Circuit Parameters of High-Frequency Transmission Lines

Laboratory 02 Manual

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1 Laboratory Objective

In this laboratory you will learn the relevant parameters of transmission lines and how they affect their behavior at high frequencies. More specifically, we will consider in detail the circuit parameters that determine the propagation factor γ and the characteristic impedance Z_0 of transmission lines, namely the R, L, G, and C of the equations

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} \tag{1}$$

and

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}\,, \tag{2}$$

where R and L are the resistance and inductance, *per unit length*, of the wires of the transmission line, respectively, G and C are the conductance and capacitance, *per unit length*, between the wires of the transmission line, respectively, and $\omega = 2\pi f$, where f is the operation frequency. To successfully complete the experiment below you will need to have studied the material covered in Chap. 9 of our textbook[1] Eqs. 1 and 2 are discussed in detail in there.

For some of the measurements below your work will become significantly easier if you download and use the NanoVNASaver software, which is freely available at the website

`https://nanovna.com/?page_id=90` .

This software allows the NanoVNA-F to interface with a PC, to make measurements, display results, as well as to download results. The software also increases the capabilities of the NanoVNA-F in several useful ways (e.g., frequency sweeps with more than 301 sample points can be performed, measured results can be saved for additional processing, etc.). Note that the above link also has operation instructions for the software.

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report should have each of its sections and subsections numbered according to this laboratory manual, and be a detailed document with all your measurement results, calculations, conclusions, drawings, plots, and relevant photos of

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[1] D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co.

all constructed components (to showcase your very important high-frequency craftsmanship).

Note that, to maximize the learning experience, the laboratory has been designed to be carried out individually, hence each person in the class received their own individual lab kits. Consequently, the experiment and the corresponding report has to be done completely individually.

2 Low-Frequency Measurement of the Capacitance and Inductance of Coaxial Cables

We will start this laboratory by measuring the capacitance and inductance, per unit length, of a coaxial cable. For this task we will use one of the two ∼200 mm long RG316 coaxial cable that came inside the NanoVNA-F box. In this section we will only deal with the reactive (as opposed to the dissipative) parameters, the dissipative parameters will be considered in the subsequent section.

1. Carefully measure the length of the RG316 coaxial cable. Note that what you need here is the length of the part of the cable that actually carries signal (if you are having problems understanding what this means, please look inside the hexagonal nut of the SMA male connectors at the end of the cables).

2. Observing that we will be doing measurements directly at the female SMA port of the NanoVNA-F, select the proper calibration to use. Make sure to verify that the stored calibration is still valid, and to recalibrate the VNA if it isn't.

3. Connect one of the two available sections of RG316 cable to the VNA port and measure its inductance L'.

   Recalling that inductance and capacitance are parameters associated with the amount of energy stored in the magnetic and electric fields, respectively, for this measurement the cable must be excited in such a way as to store energy significantly only in the magnetic field, and not in the electric field. This can be achieved by shorting the end of the cable using the short of the SMA female calibration kit and measuring the input impedance of the cable at the lowest possible frequency. The short and the low measurement frequency assure that there is no significant voltage, and hence electric field, between the two conductors of the cable (any stored energy is then practically associated only with the magnetic field). Note that the short adds an extra length to the cable and this needs to be properly accounted for.

4. Determine the measured inductance L, *per unit length*, of the NanoVNA-F provided RG316 coaxial cable and compare with the predicted result, given by[2]

$$L_c = \frac{L'}{\ell} = \frac{\mu_d}{2\pi} \ln\left(\frac{b}{a}\right) , \tag{3}$$

[2] D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., pag. 446.

where ℓ is the cable length, μ_d is the permeability of the dielectric ($\mu_d = \mu_0$, since the dielectric is non magnetic), b is the inner radius of the outer coaxial conductor, and a is the outer radius of the inner coaxial conductor. Observe that the RG316 dielectric is polytetrafluoroethylene (this dielectric name is usually abbreviated as PTFE, also referred to by the Teflon trade name) which is a non-magnetic material, therefore its permeability is the same as the one of vacuum.

Any required dimensions of the coaxial cable can be obtained by consulting the manufacturer's data sheet.

5. Measure the capacitance C, *per unit length*, of the NanoVNA-F provided RG316 coaxial cable. How does your C value compare with the RG316 data sheet value?

   Observe that for this measurement you need to store energy significantly only in the electric field of the cable. Also note that the open standard also adds an extra length to the cable, and this needs to be properly accounted for.

6. Using the measured L and C values, and neglecting the R and G contributions since they can be assumed negligible for now, determine the characteristic impedance Z_0 of the RG316 coaxial cable and compare with the expected value.

7. Determine the phase propagation velocity u_p of the RG316 coaxial cable.

8. What is the velocity factor, defined as

$$VF \equiv u_p/c_0\,, \tag{4}$$

   of the RG316 coaxial cable? Note that c_0 is the speed of light in vacuum (i.e., $c_0 = 1/\sqrt{\mu_0\epsilon_0}$). The VF is a very useful practical parameter, which you will use multiple times as we move forward; it is simply a measure of how much slower the electromagnetic wave propagates on a transmission line, in comparison to the velocity of light in vacuum.

   How does your VF value compare with the RG316 data sheet value?

9. What is the relative permittivity ϵ_r of the RG316 coaxial cable polytetrafluoroethylene dielectric? How does your measured value compares with the value available from the cable manufacturer?

3 Low-Frequency Measurement of the Resistance of Coaxial Cables

We will now consider the dissipative parameters of a coaxial cable. They are very important because they determine the attenuation of any signal propagating on the cable. For this task we will again use one of the two ~200 mm long RG316 coaxial cables that came inside the NanoVNA-F box.

The dissipative parameters of coaxial cables are made of two terms: the combined resistance of the inner an outer coaxial conductors and the resistance between the inner and outer coaxial conductors (due to the current that flows through the imperfect dielectric between them).

1. Let's start by considering the combined resistance R' of the inner and outer coaxial conductors. Use the NanoVNA-F to measure the R' of the RG316 coaxial cable at 2.5, 5, 10, and 20 MHz, and from it obtain the resistance R per unit length of the cable at these frequencies.

   For the above measurement you will be using the calibration that was previously made at the naked NanoVNA-F ports, the one that used the SMA male standards. However, since this calibration covers the 50 KHz to 1 GHz frequency range using 301 points, none of the desired frequencies is precisely available for use. Also, since 301 points are measured, the frequency sweep takes some time to run. We will then change the frequency range to 0.5 to 25.5 MHz and the number of points to 51, as this will sweep faster and yield a (25.5 - 0.5)/50 = 0.5 MHz step size, precisely making available the 2.5, 5, 10, and 20 MHz frequencies of interest. Although in principle this change would require recalibrating the NanoVNA-F, because of the low frequencies involved we can take advantage of the intrinsically available calibration interpolation and proceed with our measurements without having to recalibrate.

   To change the NanoVNA-F frequency sweep range invoke its menu tree, select "STIMULUS," and then select either "START" or 'STOP" to specify the desired sweep start and stop frequencies, respectively. To change the number of frequency points select "DISPLAY" and then "SWEEP POINTS."

   Since the coaxial cable resistance R' is quite small, its measurement is quite susceptible to uncertainties. In particular notice that the reading is not stable; the NanoVNA-F displayed resistance value keeps on changing as a result of the inherent measurement noise. Since this noise is proportional to the operation bandwidth of the measurement receiver, to some extent VNAs allow you to control their noise by adjusting their receiver bandwidths. The bandwidth currently being used is the number displayed in the lower left of the NanoVNA-F screen as "BW/HZ 1000" (in this case the bandwidth is 1 kHz). To reduce it go to "DISPLAY," "BANDWIDTH," and then select the desired bandwidth. Smaller bandwidths yield smaller noise (i.e., smaller variations on the displayed R' values), at the expense of longer measurement times.

   Another source of measurement uncertainty is the inherent error present in any calibration. You can take a look at the impact of this error in your R' measurement by connecting the SMA short standard to the calibrated port and taking a look at the displayed resistance value, which in a perfect scenario would be zero.

   Please make sure to consider the uncertainty effects just discussed to make sure that your measured R' values have acceptable accuracy.

2. After performing the R' measurements you will note that (perhaps to your surprise) the resistance R approximately doubles every time the frequency quadruples. To understand the reason behind this important behavior, recall that the

resistance per unit length of the coaxial cable conductors is analytically given by[3]

$$R = \frac{R'}{\ell} = \frac{1}{\sigma_c}\,\frac{1}{2\pi a\,\delta} + \frac{1}{\sigma_c}\,\frac{1}{2\pi b\,\delta} = \frac{1}{2\pi\sigma_c\delta}\left(\frac{1}{a} + \frac{1}{b}\right), \tag{5}$$

where ℓ is the length of the cable, σ_c the effective electric conductivity of the conductors, a is the outer radius of the inner conductor, and b is the inner radius of the outer conductor. The term

$$\delta = \sqrt{\frac{2}{\omega\mu_c\sigma_c}}, \tag{6}$$

where $\omega = 2\pi f$, and μ_c is the permeability of the conductors' materials ($\mu_c = \mu_0$, since they are made of non-magnetic materials), is very important and you will have the opportunity to encounter it often as you continue to learn applied electromagnetics. δ is called the *skin depth* (a value in meters), and takes into account the fact that, as the operation frequency increases from DC (i.e., direct current) the electric currents tend to progressively concentrate on the *skin* of conductors.

For practical purposes everything then behaves as if the electric current is only flowing in the near-surface region with depth equal to the above skin depth value.

Use the above results, together with your previously measured R values, to determine the effective conductivity σ_c of the coaxial cable conductors.

Observe that the reason for the above effective conductivity nomenclature is that the outer conductor of the coaxial cable is not a solid material; to render the cable flexible it is actually a braid. Since any braid has gaps, its conductivity can be expected to be significantly lower than the conductivity of the metal used to make the individual wires of the braid. Furthermore, and also to make the cable flexible, the center conductor is stranded. This also reduces the conductivity of the center conductor.

3. Compare your measured effective conductivity σ_c value with the conductivity of the material used to make the RG316 conductors, and comment on any observed differences.

4. Let's consider next the conductance G'_{bc} between the inner and outer coaxial conductors, which is analytically given by[4]

$$G'_{bc} = \frac{2\pi\sigma_d}{\ln(b/a)}\,\ell, \tag{7}$$

where σ_d is the electric conductivity of the coaxial cable dielectric, and the subscript *bc* stands for between conductors.

[3]D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., pag. 447.

[4]D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., pag. 446.

Since polytetrafluoroethylene is an excellent dielectric, its conductivity σ_d is incredibly small (a perfect dielectric would have zero conductivity). Its value depends on how pure the dielectric is, but at low frequencies it generally hovers around 1 aS/m (the suffix a, which is pronounced atto, corresponds to 10^{-18}). Using this value, calculate the resistance R_{bc} between the two conductors of your coaxial cable, in $\Omega\times$m.

Considering the frequency range of our experiment (i.e., between ∼2.5 MHz and ∼20 MHz) and the alternative propagation factor γ expression

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}\sqrt{\left(1 - j\,\frac{R}{\omega L}\right)\left(1 - j\,\frac{G}{\omega C}\right)}\,, \tag{8}$$

does the conductance $G = G'_{bc}/\ell$ needs to be considered in the determination of the propagation factor of our RG316 cable? Justify your answer.

Note that Eq. 8 is just a modified more convenient version of Eq. 1, since it makes clear that what impacts the right most square root of γ is how large the ratios $R/\omega L$ and $G/\omega C$ are relative to one. For a lossless transmission line $R = 0$ and $G = 0$, and Eq. 8 readily yields the expected $\alpha = 0$ and $\beta = j\omega\sqrt{LC}$. Similarly we can also write a convenient alternative form for Eq. 2, namely

$$Z_0 = \sqrt{\frac{L}{C}}\sqrt{\frac{1 - j\,\frac{R}{\omega L}}{1 - j\,\frac{G}{\omega C}}}\,. \tag{9}$$

5. Use Eqs. 8 and 9, together with a Taylor series, to derive an approximate, simple, and yet very accurate expression for the attenuation constant α of the RG316 coaxial cable at low frequencies. Conveniently provide your result in terms of R and Z_0.

6. Use the expression obtained in the previous item to determine the attenuation of the RG316 cable at 10 MHz, in dB/m, compare with the manufacturer provided data sheet value, and comment.

4 Low-Frequency Measurement of the Capacitance and Inductance of Parallel-Wire Transmission Lines

We will now measure the capacitance and inductance, per unit length, of a parallel-wire transmission line.

In contrast with coaxial transmission lines, simple parallel-wire transmission lines do not have their signal carrying volume completely shielded from the surrounding environment. They are then susceptible to undesirable interference and radiation effects, and the accurate measurement of their electrical parameters is more difficult. Furthermore, the improper shielding limits their operation frequency and in general they have inferior performance characteristics when compared to coaxial transmission lines. However, since their relative simplicity offers a significant cost advantage, they are widely used for internet, telephone, and power transmission applications.

1. Construct a section of parallel-wire transmission line using two 220 mm pieces of the 26 AWG insulated wire that came with your laboratory kit. To do this straighten and cut two pieces of dissimilar color (the different colors will help making the connections correctly later), remove 4 mm of insulation from the four wire ends (to make some required electric connections later), and then twist together the two insulated wires at the rate of about one turn per 25 mm (the actual rate and its uniformity is not critical). This process should leave you with a parallel-wire transmission line of about 210 mm length (counting only the insulated portion).

   In case you are wondering, the twists serve basically three useful and important purposes: maintain a constant close spacing between the two wires, produce an approximately equal interaction of the two wires with the environment, and minimize both radiation and incoming interference.

2. Connect the SMA-male-to-SMA-male adapter to the Port 1 of the VNA. Verify that the needed calibration is still reliable and recalibrate the VNA if it isn't.

3. Connect the provided SMA-DIP8-SMA test fixture to the Port 1 adapter of the VNA. Then connect each end of your your transmission line to the pins associated with each SMA connectors. Note that the test fixture provides a convenient way to short or open the end of the transmission line, as needed.

4. Measure the inductance L' of your transmission line. Don't forget to subtract the stray inductance associated with the test fixture. Determine the inductance L, per unit length of the parallel-wire transmission line.

5. Recalling that the diameter of a 26 AWG wire is $2a = 0.39$ mm, and the diameter of our insulated 26 AWG wire is $D' = 1.19$ mm, compare the measured L value obtained with the theoretical one, which can be determined using[5]

   $$L_w = \ell \, \frac{\mu_0}{\pi} \, \ln\left[\frac{D}{2a} + \sqrt{\left(\frac{D}{2a}\right)^2 - 1} \right] , \tag{10}$$

   where L_w is the inductance of a length ℓ of two parallel wires, D is the distance between the wires' centers, and a is the radius of the wires.

6. Measure the capacitance C'' of your transmission line. Don't forget to subtract the stray capacitance associated with the test fixture.

[5] For the derivation of this equation please take a look at pags. 165 and 445 of D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co.

7. Touch the middle of the transmission line with your hand, observe the consequence, and explain the effect you are seeing. Does this happen when you touch a coaxial transmission line? Explain why.

8. At first you may think that the capacitance C'' you measured above is just the capacitance C' between the two wires of the parallel-wire transmission line. However, it may surprise you to find out that this is not the case. To understand this fact disconnect the transmission line from the side of the test fixture that is not connected to the VNA. And then move the conductor of the transmission line that is connected to the ground to the IC socket pin that is connected to the coaxial cable center conductor, on the side of the test fixture connected to the VNA. Record the stray capacitance C_g reported by the VNA and explain where it is coming from.

   Because of the existence of C_g, our measurement has difficulties in determining the desired C' value. To determine C' what is needed is to perform a differential measurement between just the two conductors of the parallel-wire transmission line, and the Port 1 of the NanoVNA-F is unable to do this since the outside of its coaxial connector is attached to the metal case (in other words, by construction the coaxial ports of the NanoVNA-F are not differential). However, since in the future we will be operating the parallel-wire transmission line *in the presence of the surrounding environment*, and C_g can be reasonably approximated as been distributed along the length of the line, we will successfully proceed with the assumption that $C' \approx C''$.

   By the way, on a subsequent laboratory we will learn how to perform a differential measurement using both ports of the NanoVNA-F.

9. Explain why a coaxial transmission line does not have a C_g.

10. Determine the characteristic impedance Z_0, the phase propagation velocity u_p, and the velocity factor VF of the parallel-wire transmission line, *in the presence of the surrounding environment.*

11. Explain why, and this is in contrast with the coaxial cable, the above measurements are incapable to establish the relative permittivity ϵ_r of the plastic dielectric of the parallel-wire transmission line.

It is important to note that the parameters of the above parallel-wire transmission line that should be employed in future applications of this transmission line (i.e., Z_0 and VF) are the parameters measured *in the presence of the surrounding environment*, since this is the environment where the line operates.

5 High-Frequency Measurement of the Characteristic Impedance and Velocity Factor of Transmission Lines

Although the previous measurement technique is capable of yielding most of the parameters of transmission lines, it has the disadvantage of being a low-frequency technique (i.e., the parameters of the transmission line are determined at low frequencies). In this section we will explore an alternative technique that is capable of determining the parameters of transmission lines at high frequencies. This technique is based on the formula that yields the input impedance Z_i of an arbitrarily terminated transmission line of length ℓ, namely[6]

$$Z_i = Z_0 \, \frac{Z_L + Z_0 \, \tanh(\gamma\ell)}{Z_0 + Z_L \, \tanh(\gamma\ell)} \,, \tag{11}$$

where Z_L is the impedance loading the end of the transmission line and $\gamma\ell = \alpha\ell + j\beta\ell$ is the (complex) argument of the hyperbolic tangent.

In the common situation where the transmission line losses are relatively small (i.e., $\alpha\ell << \beta\ell$) and hence can be neglected, the above equation can be approximated with excellent accuracy by

$$Z_i \approx Z_0 \, \frac{Z_L + j \, Z_0 \, \tan(\beta\ell)}{Z_0 + j \, Z_L \, \tan(\beta\ell)} \,. \tag{12}$$

Neglecting losses, in this section we will measure the reactive parameters of our RG316 coaxial cable using the NanoVNA-F in association with the above Eq. 12; because this equation assumes that the cable is lossless, the resistive parameters of the RG316 can not be obtained from it. However, in a subsequent laboratory we will return again to this subject and at that time measure the resistive parameters of the cable using Eq. 11.

1. Starting from Eq. 11, derive Eq. 12.

2. If you have not already done so, please download the NanoVNASaver software and familiarize yourself with its operation.

3. Select and verify the validity of the appropriate VNA calibration and then connect the section of RG316 cable that you have been using in the previous items to the VNA port.

4. Connect the short standard to the end of the coaxial cable and determine all resonance frequencies present in the 10 KHz – 1 GHz range. Make sure to clearly indicate what are parallel (f_{0p}) and what are series (f_{0s}) resonance frequencies.

5. Connect the open standard to the end of the coaxial cable and determine all resonance frequencies present in the 10 KHz – 1 GHz range. Make sure to clearly indicate what are parallel (f_{0p}) and what are series (f_{0s}) resonance frequencies. How does the resonance frequencies with the short and the open relate to each other?

[6] See Sec. 9-4 of D. K. Cheng, *Field and Wave Electromagnetics*, Second Edition., Addison-Wesley Pub. Co.

6. Use Eq. 12 to prove that the resonance frequencies of both the open- and short-circuited transmission lines are given by

$$f_0 = n\,\frac{VF\,c_0}{4\,\ell}\,, \tag{13}$$

where n is an integer (i.e., $n = 1, 2, 3, ...$).

7. Use Eq. 13, together with the measured resonance frequencies, to determine the velocity factor VF at the resonance frequencies of the transmission line. What is the averaged VF value? Compare your results with the VF value obtained in Sec. 2.

8. To determine the characteristic impedance Z_0 of the transmission line, consider the input impedance Z_{is} of a short-circuited transmission line (line with $Z_L = 0$), the input impedance Z_{io} of an open-circuited transmission line of the same length (line with $Z_L = \infty$), and use Eq. 12 to show that

$$Z_{is} \times Z_{io} = Z_0^2\,. \tag{14}$$

9. Also derive the useful results

$$L = Z_0\,\frac{1}{VF\,c_0}\,, \tag{15}$$

$$C = \frac{1}{Z_0}\,\frac{1}{VF\,c_0}\,, \tag{16}$$

which allow the determination of the inductance and capacitance per unit length (i.e., L and C) of the transmission line, once the velocity factor VF and the characteristic impedance Z_0 are known.

10. Use the NanoVNASaver software to measure the short- and open-circuited input impedances of your RG316 coaxial cable (i.e., Z_{is} and Z_{io}, respectively) at three widely spaced frequencies in the $0-1$ GHz range of the NanoVNA-F, and from these measurements determine Z_0, L, and C.

    To determine at what frequencies the measurements should be performed, observe that to arrive at Eq. 14 you have relied in a perfect cancellation of the $\tan(\beta\ell)$ term, which in practice is impossible due to unavoidable measurement errors. You should then measure the Z_{is} and Z_{io} at frequencies where the $\tan(\beta\ell)$ term is not changing excessively fast, otherwise the unavoidable measurement errors will cause an imperfect cancellation of the $\tan(\beta\ell)$ and hence large errors in your obtained Z_0. In other words, you should stay away from making measurements in the neighborhood of the frequencies where $\tan(\beta\ell)$ is infinite. Convenient measurement frequencies are then about midway between adjacent frequencies with $\tan(\beta\ell) = 0$ and $\tan(\beta\ell) = \infty$.

11. As a last item for this laboratory, let's take some extra time to better understand the already mentioned potentially large source of inaccuracy associated with the measurements performed in the previous item.

Observe that, due to all the unavoidable errors present in our measurements, the measured Z_{is} and Z_{io} values will always be somewhat incorrect. To better understand the consequences of this, let's assume that all the measurement errors observed are being caused by some equivalent error in the calibration standards that are being used to short and open the transmission line under measurement, which cause the lengths of the shorted and opened transmission lines to not be identical. In other words, if the length of the opened line is ℓ, the length of the shorted line will actually be $\ell + \Delta_\ell$, where Δ_ℓ is an small difference in length between the two lines.

With the above in mind determine the sensitivity (i.e., the rate of change) of the $Z_{is} \times Z_{io}$ product with respect to the small Δ_ℓ, and therefore show that

$$\frac{\partial(Z_{is} \times Z_{io})}{\partial \Delta_\ell} = \frac{2Z_0^2\beta}{\sin(2\beta\ell)}\,, \tag{17}$$

and since for Δ_ℓ small we can use the first two terms of the Taylor series to write

$$Z_{is} \times Z_{io}\big|_{\text{due to }\Delta_\ell} \approx Z_{is} \times Z_{io}\big|_{\Delta_\ell=0} + \left.\frac{\partial(Z_{is} \times Z_{io})}{\partial \Delta_\ell}\right|_{\Delta_\ell=0} \times \Delta_\ell\,, \tag{18}$$

obtain a convenient expression for the error caused by the small difference Δ_ℓ in the characteristic impedance measurements, namely

$$Z_0\big|_{\text{due to }\Delta_\ell} \approx Z_0\big|_{\Delta_\ell=0}\sqrt{1 + \frac{2}{\sin(2\beta\ell)} \times \beta\Delta_\ell}\;. \tag{19}$$

This result shows that, at the frequencies where the transmission line electric length is such that $\sin(2\beta\ell)$ is near zero (i.e., $\beta\ell \approx n\pi/2$, where $n = 0, 1, 2, \cdots$), any very small difference Δ_ℓ will cause excessively large errors in the measured $Z_{is} \times Z_{io}$ product, and therefore on the Z_0 derived from it. One can then clearly see why the measurements always need to be performed far away from the frequencies where $\tan(\beta\ell) \approx 0$ and $\tan(\beta\ell) \approx \infty$.