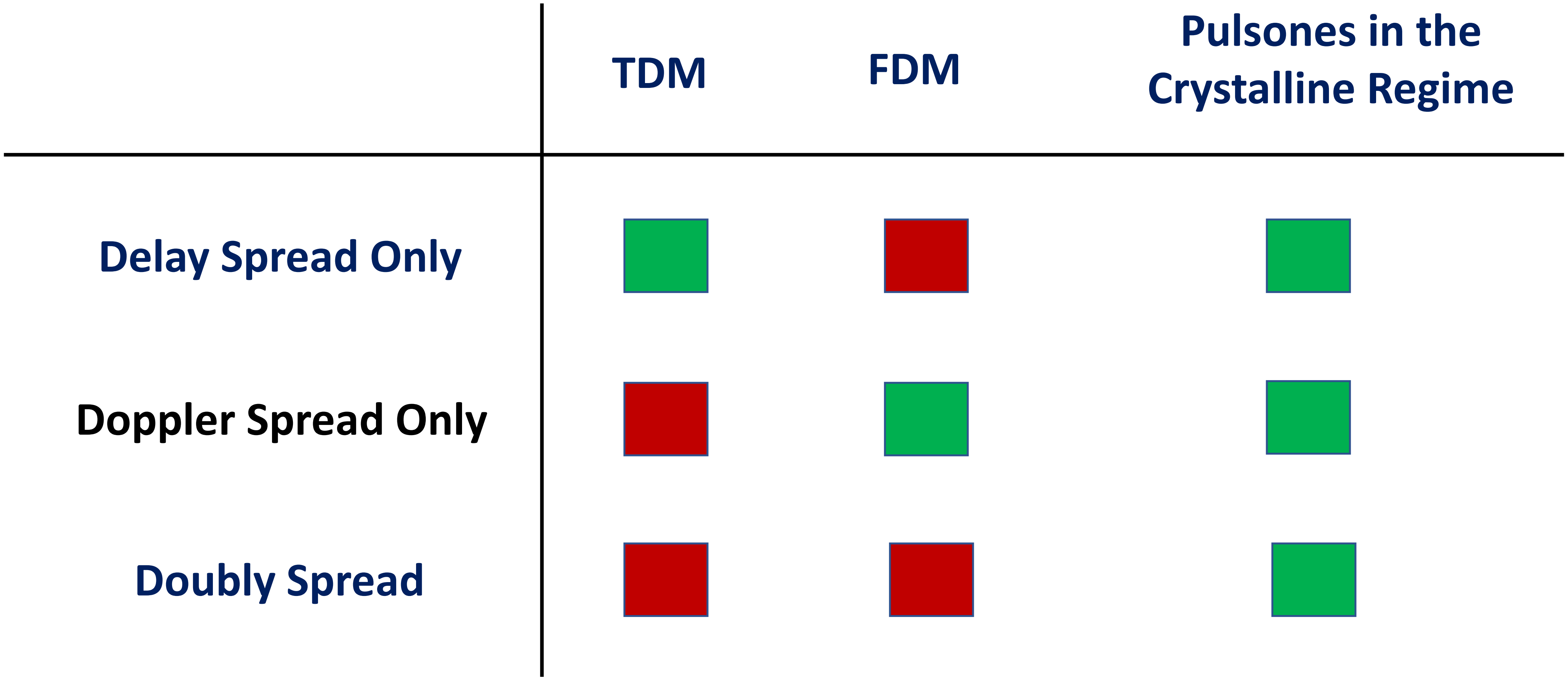
Relative Prediction Error (RPE) in dB, as a function of delay (horizontal axis) and Doppler (vertical axis) with sinc pulse-shaping filters



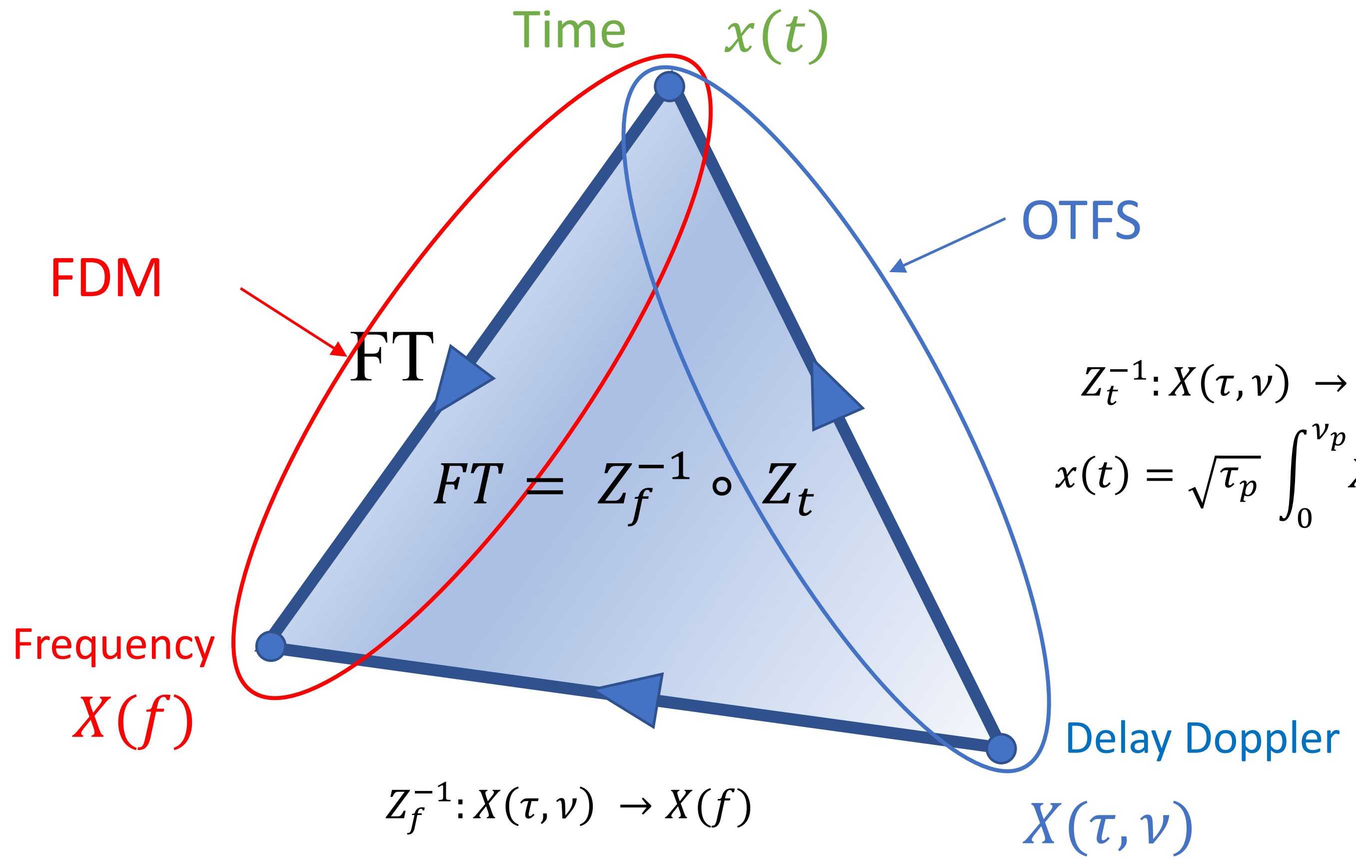
RPE minimized at the pulse location

RPE significantly smaller in the crystalline regime

Non-Fading and Predictable Operation



Signal Processing in the DD Domain



$$Z_t^{-1}: X(\tau,\nu) \rightarrow x(t)$$

$$x(t) = \sqrt{\tau_p} \int_0^{\nu_p} X(t,\nu) d\nu$$

$$Z_f^{-1}: X(\tau,\nu) \rightarrow X(f)$$

$$\mathrm{X}(f) = \sqrt{\nu_p} \int_0^{\tau_p} X(\tau,f) e^{-j2\pi f \tau} d\tau$$

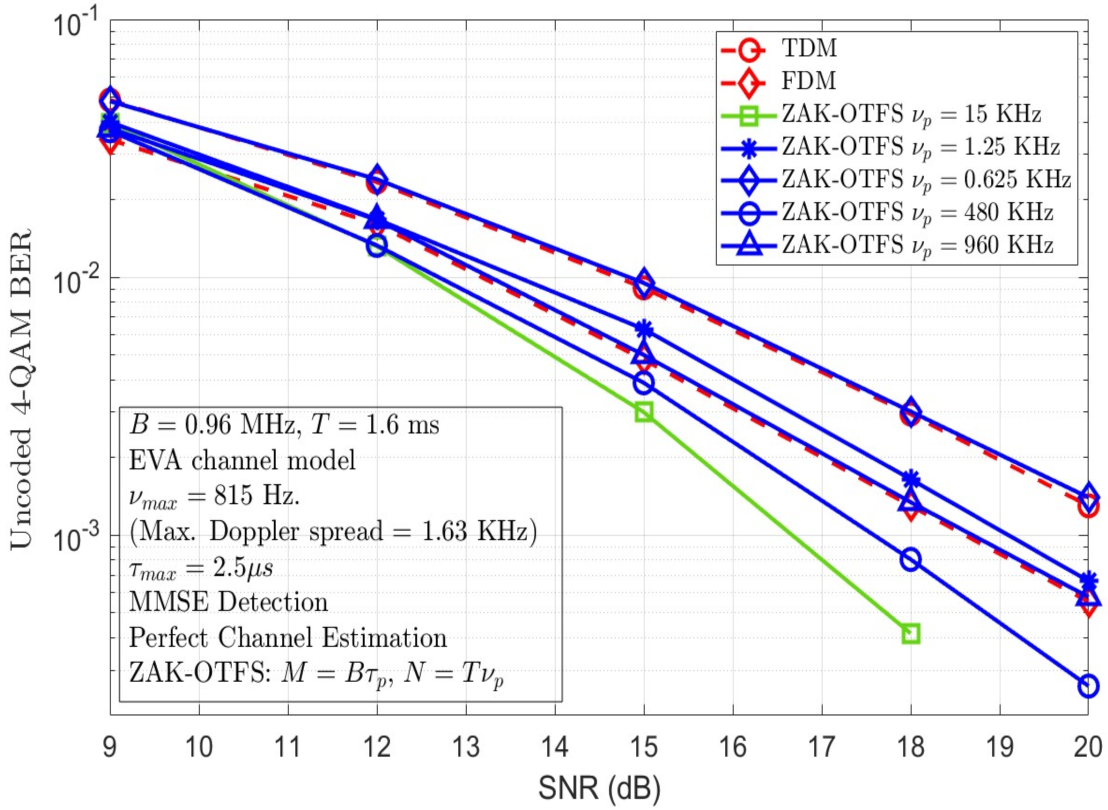
The Fourier Transform as a Composition:

First apply Z_t from TD to DD domain, then apply Z_f^{-1} from the DD domain to the FD

Not more complicated than the Fourier Transform

Impact of Fading in the Crystalline Regime

Perfect Channel Estimation: VehA Channel Model



Summary

Performance in the crystalline regime is superior

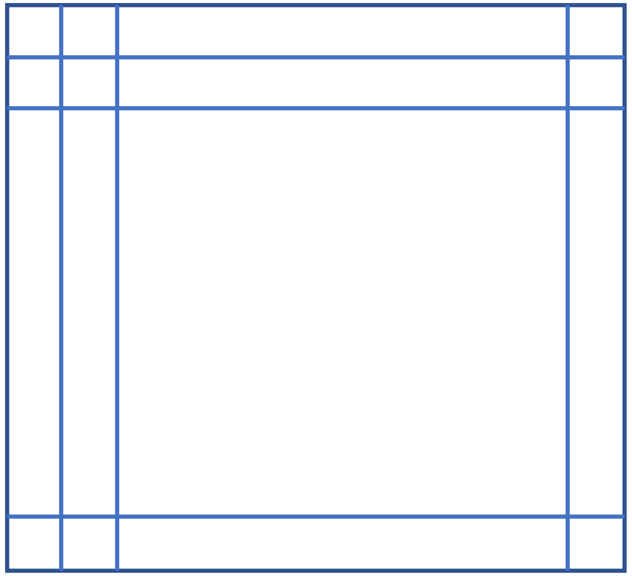
Performance approaches TDM as the delay period $\tau_p \to \infty$

Performance approaches FDM as the Doppler period $\nu_p \to \infty$

Predictable means Learnable

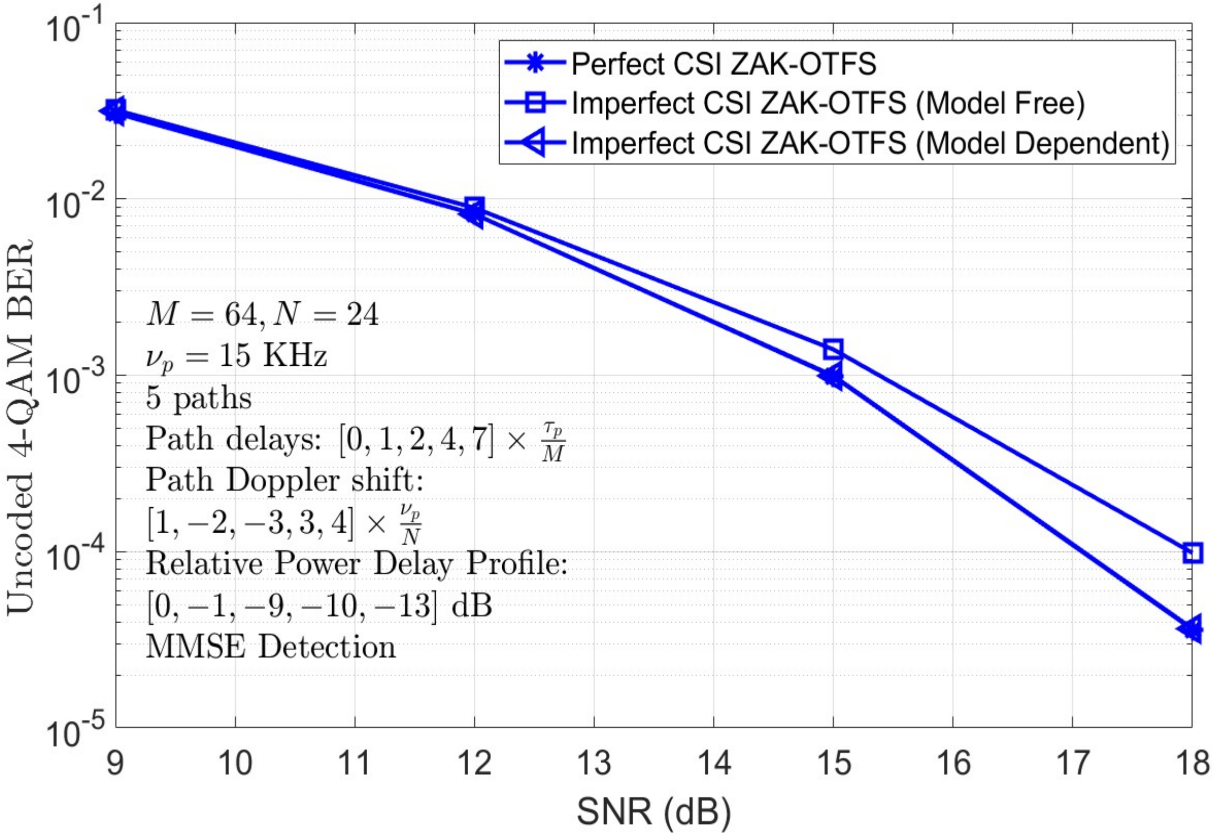
Doubly Spread Channels are becoming infinitely complicated

Input-Output Relations can be comparatively simple



Model-Free Operation: It is possible to use pulsones to learn the input-output relation directly without learning the channel

Model-Free vs Model-Dependent

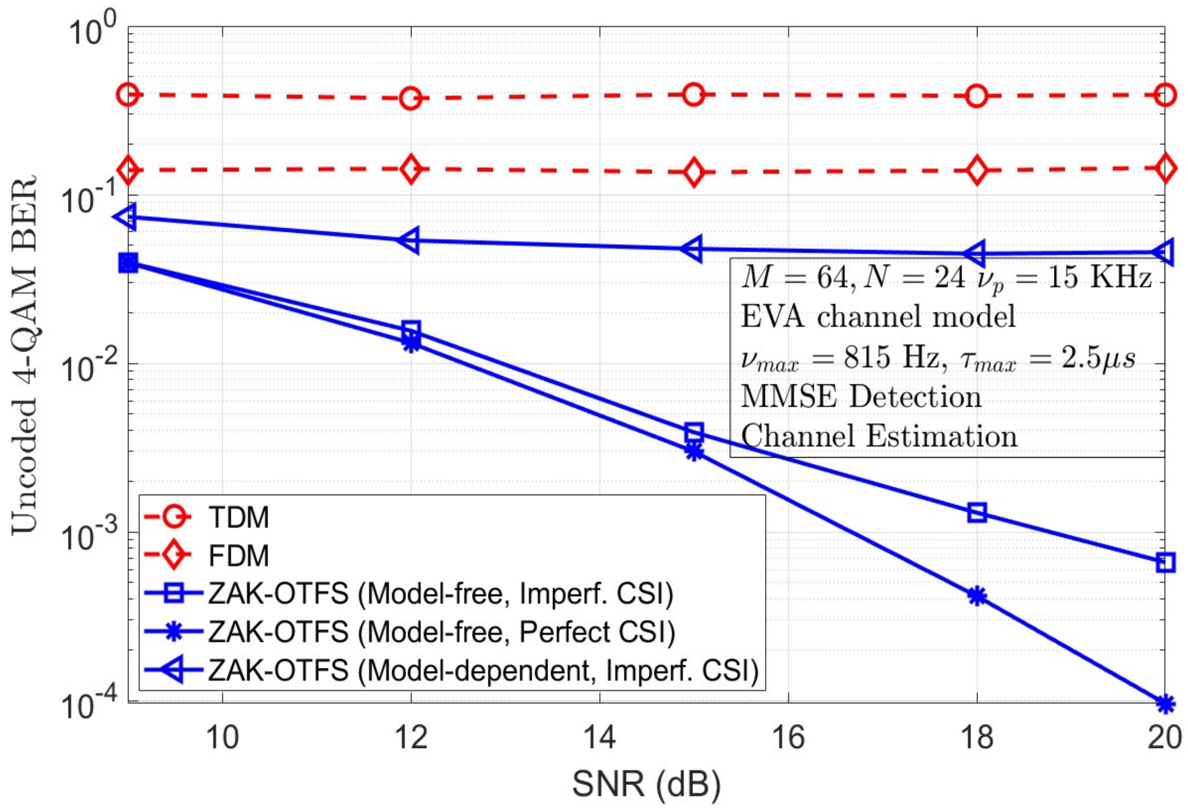


In the crystalline regime, when it is possible to learn the channel:

Model-Dependent pulsone performance coincides with ideal performance

Model-Free pulsone performance is only slightly inferior

Model-Free vs Model-Dependent

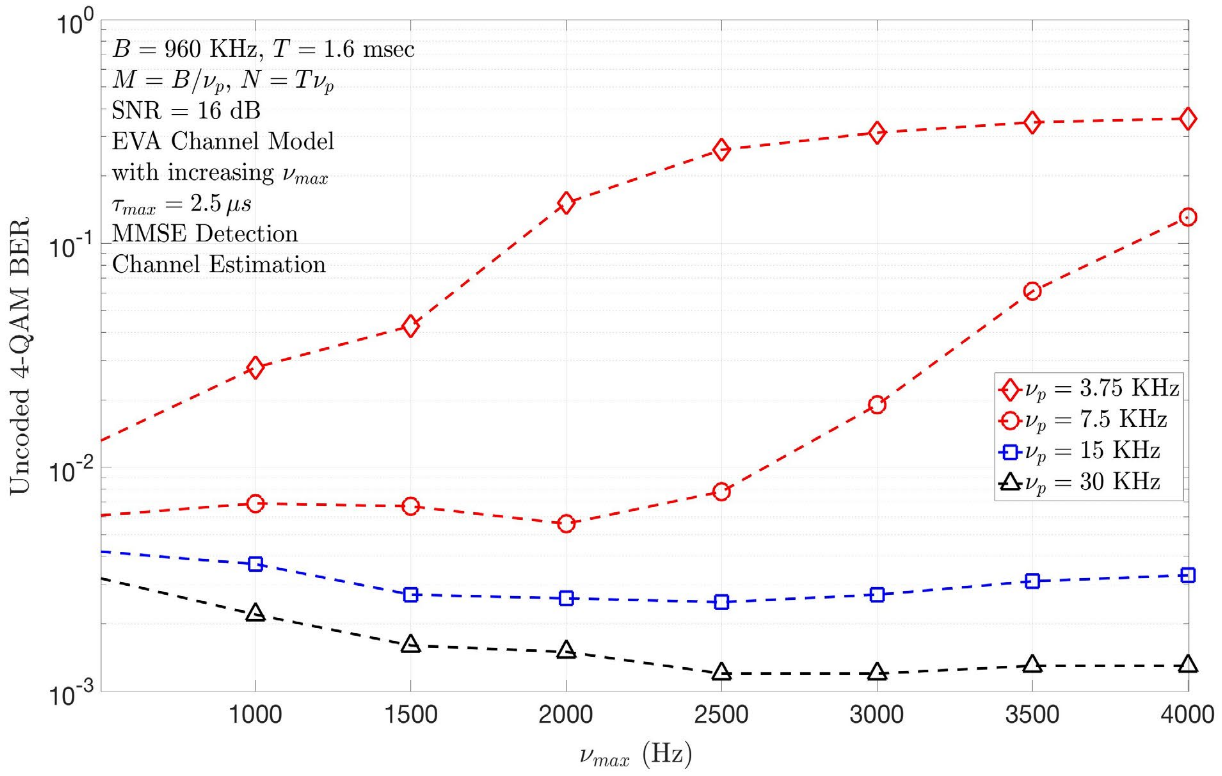


When it is not possible to learn the channel:

Pulsones support model-free operation in the crystalline regime

Not shown: Improvements in filtering – root raised cosine vs. sinc – extend the region of reliable operation

Pushing the Envelope – Impact of High Doppler



Model-Free operation in the crystalline regime:

When the Doppler spread $2\nu_{max}$ is bounded away from ν_p then pulsone performance does not degrade as ν_{max} increases

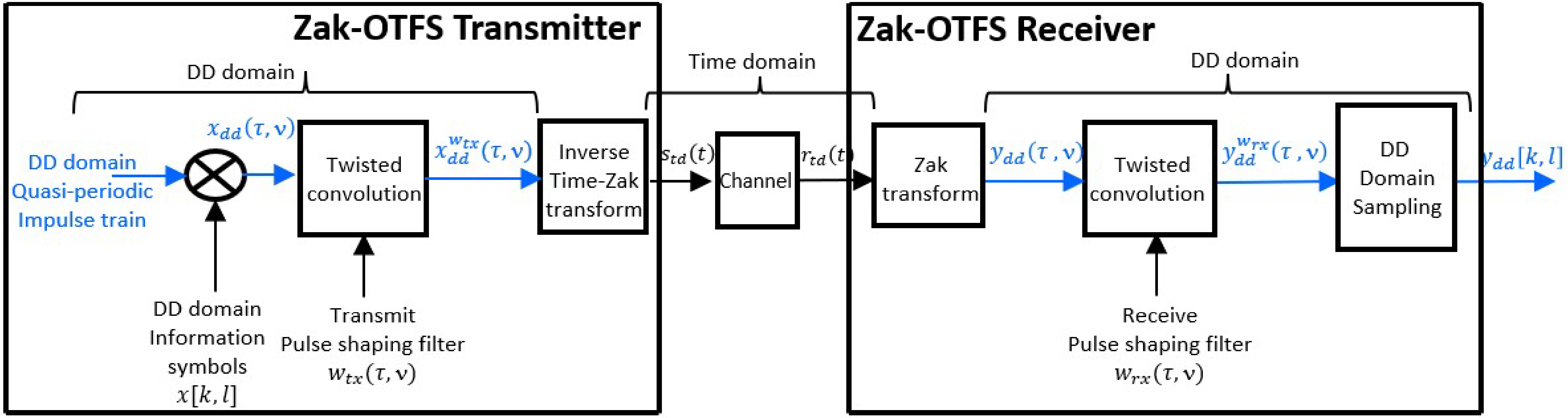
When the Doppler spread $2\nu_{max}$ is close to ν_p then performance degrades because of Doppler domain aliasing

Navigating Orders of Magnitude in Doppler Spread

$$\tau_p = 32\ \mu s$$
$$\nu_p = 31.2\ KHz$$

	Delay Spread (μs)	**Doppler Spread (KHz)**
Leo-Satellite Channel	0.8	82
UAV/Aeronautical Channel (GHz)	7.0 (Take Off) 33-60 (En-Route)	7.68 (En-Route)
mmWave Mobile Channel (28GHz)	1.0	3.0
Terrestrial Mobile Channel (GHz)	5.0	0.3
Terrestrial Pedestrian Channel (GHz)	0.41	0.005

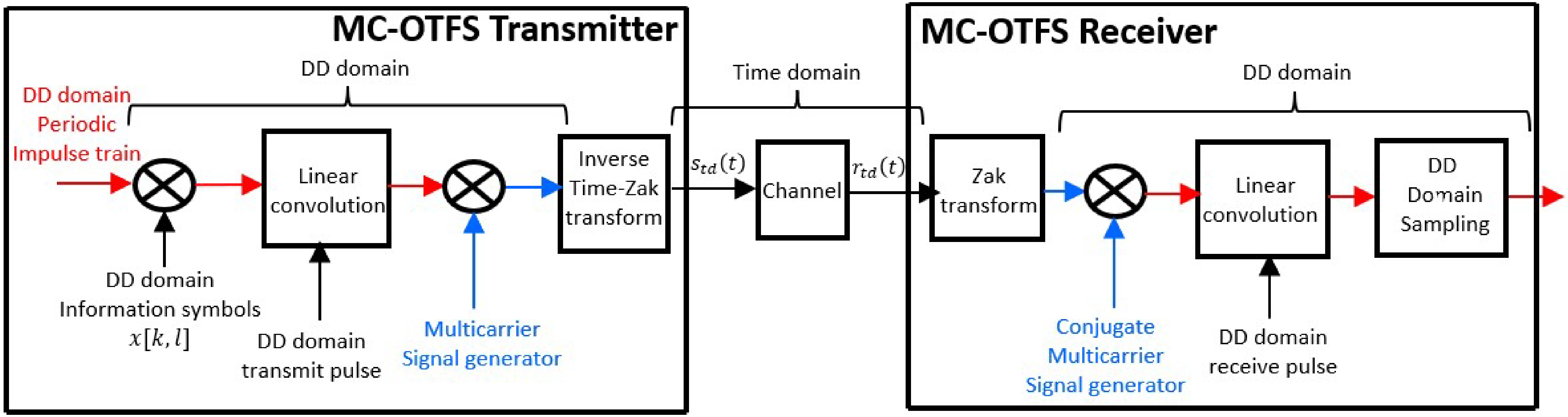
Signal Processing in Zak-OTFS



Zak-OTFS I/O Relation

$$y_{dd}^{w_{rx}}(\tau,\nu) = w_{rx}(\tau,\nu) \ast_\sigma h(\tau,\nu) \ast_\sigma w_{tx}(\tau,\nu) \ast_\sigma x_{dd}(\tau,\nu) = h_{dd}(\tau,\nu) \ast_\sigma x_{dd}(\tau,\nu)$$

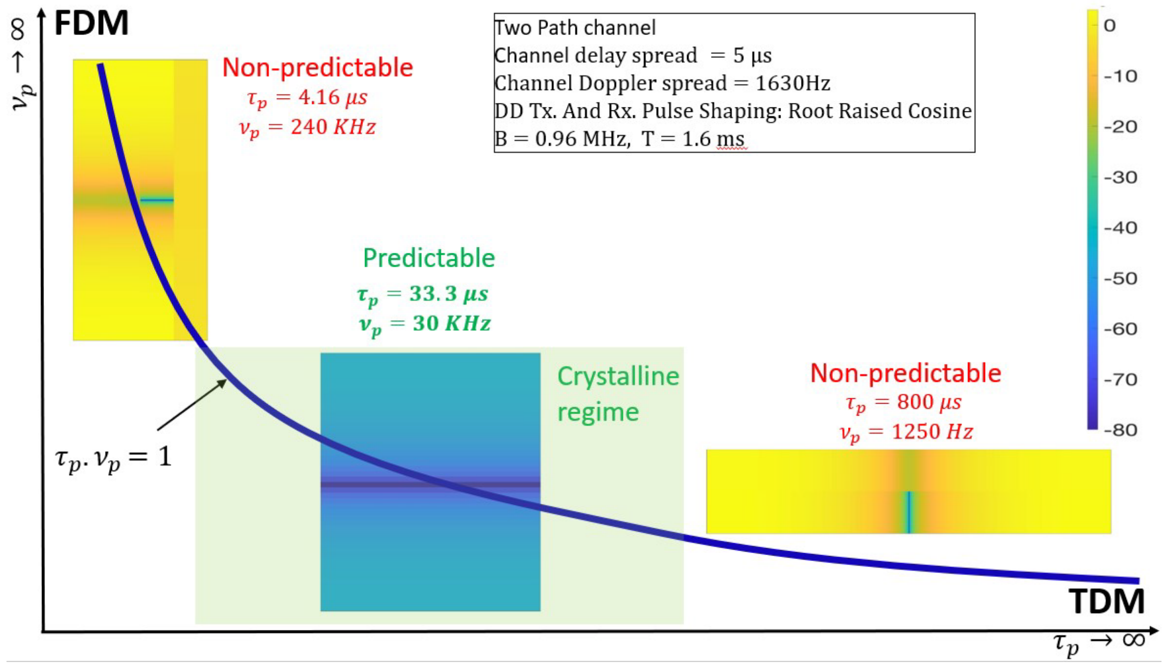
Signal Processing in MC-OTFS



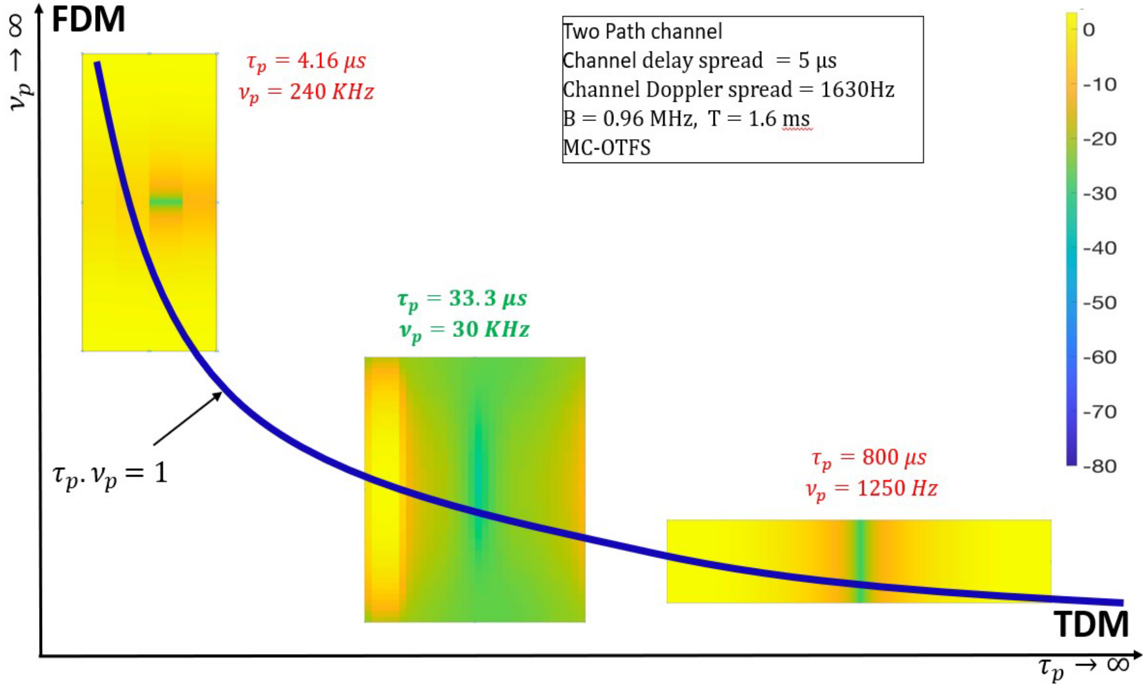
MC-OTFS I/O Relation

$$y^{w_{rx}}(\tau,\nu) = w_{rx}(\tau,\nu) \star \left[G_{dd}^{*}(\tau,\nu) \cdot \left(h(\tau,\nu) *_{\sigma} \{G_{dd}(\tau,\nu) \cdot [w_{tx}(\tau,\nu) \star x(\tau,\nu)]\}\right)\right]$$

Zak-OTFS vs MC-OTFS Predictability of the I/O Relation



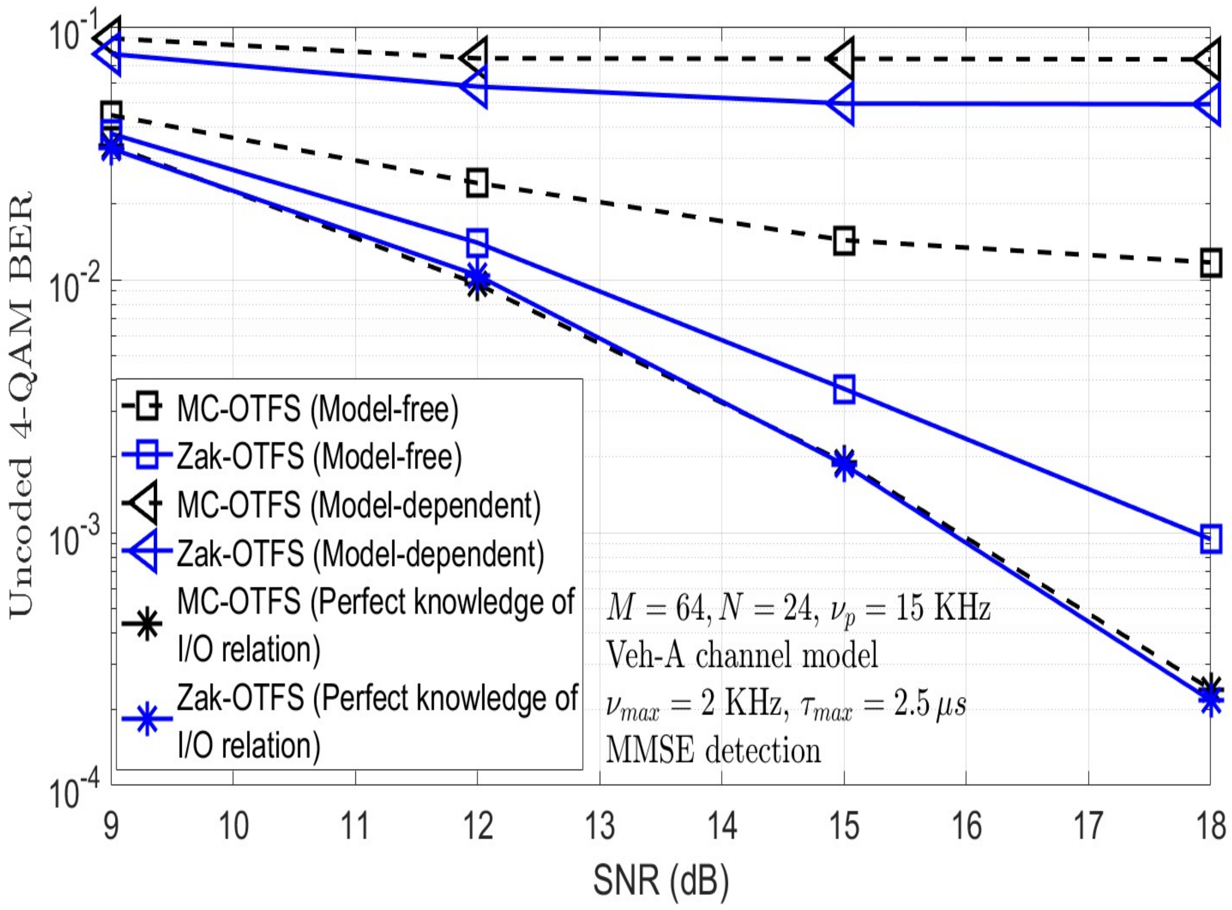
Zak-OTFS



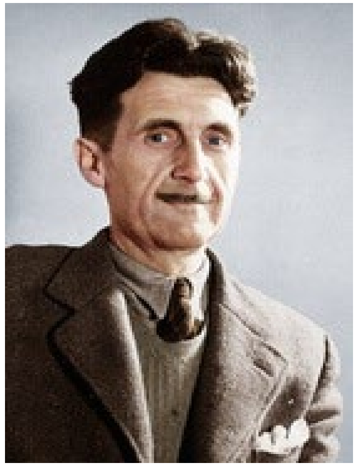
MC-OTFS

Zak-OTFS vs MC-OTFS in the Crystalline Regime

When it is not possible to learn the channel



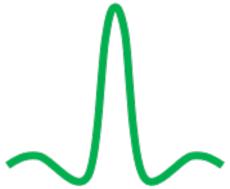
Performance of model-free Zak-OTFS is superior to that of MC-OTFS because the Zak-OTFS I/O relation is more predictable



George Orwell: *Every generation imagines itself to be more intelligent than the one that went before it, and wiser than the one that comes after it.*

2G
TDMA

Voice



Localization in Time

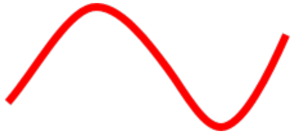
3G
CDMA

Voice +Data

Localization in Code

4G
OFDM

High Speed Data



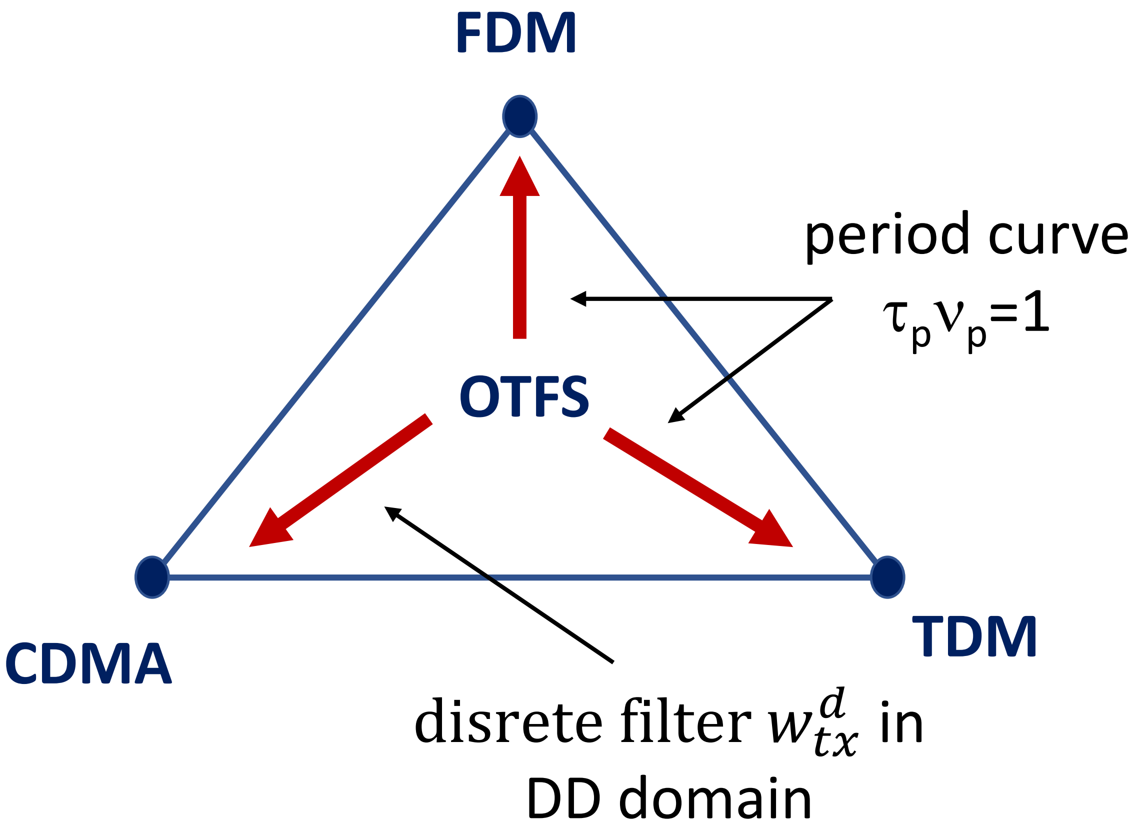
Localization in Frequency

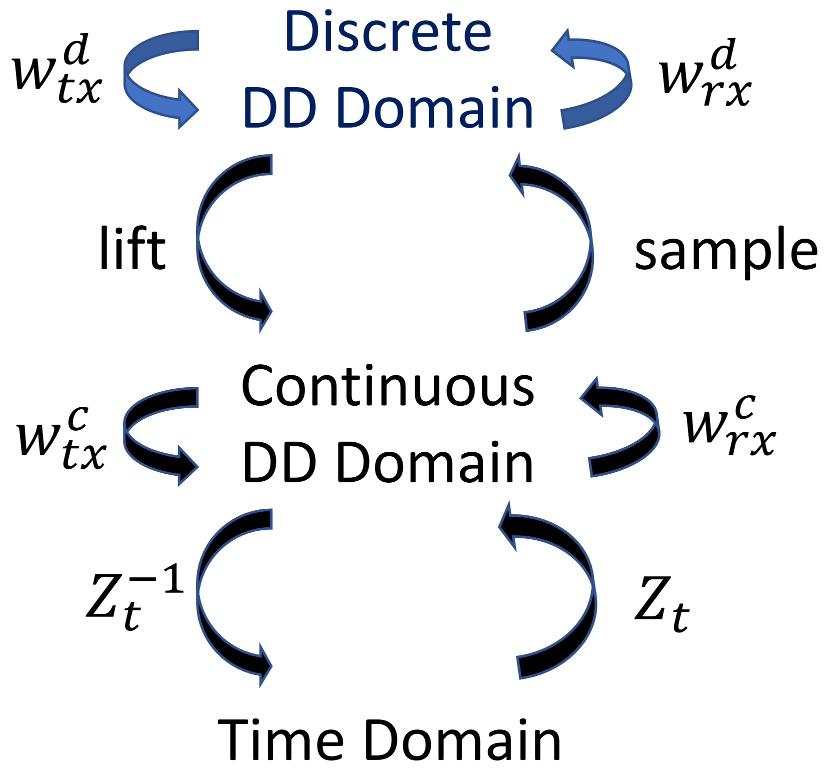
5G & Beyond
OTFS

Massive IoT
Extreme Mobility
Extreme Broadband

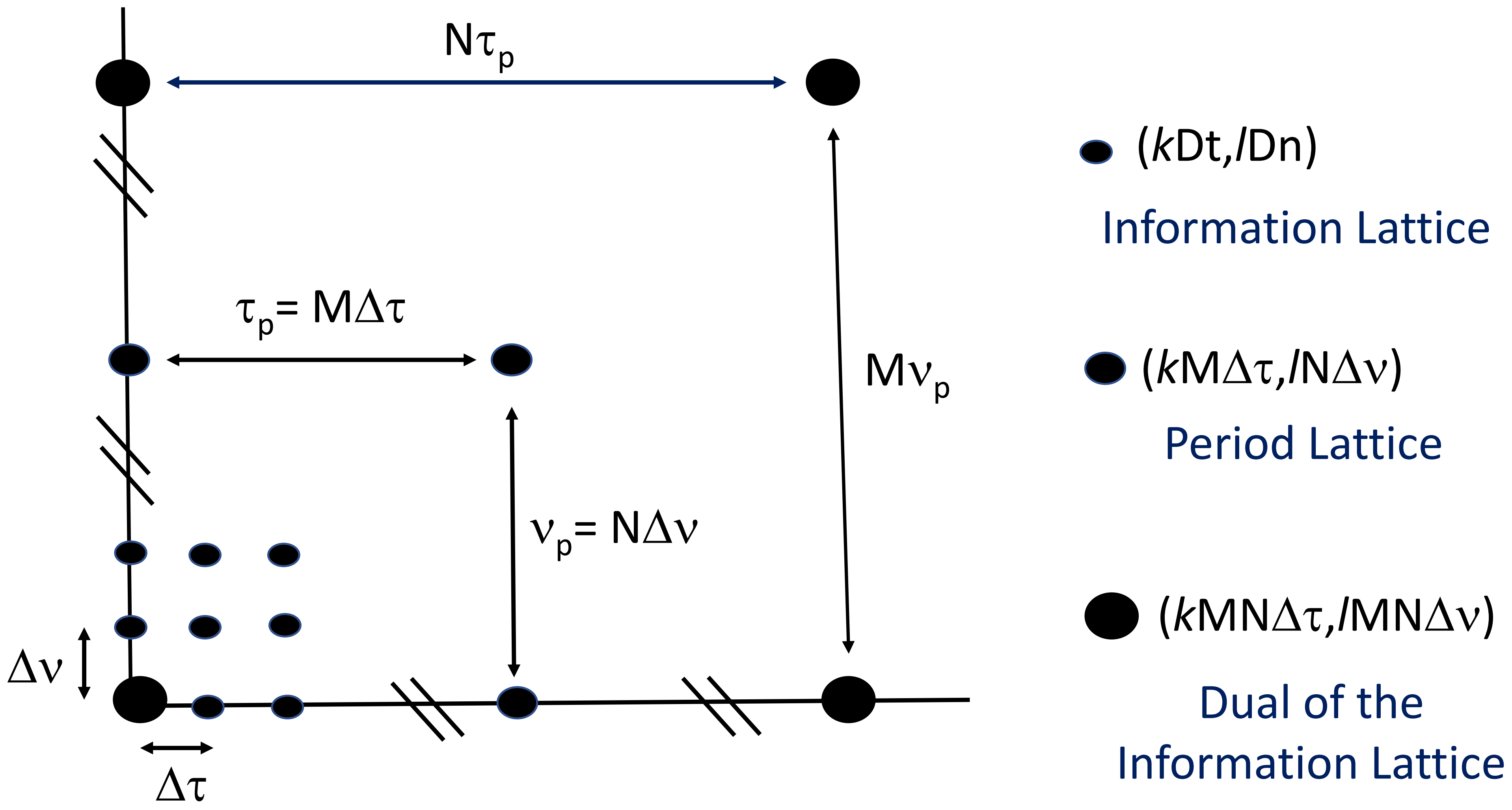
Localization in Delay / Doppler

Reconnecting with CDMA

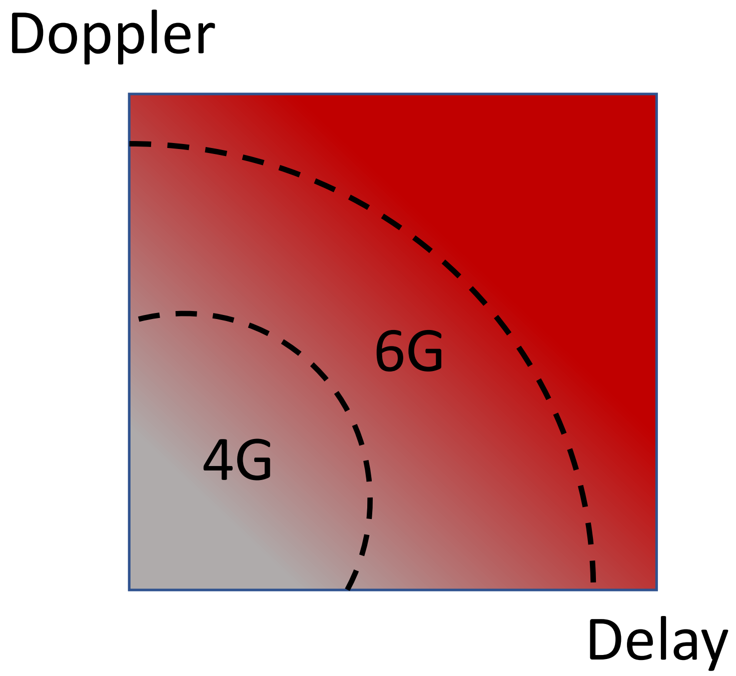




The Mathematics of Hopping and Spreading



Conclusions



Bad News: It is becoming impossible to learn channels

Good News: It is still very possible to learn input-output relations

In the crystalline regime: Pulsones enable model-free operation, opening the door to machine learning

What makes this possible? We are using the operators that define doubly spread channels both to probe the channel, and to transmit information

Live here, You must

Pulsones in the Crystalline Regime