Scattering Parameters and Wave Characteristics of High-Frequency Transmission Lines

Laboratory 03 Manual

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1 Laboratory Objective

Up to this point our laboratories have primarily handled high-frequency electromagnetics using circuit theory concepts (i.e., voltages, currents, capacitances, resistances, inductances, impedances, susceptances, etc.). Although this is very useful, since among other things it relates well to the material that you have previously learned in your circuits' classes, it must be kept in mind that circuit concepts are not always applicable. They are applicable only in situations where the overall dimensions of the device being considered are small in relation to the operation wavelength, so that propagation delays can be ignored. However, even though it is true that sometimes the applicability of circuit concepts can be extended to high frequencies, and a good example of this is the treatment of transmission lines using the telegraphers' equations and distributed parameters (i.e., inductance, capacitance, resistance, and conductance per unit length) we need to have in place tools that allow us to handle situations where circuit concepts can't be applied at all (an example or this are situations involving optical fibers).

In this laboratory you will learn about a very useful technique for handling both transmission lines and high-frequency circuits. A technique based on scattering parameters (also called S-parameters, in high-frequency parlance). This technique is conveniently based on incident and reflected waves, since this is how energy propagates in transmission lines. You will also have the opportunity to apply what you learned in class about transmission lines, and use this knowledge in conjunction with some VNA measurements. To successfully complete the experiments below you will need to have studied in detail the material covered in Chap. 9 of our textbook[1].

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report should have each of its sections and subsections numbered according to this laboratory manual, and be a detailed document with all your measurement results, calculations, conclusions, drawings, plots, relevant photos of all constructed components (to showcase your very important high-frequency craftsmanship), and printouts of any developed software.

Note that, to maximize the learning experience, the laboratory has been designed to be carried out individually, hence each person in the class received their own individual lab kits. Consequently, the experiment and the corresponding report has to be done completely individually.

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[1] D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., 1989.

2 S-Parameters and the VNA

Consider a two-port device, shown in Fig. 1 connected to a VNA for measurement. The device is depicted in generalized form as a box with two terminals on either side. The box represents any device (e.g., the laboratory test fixture, a section of transmission line, a passive electric circuit, an electronic amplifier, a wireless link between two antennas, etc.). The device box has two ports (numbered 1 and 2, at its left and right sides, respectively) and each port has two terminals (e.g., the two conductors of the coaxial transmission line that connects it to the VNA). The location of the two ports is represented by the two pairs of small circles; and these locations also define the *device two reference planes.*

In a practical scenario the ports 1 and 2 of the device may be the two SMA female connectors of the device, to which you will hook up the two calibrated coaxial test cables connected to the two measuring ports of the VNA (Fig. 1 also shows these two test cables). Because of this, *VNA calibration planes* have been added to the ports 1 and 2 of the device (shown by dashed vertical lines); in the figure the device two reference planes have been made to coincide with the two VNA calibration planes. The characteristic impedance of the two test cables (shown as Z_0 in Fig. 1) is established by the internal VNA characteristic impedance, which is almost always equal to 50 Ω. Note that in several instances the device may have just one port, and in such cases port 2 either does not exist or is physically unavailable (this is the case of a circuit element, such a capacitor. It is also the case of an antenna, where port 2 is the space surrounding the antenna).

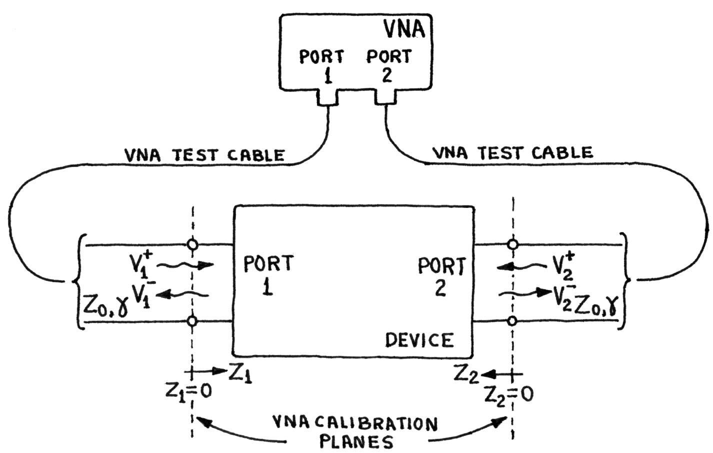

Figure 1: Wave representation of a general two-port device

Figure 1 also shows the complex amplitudes of the incident and reflected voltages of the four waves traveling along the two transmission lines, namely

$$V_1^+ = V_1^+|_{z_1=0}\ e^{-\gamma z_1}\,, \tag{1}$$
$$V_1^- = V_1^-|_{z_1=0}\ e^{+\gamma z_1}\,, \tag{2}$$
$$V_2^+ = V_1^+|_{z_2=0}\ e^{-\gamma z_2}\,, \tag{3}$$
$$V_2^- = V_1^-|_{z_2=0}\ e^{+\gamma z_2}\,, \tag{4}$$

where γ is the complex propagation factor of the transmission lines, and z_1 and z_2 are distances measured along the transmission lines connected to ports 1 and 2, respectively, with the $z_1 = 0$ and $z_2 = 0$ planes coinciding with the VNA calibration planes of Ports 1 and 2, respectively. The incident waves propagating towards the ports have a plus superscript (+), and the waves reflected by the ports have a minus superscript (−). Note that the coordinates z_1 and z_2 are different, and their positive directions (i.e., directions towards which their corresponding z-values increase) always point away from the VNA (towards the two-port device).

Assuming that the device represented by the Fig. 1 box is linear, we can write

$$V_1^- = S_{11}\, V_1^+ + S_{12}\, V_2^+ \,, \tag{5}$$

$$V_2^- = S_{21}\, V_1^+ + S_{22}\, V_2^+ \,, \tag{6}$$

where S_{11}, S_{12}, S_{21}, and S_{22} are complex constants, called S-parameters; they fully characterize the electrical behavior of the device represented by the box, at its reference planes (i.e., planes with z_1 and z_2 equal to zero, since we have made the reference and calibration planes coincide in Fig. 1). When written in matrix form these four parameters are collectively referred to as the S-matrix of the device.

There are many alternative ways to electronically represent a two-port linear device, and you should have already encountered several of them in your previous classes (e.g., impedance matrix, admittance matrix, transmission matrix, etc.). Each representation has its own applications, conveniences, and limitations. The S-matrix is just another representation, one that is very convenient to handle high-frequency circuits connected using transmission lines and waveguides. Its convenience stems from the fact that it deals with incident and reflected traveling waves, as opposed to dealing with the usual circuit voltage and currents, for example (the case of the impedance and admittance matrices).

Observe that once you have the scattering parameters in hand (determined using analysis, experiments, or both), other matrix representations can be derived from them, and vice verse. For instance, the impedance matrix, which is often convenient to use when dealing with circuits, since you can use circuit techniques on them, is given by

$$V_1 = Z_{11}\, I_1 + Z_{12}\, I_2 \,, \tag{7}$$

$$V_2 = Z_{21}\, I_1 + Z_{22}\, I_2 \,, \tag{8}$$

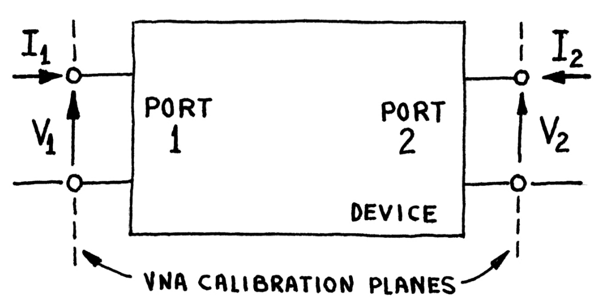

Figure 2: Impedance representation of a general two-port device

where V_1, V_2, I_1, and I_2 are the usual voltages and currents on the device ports (at the device reference planes, as shown in Fig. 2, not waves). If desired, the corresponding

Z-parameters can be obtained form the scattering matrix using the transformation formulas[2]

$$Z_{11} = Z_0 \, \frac{(1+S_{11})(1-S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \,, \tag{9}$$

$$Z_{12} = Z_0 \, \frac{2\,S_{12}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \,, \tag{10}$$

$$Z_{21} = Z_0 \, \frac{2\,S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \,, \tag{11}$$

$$Z_{22} = Z_0 \, \frac{(1-S_{11})(1+S_{22}) + S_{12}S_{21}}{(1-S_{11})(1-S_{22}) - S_{12}S_{21}} \,. \tag{12}$$

In our laboratories we will have the opportunity to make good use of both the S and the Z parameters, as you will see.

As already hinted above, an advantage of the Z parameters is that they can be readily manipulated using the circuit techniques you are already familiar with (i.e., Kirchhoff's voltage and current laws). To directly manipulate S parameters, signal flow graph techniques are needed instead. Although the ability to manipulate signal flow graphs is also an important and useful skill, which can be found discussed in the available literature, it will not be considered in the present laboratory[3,4,5]. Whenever we need to manipulate S parameters we will then have to resort to either the provided formulas, or to manipulating instead the corresponding Z parameters.

The scattering, impedance, and other two-port matrices have many useful properties. In particular, reciprocal devices have $S_{21} = S_{12}$ and $Z_{21} = Z_{12}$. Any passive circuit made with common components (e.g., circuits made with only resistors, capacitors, inductors, transmission lines, antennas, etc.) can be proven to be reciprocal. However, active electronic circuits (e.g., transistor amplifiers) are almost always non reciprocal. This reciprocity property is not at all related to symmetry; a passive circuit can be completely asymmetrical and still be reciprocal.

Observe that, when the circuit is reciprocal (i.e., $Z_{21} = Z_{12}$), Eqs. 7 and 8 are just the Kirchhoff's voltage laws for the two meshes present in the T-circuit depicted in Fig. 3 (the impedance values are shown inside the corresponding rectangles). Consequently, the circuit of Fig. 3 provides a convenient equivalent representation for any reciprocal two-port device, and hence can be used in any desired manipulations, etc. In other words, the representations provided mathematically by Eqs. 7 and 8 and schematically by Fig. 3 are equivalent.

[2] See for example D. M. Pozar, *Microwave Engineering*, Third ed., John Wiley & Sons, Inc., 2005, pag. 187.

[3] S. J. Mason, "Feedback Theory–Some Properties of Signal Flow Graphs," Proc. Inst. Radio Eng., vol. 41, pp. 1144-1156, Sep. 1953.

[4] S. J. Mason, "Feedback Theory–Further Properties of Signal Flow Graphs," Proc. Inst. Radio Eng., vol. 44, pp. 920-926, Jul. 1956.

[5] D. K. Cheng, *Analysis of Linear Systems*, Addison-Wesley Publishing Co., Inc., 1959, Sec. 9-4.

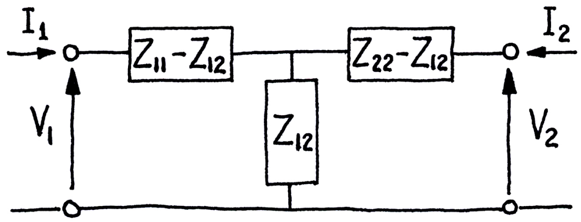

Figure 3: Equivalent circuit of a general reciprocal two-port device ($Z_{21} = Z_{12}$)

Now let's imagine that we connect a matched load Z_L to port 2 of the device (i.e., $Z_L = Z_0$). In this situation $V_2^+ = 0$, since the load is matched to the transmission line, and Eq. 5 yields

$$S_{11} = \left. \frac{V_1^-}{V_1^+} \right|_{V_2^+=0} . \tag{13}$$

However, we also know that the reflection coefficient $\Gamma^{(1)}$ of the port 1 of the device is given by

$$\frac{V_1^-}{V_1^+} = \Gamma^{(1)} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0} , \tag{14}$$

where $Z_{in}^{(1)}$ is the input impedance of the port 1 of the device and Z_0 is the internal characteristic impedance of the VNA. Therefore S_{11} is the reflection coefficient of the port 1 of the device, *with the port 2 matched*, namely

$$S_{11} = \left. \Gamma^{(1)} \right|_{\text{Port 2 matched}} . \tag{15}$$

Similarly, if we match the port 1 of the device we obtain from and Eq. 6

$$S_{22} = \left. \frac{V_2^-}{V_2^+} \right|_{V_1^+=0} . \tag{16}$$

But the reflection coefficient $\Gamma^{(2)}$ of the port 2 of the device is given by

$$\frac{V_2^-}{V_2^+} = \Gamma^{(2)} = \frac{Z_{in}^{(2)} - Z_0}{Z_{in}^{(2)} + Z_0} , \tag{17}$$

where $Z_{in}^{(2)}$ is the input impedance of the port 2 of the device and Z_0 is again the internal characteristic impedance of the VNA. Therefore S_{22} is the reflection coefficient of the port 2 of the device, *with the port 1 matched*, namely

$$S_{22} = \left. \Gamma^{(2)} \right|_{\text{Port 1 matched}} . \tag{18}$$

Observe then that it is relatively easy to measure the scattering parameters S_{11} and S_{22} of a device; all we need to do is measure the reflection coefficient (or input impedance) of a port with the other port matched.

To determine S_{21} and S_{12} we observe that, from Eqs. 6 and and 5,

$$S_{21} = \left. \frac{V_2^-}{V_1^+} \right|_{V_2^+=0} , \tag{19}$$

$$S_{12} = \left. \frac{V_1^-}{V_2^+} \right|_{V_1^+=0} . \tag{20}$$

Hence S_{21} is the forward voltage gain of the device with the port 2 matched (also called the transmission coefficient from port 1 to port 2, or the forward transmission coefficient) and S_{12} is the reverse voltage gain of the device with the port 1 matched (also called the transmission coefficient from port 2 to port 1, or the reverse transmission coefficient).

Because it is relatively easy to match the ports of a device and take measurements of the incident, reflected, and transmitted waves, even at very high frequencies (this is routinely done by VNAs, including providing the required matched conditions), S-parameters provide a very convenient way to characterize devices at very high frequencies.

Note that the above equations were derived for a two-port device. In the case of a one-port device there is only port 1, and hence just one S parameter, namely S_{11}, and it is simply given by

$$S_{11} = \frac{V_1^-}{V_1^+} = \Gamma^{(1)} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0}\,, \tag{21}$$

where Z_0 is again the internal characteristic impedance of the VNA.

Also note that the VNA will always measure the reflection coefficient $\Gamma^{(1)}$ of the port 1 of a two-port device according to Eq. 14, and it will always call it S_{11}. However, the measured value will only be the S_{11} of the device (i.e., the one of Eq. 5) if you perform the measurement with the port 2 of the device matched. Connecting the port 2 of the device to the Port 2 of the VNA will provide this required match to measure both S_{11} and S_{21}.

Armed with the above recently-acquired S-parameters knowledge, lets now take a closer look at the parameters reported on the NanoVNA-F screen (see Fig. 4). The NanoVNA-F was properly calibrated, is sweeping frequency between 50 KHz and 1 GHz with 301 points (note the yellow 301P that is displayed on the lower left of the screen), and has matched loads connected to both ports (the marker 1 is at 50 KHz).

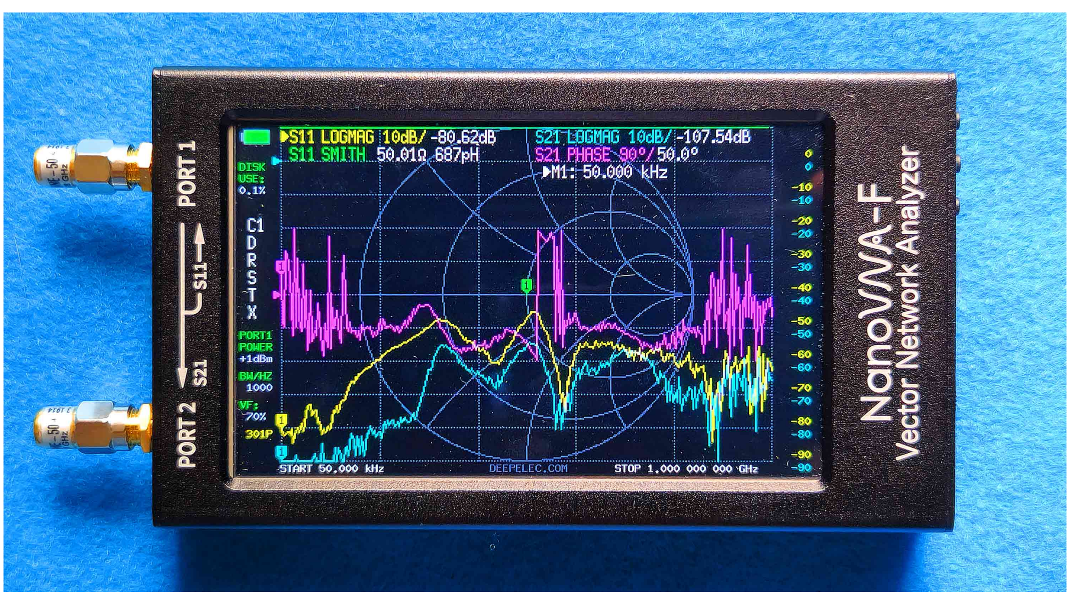

Figure 4: Display of the NanoVNA-F

The yellow legend (first line on top left) reads “S11 LOGMAG 10dB/ -80.62dB” and is associated to the yellow curve. Hence the yellow curve is displaying the magnitude of the measured S_{11} value, in dB (i.e., $20 \log |S_{11}|$) on a vertical scale with 10 dB per division (yellow scale on the right of the screen), where the 0 dB line is the second line from the top of the screen (the cyan triangle at the top left indicates the 0 dB level). The -80.62 dB value is the reflection coefficient of the load at the marker frequency.

The cyan legend (first line on the top right reads “S21 LOGMAG 10dB/ -107.54dB dB” and is associated with the cyan curve. Hence the cyan curve is displaying the magnitude of the measured S_{21} value, in dB (i.e., $20 \log |S_{21}|$) on a vertical scale with 10 dB per division, where the 0 dB line is the second line from the top of the screen (the cyan triangle at the top left indicates the 0 dB level). The -107.54dB dB value is the voltage gain (or transmission coefficient) from Port 1 to Port 2, at the marker frequency (it is reporting a very low value since the two ports have 50 Ω loads on them, and hence there is an insignificant signal going from Port 1 to Port 2).

The magenta legend (second line on top right) reads “S21 PHASE 90°/ 50.00°” and is associated to the magenta curve. Hence the magenta curve is displaying the phase of the measured S_{21} value on a vertical scale with 90° per division, where the 0° line is at the middle of the screen (the magenta triangle at the middle left indicates the 0° line). The 50.00° value is the phase of the voltage gain from Port 1 to Port 2, at the marker frequency (it is basically reading noise since the $|S_{21}|$ is very low).

The green legend (second line on the top left) reads “S11 SMITH 50.01Ω 687pH” and is associated with the green line that is displaying the S_{11} on the grey Smith Chart (in this case just a point at the center of the Smith Chart, where the green marker 1 is, since the load in Port 1 is 50.01 Ω). The two numerical values of the legend are reporting the measured resistive part of the impedance connected to Port 1 (i.e., 50.01Ω) as well as its series capacitance or inductance, at the frequency associated with marker 1 (i.e., 50 KHz). The reported value of 687 pH, which has a reactance of only 215.8 $\mu\Omega$ at 50 KHz, is associated with the VNA measurement uncertainty since the measured $|S_{11}|$ is very small. The phase associated with S_{11} is not being shown. However, the user can select to have it displayed, if desired, from the NanoVNA-F menu.

3 Direct Measurement of the Characteristic Impedance, Attenuation Constant, and Velocity Factor of Transmission Lines

In a previous experiment we used the NanoVNA-F to determine the characteristic impedance Z_0, the velocity factor VF, and the attenuation factor α of transmission lines at a few discrete frequencies. We are now going to use the acquired experience, together with what we just learned about S-parameters, to determine these same parameters across the 10 KHz to 1.5 GHz frequency range.

We will be using the NanoVNASaver software to perform the measurements and save the results to a file. And afterwards we will create a Matlab code to read the file generated by the NanoVNASaver, process the measured results, and conveniently output the desired information in graphical form.

1. Start by connecting the NanoVNA-F to a PC computer and powering it up. Then run the NanoVNASaver software and make sure it is properly communicating with the NanoVNA-F.

2. Select the desire frequency range to sweep. Since we will be sweeping the frequency over the 10 KHz to 1.5 GHz range, please make sure that the "Start" and "Stop" fields of the "Sweep Control" have the correct frequencies (top left corner of the screen).

   The NanoVNASaver software allows measurements at more that just the maximum of 301 points provided by the NanoVNA-F hardware. This is accomplished by selecting the desired number of 101 point segments in the "Segments" field. For example, with 15 segments one has $15 \times 101 = 1515$ measurement points, and hence the corresponding frequencies will be spaced by about 1 MHz ((1500-0.01)/(1515-1) = 0.990746 MHz, to be precise). I suggest that you go with 15 segments for the current measurements, since it provides a reasonable compromise between measurement time and frequency resolution.

   The "Sweep settings ..." button opens a field that allows you to select a few other parameters. Of particular interest is the "Number of measurements to average" and "Number to discard." By averaging and discarding outliers one can reduce the noise and improve the accuracy of the measurements. I suggest that you go with an average of 10 measurements and discard none (i.e., put 0 in the associated field).

3. Calibrate the NanoVNA-F directly at its SMA female ports. This is similar to what we have previously been storing in the memory position 1.

   The NanoVNA-F can obviously be directly calibrated without software assistance, the calibration saved, recalled, and the calibration will then be used by the NanoVNASaver (this is what we have been doing up to this point). However, since this calibration is hardware limited to a maximum of 301 frequencies, it is necessarily interpolated by the NanoVNASaver software. This may introduce avoidable inaccuracies. It is then better to perform the calibration from the NanoVNASaver itself, since it will be done at all points of the selected sweep. This can be done by clicking the "Calibration ..." button (at the lower left of the screen). I suggest that you use the available "Calibration assistant," as it will conveniently guide you through the entire calibration process.

   When the calibration is complete, please make sure to save it to a convenient location in your computer hard drive. This way you can recall it whenever needed. This can be accomplished by clicking on the "Save calibration" button. A convenient name for the saved calibration file is probably "SMAF-10kHz-1.5GHz" (the .cal file extension will be automatically added to the file name when it is saved).

4. For our measurements you will be using the SMAM-SMAM 2000 mm long section of RG316 coaxial cable that you have in our laboratory kit. Locate this cable,

precisely measure its length ℓ (you will be needing it below), and then connect the cable to the calibrated Port 1 of the NanoVNA-F.

Note again that what you need here is the length ℓ of the part of the cable that actually carries signal when the cable is connected and being used (if you are still having problems understanding what this means, please look inside the hexagonal nuts of the SMA male connectors at the end of the cables).

5. Connect the short to the end of the coaxial cable and tell the NanoVNASaver to measure the S-parameters of the cable input by clicking on the "Sweep" button (top left of the screen). The green bar will indicate the measurement progress and will become full when done. The results are shown on both the NanoVNA-F and computer screens.

6. Save your results to a Touchstone file for posterior processing. This is done by clicking the "Files ..." button (at the lower left of the computer screen) and then clicking the "Save 1-Port file (S1P)" (the s1p file extension stands for S-parameters one port). The other saving option includes both the S_{11} and the S_{21} data, which is unnecessary here since we just need the S_{11} parameter of the cable. Please name the saved file "Shorted_cable.s1p."

   The Touchstone file format is a de-facto industry standard ASCII file (a text file that can very conveniently be opened by pretty much any text editor). This file format was created in the early eighties to be used in a circuit simulator that is long gone. However, the format created has endured very well to this day, and is used by pretty much all VNAs and associated software.

7. Open the s1p file that was saved in the previous item and take a good look inside. You will need to create a Matlab code to open, read, and process the data in the steps below.

   Here is the beginning of a typical s1p file saved by the nanoVNAsaver software:

```
# HZ S RI R 50
10000 -0.9795324339868982 0.004383432505081511
1000746 -0.940407227282745 0.13214126046173846
1991492 -0.9096249375908326 0.2410657871089827
2982238 -0.869749029525898 0.34474471094726244
   .             .                   .
   .             .                   .
   .             .                   .
```

   The first line is a heading indicating that the subsequent lines start with a frequency (in Hz), followed by the corresponding S parameters (complex numbers, real and imaginary parts in separate columns, linear scale). The number 50 indicates that the characteristic impedance of the system used in the measurements was 50 Ω. The lines of data start after the heading line, one S-parameter per frequency, and continue all the way to the last frequency.

8. Replace the short circuit at the end of the coaxial cable by an open circuit, measure the cable S_{11} at the same frequencies, and save the data to another s1p file for posterior processing. Please name the saved file "Opened_cable.s1p."

9. Recall once again that the input impedance Z_i of an arbitrarily terminated transmission line of length ℓ, is provided by by[6]

$$Z_i = Z_0 \, \frac{Z_L + Z_0 \, \tanh(\gamma\ell)}{Z_0 + Z_L \, \tanh(\gamma\ell)} \,, \tag{22}$$

where Z_L is the impedance loading the end of the transmission line and $\gamma = \alpha + j\beta$ is the (complex) propagation factor of the transmission line.

10. Consider now the input impedance Z_{is} of a short-circuited transmission line (line with $Z_L = 0$) and the input impedance Z_{io} of a open-circuited transmission line (line with $Z_L = \infty$), use Eq. 22 to show that

$$Z_{is} \times Z_{io} = Z_0^2 \,, \tag{23}$$
$$Z_{is} \,/\, Z_{io} = \tanh^2(\gamma\ell) \,, \tag{24}$$

and from these two equations derive the useful results

$$Z_0 = \sqrt{Z_{is} Z_{io}} \,, \tag{25}$$
$$2\gamma\ell = \ln\left[\frac{\sqrt{Z_{io}/Z_{is}} + 1}{\sqrt{Z_{io}/Z_{is}} - 1}\right] \,. \tag{26}$$

11. Also derive the useful results

$$\gamma \times Z_0 = R + j\omega L \,, \tag{27}$$
$$\gamma \,/\, Z_0 = G + j\omega C \,, \tag{28}$$

which allow the easy determination of the four circuit parameters of the transmission line (i.e., R, L, G, and C per unit length), once the characteristic impedance and propagation factor are known.

12. Write a Matlab code to read the above two s1p files, process the measured data according to the above equations, and generate three separate plots to summarize your results: 1 - the (complex) characteristic impedance Z_0 as a function of frequency; 2 - the velocity factor VF as a function of frequency; and 3 - the attenuation constant α as a function of frequency. Please make sure the axis of your plots are properly labeled and the corresponding units are clearly depicted.

Observe that the above equation that yields the complex $2\gamma\ell$ values requires the calculation of the logarithm of a complex argument. This presents an interesting

[6] From Sec. 9-4 of D. K. Cheng, *Field and Wave Electromagnetics*, Second ed., Addison-Wesley Pub. Co., 1989.

difficulty since the $\ln z$ is a multivalued function[7]. In other words, for an arbitrary complex number $z = re^{j\theta}$,

$$\ln z = \ln r + j(\theta + 2n\pi)\,, \tag{29}$$

where $n = 0, \pm 1, \pm 2, \pm 3, \ldots$. This happens because unavoidably $e^{\ln z} = z$ for any of the above $\ln z$ values. Since the imaginary part of the $\ln z$ yields the phase factor β, one is then at loss of what is the correct n value to use (the correct branch of the $\ln z$ to use). The answer to this conundrum rests on the fact that at very low frequencies the $\ln z$ function of Matlab (and also of your calculator) yields the principal value of the $\ln z$ (i.e., value with $n = 0$), and this is associated with the correct β. The problem surfaces as the frequency increases though, as at some point the next value of n needs to be used, and if this is not done a 2π discontinuity will occur on β. However, because this discontinuity is unreal (after all β is a continuous function of frequency), and we are doing a frequency sweep, the 2π jumps can be easily detected and corrected. This outcome can be easily accomplished through an intrinsic Matlab function called "unwrap," and I suggest that you learn how it operates and use it on the imaginary part of $2\gamma\ell$.

To further assist you in your programming efforts, here are the Matlab lines that I usually use to read my s1p data files (in this case the file is named Shorted_cable.s1p):

```
fns = 'Shorted_cable.s1p';
[f(:),S11_real(:),S11_imag(:)] = ...
                              textread(fns,'%f %f %f','headerlines',1);
S11 = complex(S11_real,S11_imag);
```

To demonstrate the type of plot that you should be generating, Fig. 5 depicts a plot of one of my measured *VF* results.

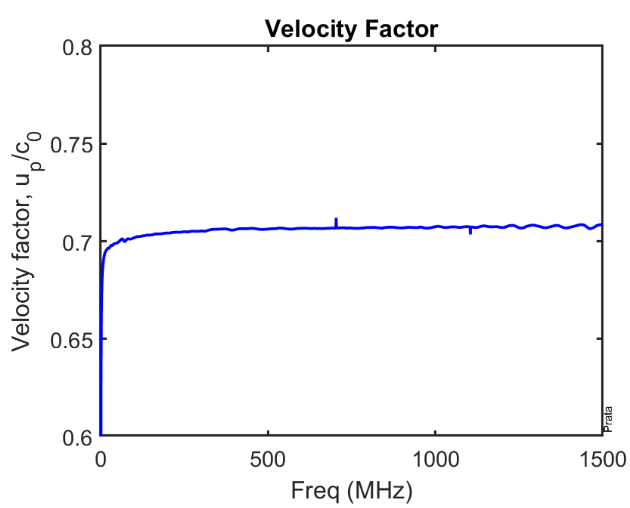

Figure 5: Measured velocity factor of a coaxial cable

[7]For much more on this important topic please see for example A. D. Wunsch, *Complex Variables With Applications*, Addison-Wesley Pub. Co., 1983, pag. 91.

13. Observing that the measured Z_0 is a complex number, make sure that your plot depicts both its real and imaginary parts.

    Also make sure to comment and explain the presence of an imaginary part on Z_0.

14. For the real part of Z_0 and the α figures only, please make sure your plots show non-connected points at each measurement frequency. This will be helpful for the subsequent item.

15. As you can tell, there is a significant amount of undesirable artifacts on your Z_0 and α plots. Reduce this problem by using the Matlab function "polyfit" to perform the least-squares fit of a polynomial to all your measured points and add the corresponding continuous curve to your Z_0 and α figures. By the way, don't get carried away and end up with a polynomial of high degree; the idea here is to get a good fit without introducing artificial ringing, and this is accomplished using the lowest-degree polynomial that will do a good job.

16. Compare the plotted Z_0, VF, and α results obtained on your experiment with the ones obtained in our previous laboratory (i.e., Lab. 02), as well as with the expected values, and provide appropriate comments and conclusions.

17. Repeat the above measurements for one of the two short RG316 coaxial test cables ($\sim$210 mm long) that come in the NanoVNA-F box, provide the corresponding plots, and comment on the results obtained.

    Observe that the measured attenuation results obtained for the short test cable is incorrect and explain why.

4 Measurement of the S Parameters of Two-Port Devices

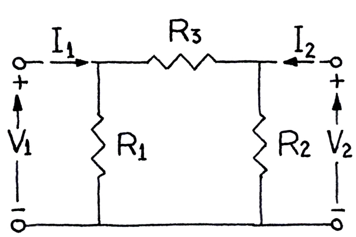

Figure 6: Π circuit

In this experiment we will measure all four S-parameters of a relatively simple two-port device and use Matlab to process the results to obtain the associated impedance matrix (parameters of Eqs. 7 and 8). The device that we will be measuring is the simple Π-circuit shown in Fig. 6, made using the 25.5, the 49.9, and the 100 Ω through-hole resistors from our laboratory kit parts.

1. Connect the two short test cables that came with the NanoVNA-F to its Ports 1 and 2 and perform a complete calibration at the SMA male connectors at the end of the cables, using the female standards. Use for thru the provided SMAF-SMAF adapter. As before, calibrate over the 10 KHz to 1.5 GHz range using 1515 frequency points.

    As we learned in a previous laboratory, common through-hole resistors only work well at relatively low frequencies. The above 10 KHz to 1.5 GHz range is then quite excessive for the Π-circuit we will be assembling and measuring below (the circuit will stop operating in any reasonable way well before the 1.5 GHz frequency is reached). However, we will still perform the NanoVNA-F calibration up to 1.5 GHz because we will be measuring the RG316 cable attenuation again in the next section, over the 10 KHz to 1.5 GHz range, using an alternative simpler yet accurate method.

2. Without cutting their leads, connect the three resistors to the SMAF-DIP8-SMAF test fixture. The R_1=25.5 Ω resistor (color coded red, green, and green) goes between the center pin and ground of the side that is connected to Port 1. The R_2=100 Ω resistor (color coded brown, black, and black) goes between the center pin and ground of the side that is connected to Port 2. And the R_3=49.9 Ω resistor (color coded yellow, white, and white) goes between the two cables' center pins, connecting the Port 1 and Port 2 coaxial center conductors.

3. Measure the S_{11} and S_{21} of the Π-circuit and save it to an appropriate s2p file (the extension s2p is automatically added and stands for S-parameters two ports). Let's name this file "Forward.s2p."

4. Observe that the NanoVNA-F has both a signal generator and a receiver on its Port 1, but it only has a receiver on its Port 2 (no signal generator). Because of this it only measures S_{11} and S_{21}. Although this is a limitation, it is not an insurmountable problem. To measure the remaining two S-parameters of the scattering matrix (i.e., S_{22} and S_{12}) all you have to do is disconnect the two test cables from the test fixture, rotate the Π-circuit by 180 degrees (to flip the connections), reconnect the test fixture to the cables, remeasure, and then save the results in another s2p file (please name this file "Reverse.s2p"). These second set of results will effectively be the desired S_{22} and S_{12} parameters.

   Measure the S_{22} and S_{12} of the Π-circuit and save it to the appropriate s2p file.

5. Write a Matlab code for reading the above two s2p files and plotting the amplitude and phase of each of the four S parameters (in dB and degrees, respectively) over the 10 KHz to 400 MHz frequency range (plenty of range to characterize your low-frequency Π-circuit). Include these four plots in your lab report.

   Note that first you will need to open one of the two s2p files with a suitable ASCII text editor and familiarize yourself with the s2p file data format (the format is similar but not identical to the s1p file format). Only after you do this you will be able to properly write the Matlab instructions to read the s2p file.

   To demonstrate the type of plot that you should be generating, Fig. 7 depicts a plot of one of my measured S_{11} results. Note that the Matlab function "yyaxis" was used to create the required plot with two distinct vertical axes.

6. Augment your Matlab code to use Eqs. 9 – 12 to calculate and plot the four impedance matrix parameters of your Π-circuit as a function of frequency (again over the 10 KHz to 400 MHz frequency range). The four plots should display the real and imaginary parts of each parameter, in Ohms.

Please make sure to discuss the measured results; in particular explain why the Z parameters vary with frequency.

To demonstrate the type of plot that you should be generating, Fig. 8 depicts a plot of one of my measured Z_{11} results.

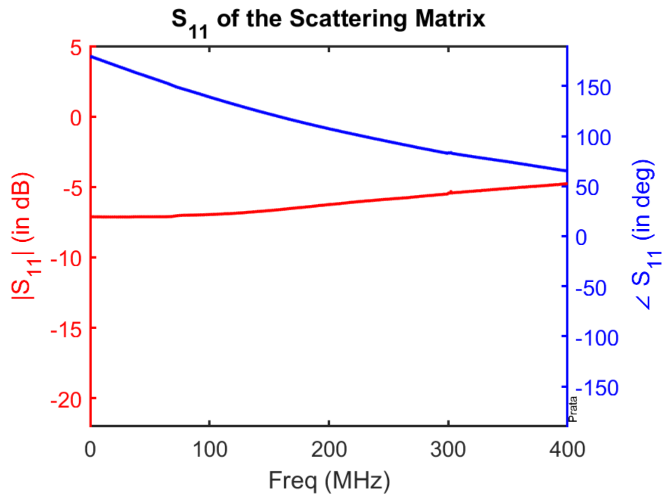

Figure 7: Measured S_{11} of the Π-circuit

7. Analytically determine the expected low-frequency values of Z_{11}, Z_{12}, Z_{21}, and Z_{22} and provide your derivation and results. Compare and comment on the accuracy of your measured results.

   As an example of how to handle this item, put a voltage source on the left side of Fig. 6, an open circuit on its right side, and then use Eq. 7, together with circuit theory, to determine Z_{11}. Similar procedures can be used to determine Z_{12}, Z_{21}, and Z_{22}.

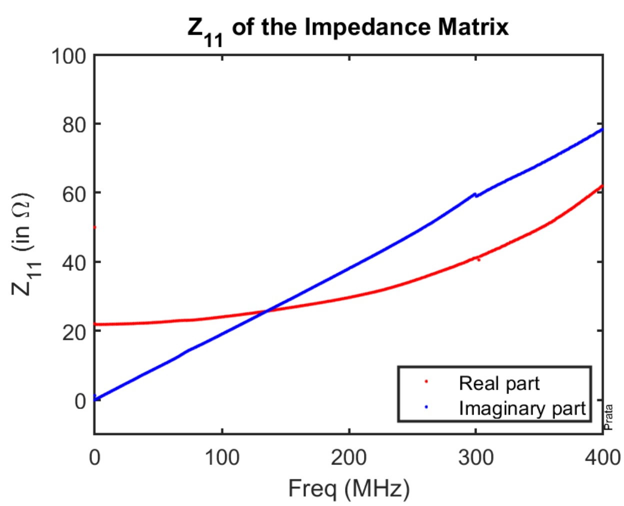

Figure 8: Measured Z_{11} of the Π-circuit

8. More sophisticated VNAs are capable to perform measurements with their internal frequency synthesizer automatically connected to either one of its test ports (the NanoVNA-F synthesizer only connects to Port 1. Its Port 2 only has a matched detector). This allows them to measure all S-parameters without resorting to the switching trick discussed previously. In addition to measurement speed and convenience, this capability also allows them to fully calibrate both ports, and hence improve the measurement accuracy (for example, the reflection coefficient provided by the internal matched termination of Port 2 is significantly improved).

   The NanoVNA-F is only capable to fully calibrate whatever is connected to its Port 1 (i.e., it will calibrate out its internal imperfections, as well as deleterious effects of test cables and adapters, only if they are connected to its Port 1). The internal imperfections of the NanoVNA-F Port 2, as well as the imperfections of any cables and adapters connected to its Port 2 are left mostly uncalibrated. To see that this is indeed the case, remove the SMAF-DIP8-SMAF test fixture, connect the two test cables together using an SMAF-SMAF adapter, and therefore measure the amplitude of the reflection coefficient Γ_2 of the SMAF-SMAF adapter, plus the test cable connected to Port 2, plus the internal impedance of the Port 2 of the NanoVNA-F. Report the measured Γ_2 through a suitable plot.

   To provide a value for comparison, note that a VNA that allows for full two-port calibration would typically be able to provide an effective Γ_2 with amplitude lower than at least -40 dB (achieved through calibration). A very low Γ_2 value increases significantly the measurement accuracy since, for example, the measurement of a device S_{21} in principle requires the port 2 of the device to be perfectly matched by the VNA (see Eq. 19).

9. The previous Γ_2 measurement shows a lot of undesirable interference between the various components connected to the Port 2 of the NanoVNA-F (the oscillations present on the plot). However, it is possible to measure the reflection coefficient of just the Port 2 of the NanoVNA-F. To do this first remove the SMAF-SMAF adapter and the test cable that is connected to Port 2. Then connect the test cable from Port 1 directly to Port 2. Report the measured Γ_2 through a suitable plot, and discuss and compare the results with the ones of the previous item.

5 Alternative Way to Measure the Attenuation and Velocity Factors of Transmission Lines

Observing that a section of transmission line is a two-port device, the knowledge and experience previously acquired from dealing with S-parameters should provide you with a relatively simple and accurate method to measure both the attenuation factor α and the velocity factor VF of transmission lines (a method that does not require any of the mathematical processing used in Sec. 3). In this section we will explore this alternative better method.

1. Start by using Eqs. 5 and 6 to derive the S-parameters of an arbitrary section of

transmission line of length ℓ.

2. How does the $|S_{21}|$ of the transmission line section relates to the attenuation α of the cable?

3. How does the $\angle S_{21}$ of the transmission line section relates to the velocity factor VF of the cable?

4. Measure the S_{21} of the same ∼2000 mm long RG316 coaxial cable you measured in Sec. 3, over the 10 KHz to 1.5 GHz frequency range, and provide a plot of the results obtained.

   Note that for this measurement it is convenient to calibrate the VNA with a 200 mm long RG316 test cable connected to Port 1 and no cable connected to Port 2. This allows you to use a single SMAF-SMAF adapter in order to be able to connect the ∼2000 mm long RG316 coaxial cable to be measured.

5. Use the results of the previous items to provide a plot of the measured attenuation of the RG316 coaxial cable, in dB/m, over the 10 KHz to 1.5 GHz range. Comment and compare with both the manufacturer specifications as well as with the results obtained in Sec. 3.

6. Use the results of the previous items to provide a plot of the measured velocity factor of the RG316 coaxial cable over the 10 KHz to 1.5 GHz range. Comment and compare with both the manufacturer specifications as well as with the results obtained in Sec. 3.

7. Repeat the above three items for the ∼200 mm long coaxial test cable that comes in the NanoVNA-F box, again over the 10 KHz to 1.5 GHz frequency range.

   Can you trust the dB/m attenuation values that you just measured for the ∼200 mm long coaxial test cable? Please make sure to justify your answer.