

Vector Network Analyzer Fundamentals and the High-Frequency Characteristics of Passive Circuit Elements

Laboratory 01 Manual

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1 Laboratory Objective

In this laboratory you will learn some of the basic operation details of Vector Network Analyzers (VNA) and the behavior of Capacitors, Inductors, and Resistors at high frequencies. Although it is assumed that you have familiarity with simple circuits and how to use impedances, some review material is provided below for completeness and convenience. If more assistance is needed, please make sure to consult a suitable circuits book¹.

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report should have each of its sections and subsections numbered according to this laboratory manual, and be a detailed document with all your measurement results, calculations, conclusions, drawings, plots, and relevant photos of all constructed components (to showcase your very important high-frequency craftsmanship).

Note that, to maximize the learning experience, the laboratory has been designed to be carried out individually, hence each person in the class received their own individual lab kits. Consequently, the experiment and the corresponding report has to be done completely individually.

2 VNA Initial Setup and Power Up

Basically, a VNA (Vector Network Analyzer) is an instrument for measuring complex impedances at high frequencies (i.e., real and imaginary parts, or amplitude and phase, of impedances). Therefore the use of the vector nomenclature, as opposed to referring to it as a scalar instrument (a scalar instrument would only measure amplitudes, for example). Currently there are commercially available VNAs capable of precisely measuring impedances at Terahertz frequencies (recall that $1 \text{ THz} = 1000 \text{ GHz} = 1,000,000 \text{ MHz}$). You can think of a VNA as a multimeter for high frequencies, a metrology device that can be used when the Kirchhoff's circuit laws may no longer be valid and the wave nature of electromagnetic phenomena needs to be considered.

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¹See for example A. J. Rosa and R. E. Thomas *The Analysis and Design of Linear Circuits*, John Wiley & Sons, Inc.

The VNA that we will be using in our experiments is called NanoVNA-F (Deep-elec Firmware 1.0.3). Due to its open source nature, there are numerous NanoVNA implementations currently available on the market². The NanoVNA-F is one of the best NanoVNA's implementations since it employs a metal enclosure, a relatively large 4.3 in display, shielded test ports, and other desirable improvements. All NanoVNAs perform their measurements tasks using miniature Wheatstone bridges excited by the Silicon Labs one-chip Si5351 CMOS frequency synthesizer. Although the NanoVNA-F only operates between 10 KHz and 2.7 GHz, it behaves in exactly in the same way as its much more sophisticated (and expensive) cousins, albeit with a little less accuracy.

In the NanoVNA-F the Si5351 frequency synthesizer is pushed to operate as high as 300 MHz. Although this is well beyond the 200 MHz manufacturer maximum recommended frequency, due to the unavoidable Silicon foundry variability the Si5351 usually can handle the overclocking without significant difficulties. To reach beyond 300 MHz, the NanoVNA-F relies on the progressive higher orders harmonics present in the of Si5351 near square wave output (e.g., 2.7 GHz is reached using the 9th harmonic, as $9 \times 300 = 2,700$). Note however that usually the higher the harmonic the lower its associated power, and hence the performance of the NanoVNA-F tends to degrade progressively at higher frequencies. Also, although a square wave only has odd harmonics, because the Si5251 output is a highly distorted square wave, even harmonics are also present and usable.

Documentation about the NanoVNA-F operation can be found in the attached manual, which is the document labeled

`NanoVNA-F_Product_Manual_v2.1.pdf` .

The electronic circuit of the NanoVNA can be found in another attached document, which is labeled

`NanoVNA-schematic.pdf` .

By the way, are you able to locate the two Wheatstone bridges, as well as the two SMA coaxial connectors of the NanoVNA-F, in the above schematic? SMA is the name of the gold-plated coaxial connector used on the two NanoVNA test ports (labeled PORT 1 and PORT 2); the name stands for *SubMiniature version A*.

It is recommended that you take an initial look at the above two documents. However, note that the best way to familiarize yourself with the operation and capabilities of VNAs is by using it in experiments and correlating the obtained results with the theoretical material that you have been learning in class. So please don't feel intimidated by the above hardware and documents as we will be discussing the basic operation of the NanoVNA-F in our laboratory sessions.

²The NanoVNA was created around 2016 by the Japanese amateur radio enthusiast Tomohiro Takahashi. To our gratitude he developed the original VNA hardware and software, created a kit for the original NanoVNA, and opensourced everything. By doing so he made affordably available to all of us what was previously a class of very expensive instruments; you can find some of his original material at <https://github.com/ttrftech/NanoVNA> .

For now please locate the provided USB cable, connect its USB-C end to the NanoVNA-F, and the USB-A end to your computer port, so that the NanoVNA-F battery gets charged (please do not remove the screen protector film). You are also welcome to power the NanoVNA-F on and familiarize yourself with its menu structure. Note that the battery charge status is indicated by the green rectangular symbol on the top left of the screen. The menu structure is detailed in a paper that should have come with the instrument, and is also included attached as the document labeled

NanoVNA-F_Quick_start_guide-Firmware_1.0.3.pdf .

However, please make sure to not change any of the existing NanoVNA-F settings at this point.

The display of the NanoVNA-F is quite complex and shows several colored labels, plots, and values (see Fig. 1). We will learn more details about the display as we progress. But for now please note the frequency associated with Mark 1 (the upside down triangular marks with the number 1 in their interior), displayed in white on the top right of the screen (the value shown in the figure is M1: 50.000 kHz). Also note the two values (in white) provided in the line that starts with the green words “S11 SMITH.” These values are the resistive part of the measured complex impedance of the component connected to the Port 1 of the NanoVNA-F (-222 kΩ in the figure), at the frequency of the Mark 1, and the series capacitance or inductance associated with the measured impedance (278 mH in the figure). You will be using these two values extensively in this laboratory, which in the figure are associated with a very large 50 kHz impedance, since nothing is connected to Port 1 of the VNA (the readings are then just some very large spurious values).

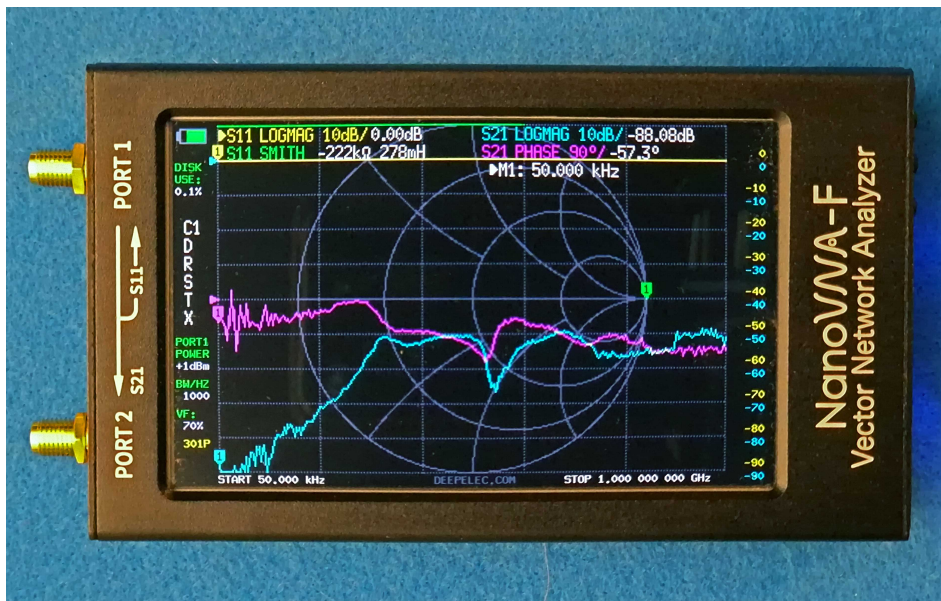


Figure 1: Display of the NanoVNA-F with nothing connected to Ports 1 and 2

3 VNA Calibration

Before a VNA can perform accurate measurements it needs to be calibrated. This is done by connecting well-characterized standards to the VNA measurement ports and letting the VNA determine and store its own inherent measurement errors. Once this is done the VNA will be ready to perform very accurate measurements at any frequency range of your choice, by mathematically correcting the determined measurement errors. In our case the calibration standards, stated in the order that will be used by the NanoVNA, will be an open circuit, a short circuit, a $50\ \Omega$ load, and a through connection (referred to as open, short, load, and “thru” in the NanoVNA menu, respectively)³.

Figure 2 depicts the calibration standards that we will be using in our laboratories. When handling and using the calibration standards always keep in mind that they are precision parts and hence must be treated with great care and respect (e.g., dropping the standards on a hard floor can damage the SMA connector threads). Always keep in mind that the accuracy of all your measurements are ultimately limited by the accuracy and physical condition of the calibration standards. The SMA male standards come with the NanoVNA-F and can be found inside its plastic box. The SMA female standards were specially made for our laboratory and can be found inside the provided bag of components. For the thru standard we will be using a coaxial cable to connect the Port 1 to Port 2 of the NanoVNA-F (you can find two suitable 210 mm long RG-316 coaxial cables inside the NanoVNA plastic box).



Figure 2: SMA male and female calibration standards (left and right, respectively)

Please take some time to carefully familiarize yourself with the details of the calibration standards. Note the two short standards, which simply connect the inner and outer conductors of the coaxial cable with flat short circuits (the flat short circuits provide a connection with negligible inductance). Then note the SMA male shielded open standards, which simply leave the center conductor of the coaxial cable unconnected while simultaneously providing a shield around it (the shield prevents undesirable electromagnetic radiation). Since the frequencies used in our laboratory are not excessively high, the SMA female open standard has been left only partially shielded by the four studs, which suffices. And finally note the $50\ \Omega$ termination (or load) standards. For protection the SMA male load is completely enclosed by a metal can, and hence unfortunately you can't see its $50\ \Omega$ resistor. However, the provided SMA female load allows you to clearly see its two $100\ \Omega$ 0805 chip resistors connected in parallel between

³In VNA parlance a calibration that uses these four standards is routinely referred to as a SOLT calibration, for the initials of short, open, load, and through.

the coaxial center and outer conductors⁴. Note that chip resistors are used because the smaller they are the smaller their associated parasitic reactances are. Consequently the higher in frequency they can reliably operate according to circuit laws (i.e., Kirchhoff's voltage and current laws). The 0805 resistors provide a very pure 50 Ω resistance value well above 1 GHz. Furthermore, the 100 Ω 0805 chip resistors behave even better than the 50 Ω resistors at higher frequencies. Therefore, two 100 Ω resistors are then used in parallel to implement the needed 50 Ω calibration standard.

When using coaxial cable connectors (e.g., SMA connectors), always make sure to firmly hold on to the connector body and to rotate only its hexagonal nut to engage and tighten its threads. *The body of the connector should never be allowed to rotate*; the only part that should rotate is the connector nut. Rotating the connector body will cause the coaxial center pin to rotate on its engaging mechanism. This irreparably damages the engaging mechanism and also wears out its thin gold plating!

The recommended tightening torque for the SMA connector nut is 0.5 Nm, and calibrated torque wrenches are commonly available to assure this value. It is then good practice to always use a torque wrench to tighten connectors; this provides accurate electric contact repeatability without any possibility of connector damage. However, on all our laboratories we will instead always be firmly hand tightening the connectors (i.e, no wrench assistance will ever be used). It turns out that this suffices for our purposes and, no matter how hard you try, hand tightening always safely provides a torque smaller than 0.5 Nm.

We will calibrate the NanoVNA-F over a 50 KHz to 1 GHz sweep frequency range (this should be the START and STOP frequencies shown on the bottom of the screen), over a total of 301 frequency points. This is the number shown in the lower left of Fig 1 as 301P, and yields a $(1000 - 0.05)/300 = 3.333167$ MHz step size, To calibrate the NanoVNA-F please perform the steps below.

1. Start by deleting all previously stored calibrations and resetting the NanoVNA to its pristine state; *note that you will be doing this only at this time*. Access the main menu by tapping anywhere on the screen (you may want to use the provided stylus for this, but your finger may also work). From the main menu, select "CALIBRATE," then click on "CLEAR ALL," and follow the screen instructions to complete the reset process. The NanoVNA will reboot indicating a successful reset.
2. To calibrate the NanoVNA again invoke its main menu, select "CALIBRATE," and then click on the field "CALIBRATE" to access the appropriate second menu layer. Then connect the open standard to Port 1 and click on "OPEN." After this, sequentially follow the screen amber labels to calibrate the two SMA female ports of the NanoVNA-F using the remaining pieces included SMA male calibration kit. As you progress, observe that check marks appear on the left side of each step, indicating its completion. For the isolation step you leave the load of the previous step connected to Port 1 and connect the provided SMA-F load standard

⁴Chip resistors have different sizes and the number 0805 corresponds to the imperial dimensions of the 0805 series, in inches. The 0805 chip resistor has a 2 mm length and a 1.2 mm width (0.08 x 0.05 inches).

to Port 2 using the provided SMA-male-to-male adapter (SMAM-SMAM), and for the through step you simply connect the Ports 1 and 2 together using one of the two provided 210 mm long RG-316 coaxial cables. The last calibration step is to click on “DONE” and to save (or store) the calibration in the location 1 (i.e., click on “SAVE 1”).

Note the letter C immediately followed by the number 1, on the left side of the NanoVNA-F screen (it reads C1). This indicates that the NanoVNA is now using the calibration that was just stored in the memory position 1. A total of seven independent calibrations can be stored, and when invoked will show as C0, C1,...C6 on the screen. The position 0 usually comes with a factory calibration and hence we are going to leave it alone.

3. Verify that your SMA female port calibration is correct by confirming that it is yielding the expected Open, Short, Load, Isolation, and thru results. If this is not the case some mistake was made during the calibration and you will have to calibrate the NanoVNA-F again.

To recalibrate invoke the main menu, click on “CALIBRATE,” and then click on “RESET.” Note that the “RESET” will not erase any of the already stored calibrations, it just resets the NanoVNA calibration (note that the check mark on the left of the “APPLY” field is removed). However, *this is a important step and should never be forgotten; without it the calibration may not work properly.* After the recalibration is complete you should store it again on position 1. This will replace what was in there by the new calibration.

4. Connect the included SMA-male-to-SMA-male (SMAM-SMAM) adapter to the Port 1 of the NanoVNA-F, calibrate the resulting SMA male port of the NanoVNA-F using the included SMA female calibration kit, and store the calibration in the location 2 (i.e., click on “SAVE 2”). For the isolation step you leave the load of the previous step connected to Port 1 and connect the provided SMA-male load standard to Port 2, and for the through step you simply connect the Ports 1 and 2 together using one of the two provided 210 mm long RG-316 coaxial cables and the included SMA-female-to-SMA-female (SMAF-SMAF) adapter.
5. Verify that your SMA male port calibration is correct by confirming that it is yielding the expected Open, Short, Load, and Isolation results. If this is not the case you will have to recalibrate the NanoVNA-F.

After you perform a calibration, and start using it, the VNA will report its measured results at the corresponding calibration plane (i.e., the physical location of the short circuit used). It is then very important that you identify a desirable calibration plane and proceed accordingly (e.g., if you need measurements at the end of a cable, the calibration should be done on the connector located at the end of the cable).

The VNA uses the calibration procedure to determine the very precise corrections that are numerically applied to its measured results, to remove the deleterious effects of its own internal imperfections, as well as effects of any external connecting cables, connectors, adapters, etc. However, because of unavoidable temperature variations and

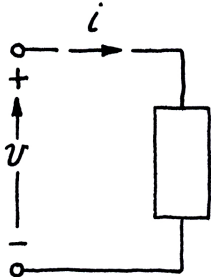
other effects, the stored calibrations have a tendency to progressively degrade with time. This is particularly true when there are many components between the calibration plane and the VNA ports. It is then very important to verify the quality of any stored calibration before using it, and to recalibrate as necessary. But don't recalibrate every time you use the NanoVNA-F, and as a rule, since this is more often than not unnecessary and causes avoidable wear and tear on the SMA connectors. Having said all this, and partially because the NanoVNA-F operation frequencies are relatively low, experience indicates that the NanoVNA-F calibrations will hold their accuracy very well over at least several months.

You now have two stored calibrations, and they will be used frequently as we move forward. By the way, whenever required you can invoke any of the stored calibrations by tapping on the NanoVNA screen and selecting "RECALL." This will take you to a menu that indicates which calibration is currently being used (calibration with the check marker on its left side), and also allows you to tap and select for use any of the available calibrations.

4 Short review of Passive Circuit Elements (i.e., Capacitors, Inductors, and Resistors)

Circuit theory teaches us to use voltages and currents when dealing with circuits, and in this process we learn to regard voltages and currents as the parameters associated with energy transport. However, it turns out that energy is actually transported by the electromagnetic field, and from a basic general standpoint, capacitors are devices that ideally store energy only in the electric field, inductors are devices that ideally store energy only in the magnetic field, and resistors are devices that ideally only dissipate energy. Ideal inductors would then not store energy on the electric field, and also would not dissipate energy. Similarly ideal capacitors would then not store energy on the magnetic field, and also would not dissipate energy. And in the same way ideal resistors would not store any energy. However, electromagnetic theory shows that it is an unavoidable fact of life that there is no such thing as an ideal capacitor, inductor, or resistor; any practical capacitor, inductor, or resistor must unavoidably be regarded as combinations of capacitors, inductors, and resistors.

Maxwell's equations show that the arbitrary time-varying voltages and currents (i.e., v and i , respectively, see Fig. 3) on the terminals of ideal passive circuit elements are related to each other by



$$i = C \frac{dv}{dt}, \quad (1)$$

$$v = L \frac{di}{dt}, \quad (2)$$

$$v = R i, \quad (3)$$

Figure 3: Voltage and current on a device, arbitrary time variation

where C , L , and R and the inductance, capacitance, and resistance values of the ideal components, respectively (measured in Farads, Henries,

and Ohms, respectively). When using these equations it is important to always respect the polarity shown in Fig. 3, and just let the involved math correctly handle the signs. Using a capacitor as an example, for a voltage $v = -3$ V decreasing with time, Eq. 1 yields a negative i , indicating that i will actually be flowing to the left on the device top terminal, while v will actually be negative at the top terminal.

In time-harmonic regimen (i.e., cosinusoidal time dependence) it is easier to make v and i complex variables, and use instead the corresponding phasors V and I for them (note the judicious usage of lower and upper case variables), where $v = \mathcal{R}e\{Ve^{j\omega t}\}$ and $i = \mathcal{R}e\{Ie^{j\omega t}\}$, respectively, and ω is the angular frequency of the voltage and current (i.e., $\omega = 2\pi f$, where f is the frequency, in Hertz). In such case Eqs. 1 – 3 conveniently reduce to

$$V = \frac{1}{j\omega C} I = X_C I, \quad (4)$$

$$V = j\omega L I = X_L I, \quad (5)$$

$$V = R I, \quad (6)$$

where $j = \sqrt{-1}$ and $X_C = 1/(\omega C)$ and $X_L = \omega L$ are the reactances of the capacitor and inductor, respectively. With phasors it is also important to always respect the polarity

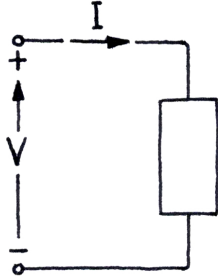


Figure 4: Voltage and current on a device, time-harmonic variation

shown in Fig. 4, and let the math handle the signs. Observe that V and I are in general complex variables, but for the polarity depicted in Fig. 4 it is assumed that V and I are real positive variables. As an example, if the voltage v is changing time-harmonically accordingly to $v = \cos(\omega t - \phi_V)$, where the phase ϕ_V is an arbitrary real constant, then $V = e^{-j\phi_V}$ and Eq. 4 yields $I = e^{-j\phi_V} j\omega C = \omega C e^{-j(\phi_V - \pi/2)}$, which corresponds to $i = \omega C \cos(\omega t - \phi_V + \pi/2)$.

Probably the best way to understand what is capacitance, inductance, and resistance is by recalling that the time-averaged energy stored in a capacitor and inductor, and the time-averaged energy dissipated in a resistor, can be shown to be given by⁵

$$W_e = \frac{1}{4} C V V^*, \quad (7)$$

$$W_m = \frac{1}{4} L I I^*, \quad (8)$$

$$W_d = \frac{1}{2} R I I^*, \quad (9)$$

respectively (the asterisks denote the complex conjugate operation). The capacitance C , inductance L , and resistance R are then parameters that depend only on the construction details of the device, and measure its ability to store energy in the electric field, store energy in the magnetic field, and dissipate energy, respectively.

⁵If you are interested in the derivation of these equations, please take a look at pages 139, 278, and 211 of D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co.

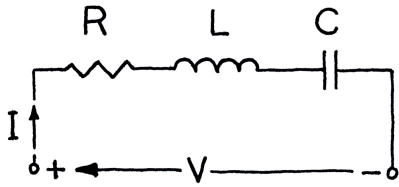


Figure 5: Series combination of ideal elements

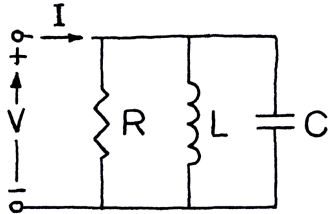


Figure 6: Parallel combination of ideal elements

In a practical circuit element, instead of the pure reactance or resistance provided by Eqs. 4–6, one always end up with a complex impedance $Z = V/I$ that corresponds to a combination of capacitors, inductors, and resistors connected together. As simple examples, one can have a purely series combination (see Fig. 5) that yields an impedance

$$Z = \frac{V}{I} = R + j\omega L + \frac{1}{j\omega C}, \quad (10)$$

or a purely parallel combination (see Fig. 6) that yields an admittance

$$Y = \frac{I}{V} = \frac{1}{R} + \frac{1}{j\omega L} + j\omega C, \quad (11)$$

but of course other alternatives, as well as more complicated combinations of ideal elements, are also possible, and frequently occur in practice. The particular combination that properly represents a practical element is established by the specific behavior of the impedance with frequency. For example, and as can be seen from Eqs. 10 and 11 with $\omega \rightarrow 0$, the circuit of Fig. 5 behaves as a capacitor at very low frequencies, while the circuit of Fig. 6 behaves as an inductor at very low frequencies.

Whatever combination of ideal elements properly represents an actual physical circuit component, resonance frequencies will always be present. At these important frequencies the energy stored in the electric and magnetic fields are equal, and alternate between each other. This is a general effect, present in other natural phenomena, it is not a particular property of electromagnetic fields. For example, a clock pendulum oscillates at its resonance frequency, and its total stored energy (called Hamiltonian) alternates between potential and kinetic energies.

Returning to the equivalent circuit of a practical device, the particular frequencies where the reactance part of either Z or Y goes to zero correspond to the frequencies in which the time-averaged energy stored in the electric and magnetic fields are equal. These are the resonance frequencies of the device. In either Eqs. 10 or 11 there is only one resonance frequency. Eq. 10 has a series resonance and Eq. 11 has a parallel resonance, and both occur at the frequency $\omega_0 = 2\pi f_0$, given by the condition

$$\frac{1}{\omega_0 C} = \omega_0 L, \quad (12)$$

which yields

$$f_0 = \frac{1}{2\pi\sqrt{LC}}. \quad (13)$$

Series and parallel resonances can be easily identified by observing the behavior of the impedance just before and after the resonance frequency. On a series resonance the impedance is capacitive below the resonance frequency and becomes inductive above

the resonance frequency. Conversely, on a parallel resonance the impedance is inductive below the resonance frequency and becomes capacitive above the resonance frequency. Note also that the magnitude of the impedance reaches a minimum and a maximum at the series and parallel resonances, respectively, and is purely resistive at f_0 .

In this laboratory we will use the above circuit concepts and resonance frequencies to understand and determine the equivalent circuits of some practical (i.e., non ideal) devices. And in several future laboratories we will continue to successfully deal with resonance frequencies, even though the associated circuit concepts may no longer be as clear and simple. However, and regardless of the situation at hand,

electric and magnetic fields always store energy, and a resonance occurs at the time-harmonic frequency where the energies stored in the electric and magnetic fields are equal.

This is a useful universal characteristic of electromagnetic fields.

5 SMA-SMA Test Fixture Stray Parameters Measurement

We will now proceed to understand and measure the residual (also called stray or parasitic) impedances associated with the SMA-DIP8-SMA test fixture that we will be using in our future measurements, and from this develop a convenient lumped element equivalent circuit for the test fixture. This circuit is needed for us to be able to properly account for the undesirable effects of these parasitics.

1. Locate the provided two SMA-DIP8-SMA test fixtures (see Fig. 7) and use one of the two provided SMAM-SMAM adapters to connect the test fixture to the Port 1 of the NanoVNA. DIP8 stands for the *Dual Inline Package* 8-pin integrated circuit (IC) socket to which the two SMA female printed-circuit board connectors are soldered.

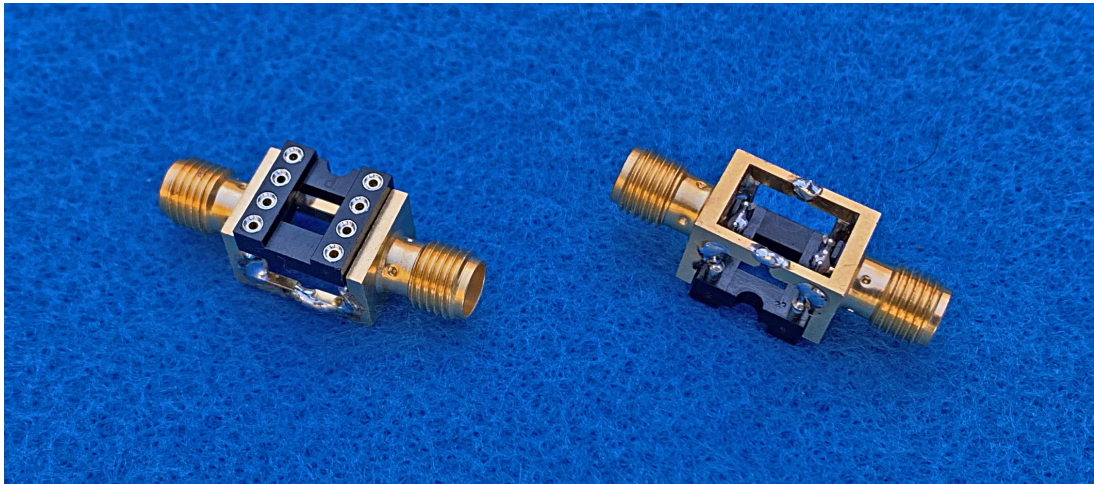


Figure 7: SMA-DIP8-SMA test fixtures

Note that the test fixture allows you to conveniently connect components to the SMA connector by inserting their leads into the pins of DIP 8-pin IC socket.

Please take a good look at the soldered connections underneath the DIP-8 IC socket and make sure you understand them; the four corner pins of the DIP-8 IC socket are connected to the coaxial cables' outer conductors, and the remaining inner four pins are connected to the center conductors of the two coaxial cables of each side.

You have two test fixtures, and each has two coaxial ports. Although the electrical circuits of each port are identical, the associated workmanship isn't. Because of this you can expect some variation between the measure strays of each of the four ports. You are welcome to measure the strays of each port and hence see how much variability is actually present. However, in what follow you are welcome to select and concentrate in just one port, and subsequently assume that whatever variability is there can be neglected.

2. Since the test fixture has SMA female connectors, make sure that the VNA is using the calibration 2 (i.e., shows C2 on the left of the VNA startup screen). Otherwise your results will be incorrect.

Note that the location of the short circuit plane of your calibration 2 allows the VNA to correct for practically all the deleterious effects of the test fixture SMA connector, but does not correct for any of the deleterious effects of the DIP-8 IC socket.

3. Measure the residual (or stray, or parasitic) capacitance C_s of the test fixture (the s subscript refers to the word stray), and report it in pF, in the 50 KHz – 1 GHz range, using 200 MHz increments (use for this a table format, with frequency in the left most column and C_s on the adjacent right column).

Please note that, because the residual capacitance C_s is very small (very high reactance X_C at low frequencies), of the order of 1 pF, the NanoVNA-F will be unable to accurately measure C_s down to 50 KHz. A 1 pF capacitance has a reactance $X_C = 3.2 \text{ M}\Omega$ at 50 kHz, which is simply excessive to allow for a sufficiently large current to flow on the NanoVNA-F measurement bridge. I then suggest that from now on you consider ~ 10 MHz your lowest somewhat reliable measurement frequency for the test fixture stray reactances.

What is the average of the measured values of the test fixture stray capacitance C_s at the 200, 400, and 600 MHz frequencies? Going forward we will consider this important average value as the C_s to be used in any measurements requiring accounting for C_s .

Although not particularly relevant for this item, you may find useful to know that the NanoVNA-F can be conveniently set to report either capacitance or inductance values or the corresponding reactances. To get the NanoVNA-F to report series impedance values (i.e., $R + jX$) invoke its menu tree, go to "MARKER," "SMITH VALUE," and select "R+Xj." Note that the also available alternative "R+L/C" selection is the one that reports capacitance or inductance values.

4. Measure the residual (or stray, or parasitic) inductance L_s of the test fixture (the s subscript refers to the word stray), and report it in nH, in the 50 KHz – 1 GHz range, using 200 MHz increments. To perform this measurement cut a small piece of the provided 26 AWG wire, remove its insulation, and connect it flush between two closest pins of the test fixture (make the wire piece as flush as possible to the two DIP-8 socket pins, and the two bends approximately 90° , otherwise you will be adding unnecessary undesirable spurious inductance to the stray inductance that you are trying to measure). The measured L_s value should be of the order of 3 nH.

What is the average of the measured values of the test fixture stray inductance L_s at the 200, 400, and 600 MHz frequencies? Going forward we will consider this important average value as the L_s to be used in any measurements requiring accounting for L_s .

5. From the previous measurements we see that the SMA-DIP8-SMA test fixture has both parasitic capacitance and inductance. Since over our frequency measurement range its dimensions are small when compared to the associated free-space wavelengths (the free-space wavelength corresponding to the frequency $f=1$ GHz is $\lambda_0 = c_0/f = 299.8$ mm), the behavior of its parasitics can be well represented by an equivalent lumped-elements' circuit. The question now is then what is a convenient (i.e., simple but yet accurate) topology for this circuit that simultaneously involves both C_s and L_s .

Taking another good look at the physical connections under the DIP-8 IC socket of the SMA-DIP8-SMA test fixture, one observes that the connections between the input SMA coaxial connectors and the DIP-8 IC output socket pins allow *DC* currents to flow (i.e., they work at zero frequencies). Hence the inductance L_s must be considered as connected in series between the input SMA connector and the corresponding DIP-8 IC output socket pin. With this in mind, together with the measured results obtained when open- and short-circuits were applied to the DIP-8 socket pins, you should be able to infer that suitable candidates for the equivalent circuit of the test fixture are either one of the two L circuits depicted in Fig. 8. As you will see below, within the accuracy available in your measurements, either one of these two equivalent circuits reproduce reasonably well the measured results, and hence characterize well the strays of the test fixture. Which specific circuit to use over the DC – 1 GHz frequency range is then largely just a matter of application convenience.

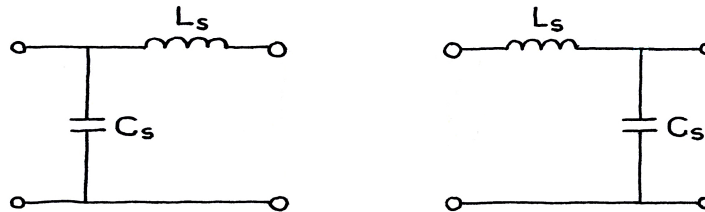


Figure 8: Possible equivalent L-circuits for the SMA-DIP8-SMA test fixture

Derive an equation for the equivalent capacitance C_i provided by the two circuits depicted in Fig. 8, at their input terminals (i.e., terminals on the left), when their output terminals (i.e., terminals on the right) are left open. Use this equation to determine C_i at 200 MHz and 1 GHz.

Also derive an equation for the equivalent inductance L_i provided by the two circuits depicted in Fig. 8, at their input terminals, when their output terminals are short circuited. Use this equation to determine L_i at 200 MHz and 1 GHz.

Create two tables comparing the C_i and the L_i values obtained above with the previously measured results (one table for the circuit on the left of Fig. 8 and one for the circuit on the right). Use the percentual differences between the calculated and measured results to determine which of the two equivalent circuits better represent the test fixture under the above two test cases and comment.

When using the SMA-DIP8-SMA test fixture note that, for proper and reliable electrical contact, the leads of the components have to be completely inserted into the IC socket pins. In other words, they should go in until they bottom out (they go in about 4 mm).

Also, please note that flexible fingers inside the pins of the DIP-8 IC socket are very delicate. They are designed to open up and grab wires with smooth surfaces and extremities, and only diameters falling in the 0.381–0.635 mm range (approximately 26–23 AWG, respectively). They will provide unreliable electric connections if the inserted wire diameter is too small and *they will get permanently damaged*, and thereafter provide unreliable electric connections, if any inserted wire is not sufficiently smooth or has a diameter outside the above allowed range! In case you are wondering, all components of the laboratory kit have been carefully selected so that their lead diameters fall safely in the allowed range.

6 Capacitor Measurement

We will now proceed to examine the behavior of two common ceramic disk capacitors at high frequencies. We will be using this type of capacitor because it is constructed to work well at high frequencies (it is basically a parallel plate capacitor with the two plates separated by a very low loss ceramic dielectric). We will also safely ignore any losses that are unavoidably present in the capacitors, since they are quite small. Note then that, since these losses are represented by a resistance, there will be no resistors in your capacitor circuit. As you perform the measurements below, please make sure to not clip the length of any of the components' leads, as we will need the intact components for subsequent experiments.

1. Locate the provided 25 pF capacitor and, without trimming the length of its leads, connect it to the SMA-DIP8-SMA test fixture.
2. Measure the low-frequency capacitance C_L of the 25 pF capacitor. Don't forget to factor out the previously measured residual capacitance of the test fixture, as it is not part of the C_L value.

3. Observe that your capacitor becomes an inductor at high frequencies (use the impedance readout on the top left of the display to verify this). Determine the frequency where the capacitor becomes an inductor (its resonance frequency f_0), and explain why this is happening. Is this frequency a series or a parallel resonance?

You should have already learned about resonant circuits in your classes⁶, and some relevant material has also been provide earlier on this laboratory manual. But please make sure to further review the subject if you need a deeper refresher.

4. To confirm your previous item hypothesis, make a hairpin shaped piece of wire with the same overall shape and dimensions of the capacitor leads and measure its inductance L_p .
5. Observe that the inductance L_w of a length ℓ of two parallel wires located in free space is given by⁷

$$L_w = \ell \frac{\mu_0}{\pi} \ln \left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1} \right], \quad (14)$$

where D is the distance between the wires' centers and $2a$ is the diameter of the wires. Use this result to predict the inductance of your hairpin, compare with the measured result, and comment.

6. Use the previously measured C_L and L_p values to develop an equivalent circuit for the capacitor, derive an equation to predict its resonance frequency f_0 when mounted on the test fixture, and compare the numerical f_0 value it produces with the measured result. Your f_0 equation should clearly show what is been used for the capacitance and the inductance.
7. Provide a drawing of the equivalent circuit of your capacitor, when it is operating mounted on the test fixture (the equivalent circuit should then include the test fixture parasitic reactances), valid from direct current (i.e., DC) up to 1 GHz (the upper limit of your NanoVNA-F measurements), clearly labeling all its components and corresponding values.

From the above measurements you are going to be forced to come to terms with the previously mentioned fact that there is no such thing as a pure capacitor (i.e., a circuit element that only provides capacitance). Every circuit element will always have capacitance, inductance, and resistance together. What is done is simply to construct circuit elements in such a way as to minimize what is undesirable parasitics; this is all that can be done. However, no matter how well this is done, if the frequency gets high enough the undesirable parasitics will eventually show up.

⁶See for example A. J. Rosa and R. E. Thomas *The Analysis and Design of Linear Circuits*, John Wiley & Sons, Inc.

⁷D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., pags. 165 and 445.

In the above capacitor we kept the connecting leads uncut, and hence we lived with more inductance than needed. If the capacitor was to be used in an actual circuit, the connecting leads would have been trimmed as short as possible. However, if the frequency of operation is high enough even the small inductance associated with the shortest possible leads will eventually become a problem. Furthermore, present is always the unavoidable inductance associated with the capacitor body itself (i.e., the two capacitor plates).

Circuit theory is just an extremely convenient and wonderful simplification of Maxwell's equations. In circuit theory Maxwell's equations are conveniently approximated into what is called Kirchhoff's voltage and current laws (KVL and KCL, respectively), and hence the general form of Maxwell's equations are not needed (complicated vectors and four-dimensional differential equations are then not needed!). Without this wonderful approximation electronics as we know it would be impossible and our world would be a very different place.

The very important question to ask is then when can we safely use Kirchhoff's laws. The answer is dictated by the dimensions of the circuit in relation to the free-space wavelength λ_M associated with the circuit maximum operation frequency. It can be shown that a reasonable rule of thumb is (this result is very important, so it deserves its own special box):

You can use KVL and KCL, and therefore ignore the more general form of Maxwell's equations, whenever the linear dimensions of the volume that just encloses the circuit under consideration are smaller than at least about $\lambda_M/50$.

Note that an obvious corollary of the above result is that, by simply reducing the size of a circuit, you can increase its maximum reliable frequency of operation (we resorted to this when we used chip resistors to implement our precise SMA female calibration load). And, as long as you do not reach any quantum electrodynamics limit (which happen when you approach molecular sizes), there is really no limit for how high in frequency you can go with a circuit.

Please make sure to always keep fresh in your mind the above facts as we move forward with our laboratories.

7 Resistor Measurement

We will now proceed to examine the electrical behavior of a common through-hole (or axial-lead) 1/4 W metal film resistor at high frequencies. Common through-hole metal film resistors are made by vacuum depositing a metallic film over a ceramic dielectric cylinder. After the deposition the direct current (i.e., DC) resistance value is measured and adjusted to the desired value by cutting a helix on the metallic film (the length of the cut depends on the starting resistance to be reduced). These and other details invariably introduce undesirable parasitic inductance and capacitance to the resistors, complicating their high-frequency behavior and rendering many of them pretty much useless at high frequencies (e.g., frequencies above 100 MHz). Furthermore, since different companies tend to manufacture their resistors differently, the high-frequency

behavior of resistors of identical DC resistance value, but different manufacturers, most probably will be different. The main point to always keep in mind is that at high frequencies just the DC resistance value is usually not sufficient to characterize a resistor.

1. Locate the provided 49.9 Ω resistor (color coded yellow, white, white, and gold) and, without trimming the length of its leads, bend and connect them to two adjacent pins of the SMA-DIP8-SMA test fixture. I suggest that you sharply bend the two leads very close to the resistor body; bend them by 90 degrees, so the two leads will run conveniently parallel to each other.
2. Measure the low-frequency resistance R_L of the 49.9 Ω resistor.
3. Measure the low-frequency inductance L_L of the 49.9 Ω resistor.
4. Using the hairpin wire technique developed for the capacitor measurements, confirm that the measured inductance L_L is associated with the lengths of the resistor leads and body.
5. Observe how the resistor resistance R changes significantly with frequency, which is a very undesirable effect for a resistor.
6. Observe that the resistor also has a resonance frequency f_0 and measure its value. Is this frequency a series or a parallel resonance?
7. In case you have not noticed, the hairpin wire alone has a similar resonance frequency as the resistor. Please measure again the hairpin wire, look for its resonance frequency (let's call it f_{0p}), and determine its value.
8. Hypothesize what is causing the resistor resonance frequency, calculate its expected value (let's call this value f_{0e}), and therefore confirm your hypothesis.
9. Notice that the measured resonance frequency f_0 is somewhat lower than the calculated value f_{0e} , which then indicates that perhaps there is some additional capacitance unaccounted for in your model.

The difficulty here is that the resonance is pretty much being caused by the test fixture stray capacitance, which is quite small, and hence any other small capacitance can affect it significantly.

If you haven't yet figured out, the missing capacitance is caused by the capacitance between the two resistor leads (let's call it C_w), which we have safely been ignoring up to this point and is now catching up with us.

Determine the value of C_w and use it to refine your f_{0e} value. This can be done both theoretically and experimentally, so please do it both ways.

By the way, the capacitance C_w of a length ℓ of two parallel wires located in free space is given by⁸

$$C_w = \ell \frac{\pi \epsilon_0}{\ln \left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1} \right]}, \quad (15)$$

where D is the distance between the wires' centers and a is the radius of the wires ($2a = 0.55$ mm for the resistor leads).

10. Provide a drawing of the equivalent circuit of your resistor, when it is operating mounted on the test fixture (the equivalent circuit should then include the test fixture parasitic reactances), valid from direct current (i.e., DC) up to 1 GHz (the upper limit of your measurements), clearly labeling all its components and corresponding values.

8 Inductor Measurement

To complete this laboratory we will now investigate the behavior of inductors at high frequencies. Instead of using available components, as previously, and considering that we will only be needing relatively small inductance values (of the order of tenths of nH), it is more convenient to construct the needed inductors. The developed skill will also prove valuable to future laboratories. Again here we will ignore any losses present in the constructed inductors, as they are quite small.

1. Remove 4 mm of the insulation of each extremity of a 60 mm long insulated 26 AWG wire (conductor diameter $2a = 0.39$ mm) and tightly coil the wire around a small form to make a closely spaced three-turn air-core inductor of about 40 nH. Use for form the sharpened conical end of a pencil. Do not remove the entire wire insulation, as it prevents short-circuiting the inductor turns, and keep the length of the two connecting exposed wire leads as small as possible (about 4 mm long exposures are needed though, since this is what goes into the SMA-DIP8-SMA test fixture).

If you are wondering, the inductance of low-frequency air-core inductors can be precisely calculated, and equations to do so have been derived and are available in the literature. For the current purposes a very useful and accurate result can be obtained from Nagaoka's formula⁹, assuming a cylindrical single-layer inductor of length not large compared to its diameter. This yields

$$L = \frac{\mu_0}{2} 2b N^2 \left[\ln \left(4 \frac{2b}{h} \right) - \frac{1}{2} \right], \quad (16)$$

where L is the inductance value, $2b$ is the inductor diameter (the cylinder diameter, measured relative to the wire center), h is the inductor length (the cylinder

⁸D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., pag. 165.

⁹F. W. Grover, *Inductance Calculations*, Dover Pub., Inc., 1962, pag. 143.

height, again measured relative to the wire center), and N is the number of turns. Although Nagaoka's original work is quite general, to arrive at the above formula it has been assumed that $h/2b \lesssim 1$.

2. Use Nagaoka's formula to predict the low-frequency inductance L_L of the three-turns inductor that you constructed. Provide the L_L value in nH.

Observe that you can only apply Nagaoka's formula to precisely made inductors, as only in such cases $2b$, h , and N are well defined. Hence, please make sure to excel on your craftsmanship.

3. Measure the low-frequency inductance L_L of the three-turns inductor and discuss any discrepancies observed between the predicted and measured values. Don't forget to factor out the previously measured residual inductance of the test fixture, as it is not part of the L_L value.
4. Observe how the inductor inductance L changes significantly with frequency, which is a very undesirable effect for an inductor.
5. Note that the inductor also have a resonance frequency. Measure its value f_{0m} . Provide its value in MHz. Is it a series or parallel resonance?
6. Hypothesize what is causing the inductor to have a resonance frequency. Then use what you concluded to calculate the expected resonance frequency value (let's call this value f_{0e}), and therefore confirm your hypothesis.
7. Tightly coil a 105 mm length of 26 AWG wire around a small form to make a closely spaced five-turns air-core inductor of about 100 nH. Again use for form the sharpened conical end of a pencil. Fig. 9 depicts examples of three- and five-turns inductors. Please make sure to also include pictures of your two inductors in your laboratory report.

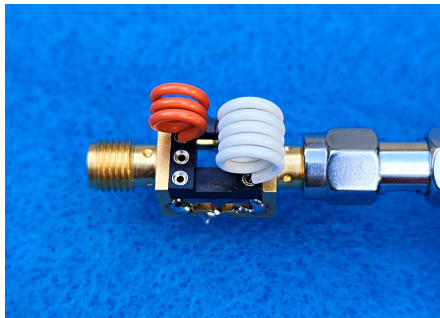


Figure 9: Three- and five-turns inductors mounted for testing on the SMA-DIP8-SMA test fixture

8. Use Nagaoka's formula to predict the low-frequency inductance L_L of the five-turns inductor.
9. Measure the low-frequency inductance L_L of the four-turns inductor and compare with the predicted value.

10. Observe how the three-turns inductor inductance L also changes significantly with frequency, which is a very undesirable effect for an inductor.
11. Observe that the five-turns inductor also have a resonance frequency. Measure its value f_{0m} . Is it a series or parallel resonance?
12. Hypothesize what is also causing the four-turn inductor to have a resonance frequency, calculate its expected value (let's call this value f_{0e}), and therefore confirm your hypothesis.
13. Observe that the measured resonance frequency of both inductors is 5 – 10 % lower than expected. This indicates that some additional capacitance C_t is not being accounted for. Use the measured resonant frequencies f_{0m} to determine the value of C_t for both the three- and the five-turns inductor, in pF.
14. The observed capacitance C_t is primarily coming from the energy stored in the electric field that unavoidably exists surrounding the inductors' wires; a consequence of the voltage drop that exists between the two end terminals of an operating inductor, and this energy manifests itself as the self-capacitance C_t (according to Eq. 7). Any inductor, even operating alone (i.e., not mounted in a test fixture), will then unavoidably have a resonance frequency (its so called self-resonant frequency).

The self-resonant frequency of inductors has been subject of research for close to 100 years now, and it can be used to estimate the self-capacitance C_t . A particularly useful self-resonant frequency derivation for our purposes yields the Miranda-Pichorim's formula¹⁰

$$C_t = \frac{\mu_0 \epsilon_0}{0.4 L} (N 2b)^2 \left(\frac{2b}{h} \right)^{0.4}, \quad (17)$$

where L is the inductor inductance.

Note that the wire dielectric sheath can be expected to affect the capacitance C_t . However, since the volume occupied by the sheath is relatively small compared to the inductor overall volume, its effect can be neglected for the current purposes.

Use Miranda-Pichorim's formula to predict the C_t of both the three- and the five-turns inductors and compare the values obtained with your two measured results.

15. Provide a drawing of the measured equivalent circuit of each one of your inductors, valid from direct current (i.e., DC) up to about 1 GHz (the upper limit of your measurements), showing the values of all its components. In particular make sure to clearly indicate what components are being introduced by your measurement setup, and hence are not really part of the inductors themselves.

¹⁰C. M. de Miranda and S. F. Pichorim, *Self-resonant frequencies of air-core single-layer coils calculated by a simple method*, Electrical Engineering, vol. 97, No. 1, Mar. 2015, pp. 57–64.

16. What is the measured resonance frequency f_{0a} of the three-turn inductor operating alone (not connected to anything)? This is the important inductor self-resonant frequency, as it ultimately determines how high in frequency your inductor can be used successfully (see next item).
17. Observing that as a guideline the inductors' inductance will stay within $\sim 20\%$ of the low frequency value as long as the inductor operates at frequencies below about half of their self-resonant frequency, determine the maximum practical operation frequency f_{max} of the three-turns inductor.

