Transmission Lines in the Time Domain

Laboratory 05 Manual
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1 Laboratory Objective

Up to this point we have been considering primarily the characteristics of transmission lines in the frequency domain; pretty much all our measurements and analysis have always been performed using time-harmonic signals at specific frequencies. Even though we have been using incident and reflected waves to learn and handle scattering parameters, the time that these waves take to propagate along transmission lines so far was not extensively used to our advantage.

In this laboratory we will consider explicitly the time waves take to propagate along transmission lines, and will use it to perform accurate measurements. We will also use our frequency-domain measurements to synthesize pulses, propagate them in transmission lines, and make measurements. To be successful you will need to have studied in detail the material covered in Sec. 9-5 of our textbook[1].

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report should have each of its sections and subsections numbered according to this laboratory manual, and be a detailed document with all your design efforts, calculations, associated Smith Charts, measurement results, conclusions, drawings, plots, relevant photos of all constructed components (to showcase your very important high-frequency craftsmanship), and printouts of any developed software.

Note that, to maximize the learning experience, the laboratory has been designed to be carried out individually, hence each person in the class received their own individual lab kits. Consequently, the experiment and the corresponding report has to be done completely individually.

2 Moving the Position of the VNA Calibration Planes and Some of its Applications

As previously discussed, when a VNA is calibrated in principle it will only report correct S-parameter results at the two calibration planes locations (the two dashed vertical lines in Fig. 1). The device being measured has two reference planes (shown by the circles on each of its two ports). Note that *the calibration planes are associated with the VNA and the reference planes are associated with the two-port device*; in the figure the calibration and reference planes are not coinciding and a coordinate system is shown with its origin at the Port 1 VNA calibration plane. For S_{11} the Port 1 calibration plane location is the

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[1]D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., 1989.

physical plane of the calibration short, open, and load (note that it is very important to have the three coincide). For S_{21} the Port 2 calibration plane location is the physical plane where the calibration plane of the Port 1 of the NanoVNA-F was connected to the Port 2 of the NanoVNA-F when the thru calibration was done.

During an S-parameters' measurement the calibration and reference planes should be made to coincide. However, unfortunately this is not always possible. A common example of this not happening is when the two-port device has female SMA connectors and the two VNA test cables end in male SMA connectors. Since for the thru calibration an SMAF-SMAF adapter is required to connect the ends of the two test cables together, this adapter becomes part of the thru calibration. The problem is that, when the two-port device is measured, this adapter has to be removed. Because of the absence of the SMAF-SMAF adapter, the calibration and reference planes do not coincide on the Port 2 of the device, and this introduces an error in the S_{21} measurement.
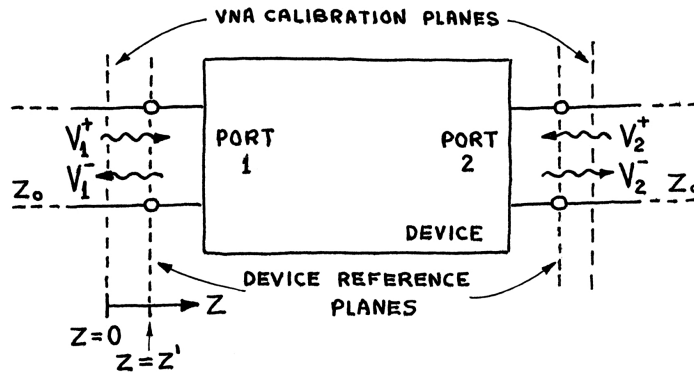

Figure 1: Wave representation of a general two-port device

In the experiment below we will investigate a useful VNA feature that allows movement of the VNA calibration planes location along the transmission line. This allows for partial correction of the SMAF-SMAF adapter error just mentioned (by making the VNA calibration planes and the device reference planes coincide), and also provides useful additional flexibility.

To see how the calibration plane movement can be accomplished, let's assume that the location of the calibration plane of Port 1 is at $z = 0$ and the $+\hat{z}$ direction is towards the device (as shown in Fig. 1). We can then write the incident and reflected waves on Port 1, at any z location, as

$$V_1^+ = V_1^+ \, |_{z=0} \; e^{-\gamma z} \,, \tag{1}$$
$$V_1^- = V_1^- \, |_{z=0} \; e^{+\gamma z} \,, \tag{2}$$

where the transmission line has a propagation factor $\gamma = \alpha + j\beta$.

To move the calibration plane of Port 1 from $z = 0$ to a new location with $z = z'$ we rewrite the above equations as

$$V_1^+ = V_1^+ \, |_{z=z'} \; e^{-\gamma(z-z')} \,, \tag{3}$$
$$V_1^- = V_1^- \, |_{z=z'} \; e^{+\gamma(z-z')} \,, \tag{4}$$

where

$$V_1^+\big|_{z=z'} = V_1^+\big|_{z=0}\, e^{-\gamma z'}\,, \tag{5}$$

$$V_1^-\big|_{z=z'} = V_1^-\big|_{z=0}\, e^{+\gamma z'}\,, \tag{6}$$

and $V_1^+\big|_{z=z'}$ and $V_1^-\big|_{z=z'}$ are the complex amplitudes of the traveling waves at the moved calibration plane location (now located at $z = z'$).

From the above Eqs. 5 and 6 we see that the movement of the calibration plane of Port 1 by a distance z' will cause the $S_{11}\big|_{z=z'}$ to become

$$S_{11}\big|_{z=z'} = \frac{V_1^-}{V_1^+}\bigg|_{z=z'} = \frac{V_1^-}{V_1^+}\bigg|_{z=0} e^{+2\gamma z'} = S_{11}\big|_{z=0}\, e^{+2\gamma z'}\,. \tag{7}$$

The process of moving the calibration plane location by a distance $\ell = z'$ is usually referred to as "port extension," in VNA parlance. However, in the NanoVNA-F it is referred to instead as "electrical delay." Regardless of how it is called, VNAs allow you to provide the desired displacement of the Port 1 location by giving the time τ that the wave V_1^+ will take to go to the plane with $z = z'$ and come back to the $z = 0$ plane (travel a distance 2ℓ), together with the transmission line velocity factor VF and attenuation factor α. And from these three parameters it will conveniently implement the desired S_{11} parameter change given by Eq. 7 automatically (i.e., do the required complex phase adjustment). Note that τ can be either positive or negative, a negative τ value corresponds to a negative z' movement of the calibration plane.

Assuming that the transmission line losses are small and hence can be safely neglected (i.e., $\alpha = 0$), the above Eq. 7 simplifies to

$$S_{11}\big|_{z=z'} = S_{11}\big|_{z=0}\, e^{+j2\beta z'}\,, \tag{8}$$

and this is what is implemented inside the NanoVNA-F (in other words, transmission line losses are ignored in the NanoVNA-F electrical delay implementation).

Recalling that

$$2\ell = u_p\, \tau = c_0\, VF\, \tau\,, \tag{9}$$

where $c_0 = 299,792,458$ m/s is the speed of light in free space, we have a convenient formula for specifying the time delay τ:

$$\tau = \frac{2}{c_0\, VF}\, \ell\,. \tag{10}$$

A convenient rule of thumb to remember is that, since the VF of most coaxial cables is around 0.7, every 1 mm of S_{11} calibration plane movement corresponds to $\tau \approx 10$ ps.

Because of the factor 2 present in the above equations, when moving the calibration plane for S_{11} measurements, the delay τ given by Eq. 10 corresponds to a length ℓ of the calibration plane movement, since the wave V_1^+ that is coming from the NanoVNA-F generator will have to perform a round trip to return to the NanoVNA-F Port 1 as V_1^-.

In a similar manner, the calibration plane of Port 2 can also be moved. However, when moving the calibration plane for S_{21} measurements, the same delay τ corresponds to a length 2ℓ of the calibration plane movement, as there are no round trips involved; the wave V_1^+ coming from the NanoVNA-F generator will simply travel an extra length 2ℓ to surface at the NanoVNA-F Port 2 as V_2^-).

In summary, *for S_{11} a positive value for τ will cause the calibration plane of Port 1 of the NanoVNA-F to be moved a positive distance ℓ towards the device (i.e., away from the Port 1 of the VNA), according to Eq. 10. And for S_{21} the same τ value will cause the calibration plane of Port 2 of the NanoVNA-F to be moved a positive distance 2ℓ towards the device (i.e., away from the Port 2 of the VNA).*

> Another convenient rule of thumb to remember is that, since the *VF* of most coaxial cables is around 0.7, every 2 mm of S_{21} calibration plane movement corresponds to $\tau \approx 10$ ps.

Let's now exercise the above newly learned port-extension capability in a useful experiment. We will measure the velocity factor VF of two devices: the RG 316 coaxial cable and the SMAM-to-SMAM adapter, which has a very small length.

1. Carefully measure the length of the $\sim$210 mm long RG316 NanoVNA-F coaxial test cable. Note again that what you need here is the length of the part of the cable that actually carries signal (if you are still having problems understanding what this means, please look where the dielectric starts, inside the hexagonal nut of the SMA male connectors, at the end of the cables).

2. Observing that we will be doing measurements directly at the female SMA port of the NanoVNA-F, and without using the NanoVNASaver software, select the proper calibration to use. Make sure to verify that the stored calibration is still valid, and to recalibrate the VNA if it isn't.

3. Connect one of the two available sections of RG316 NanoVNA-F test cable to the Port 1 of the VNA, connect the calibration female short to the other end, and observe the displayed complex values of its S_{11} in the Smith Chart (green trace). Note that the phase of the S_{11} is going through a bit more than two full turns (i.e., a bit more than $2 \times 360°$ of phase change with frequency), starting from the short-circuit location. Explain why this is happening.

4. Let's now move the location of the Port 1 of the NanoVNA-F (currently located at the calibration plane) all the way to the end of the cable, to the precise location of the short circuit plane. To do this used the previously boxed results to estimate the required round-trip propagation delay τ and input it into the NanoVNA-F.

   The location where the τ value needs to be stored is located deep inside the NanoVNA-F menu tree, and at a somewhat unexpected place. More specifically, select "DISPLAY," "SCALE," and "ELECTRICAL DELAY." A white numerical keyboard will then appear on the screen and you can then type in the desired time delay value τ, in either ns (nano seconds) or ps (pico seconds). The τ value can be either positive or negative and the left arrow erases typed digits. When

done typing the desired τ value, press either n or p to enter the delay into the NanoVNA-F memory. The value of the entered τ will then show in white letters on the top left of the NanoVNA-F screen (third line, labeled "Edelay."). When $\tau = 0$ no values are shown. Also shown to the right of the delay number will be the corresponding transmission-line distance traveled by a wave during the time τ, for the velocity factor shown in percent. You can enter the velocity factor of your cable (in percent of the velocity of light in vacuum) by selecting "DISPLAY," "TRANSFORM," and "VELOCITY FACTOR" on the menu.

By the way, you can also move the port location inside the NanoVNASaver software. However we are not exercising this option at this time.

5. Take a look at the S_{11} plot on the Smith Chart. Is it what you expect to see with the measurement plane on top of the short-circuit plane? If not iterate the τ value until you get the expected result. When the expected result is achieved the τ value will be the precise round-trip delay from the calibration plane to the short.

6. Use the previously determined delay τ to determine the velocity factor VF of the coaxial cable, and compare it with the previously obtained values. Observe that in principle when you move the calibration plane to the end of the coaxial cable it is almost as if you had calibrated the VNA at the end of the coaxial cable; almost but not really true. When you actually calibrate the VNA at the end of the cable, you are calibrating out all the imperfections of the cable and connectors up to the calibrating plane, and this does not occur when you simply move the calibration plane. Whenever possible, calibrating at the end of the cable will then yield a much higher measurement accuracy. You should then consider moving the calibration plane with prudence, avoid using it, and only using it whenever no better alternative is available.

7. To conclude this section, use what you just learned to measure the velocity factor VF of the SMAM-to-SMAM adapter, and hence determine the relative permittivity ϵ_r of the dielectric used inside the adapter. Observe that the physical length of the adapter is quite small, and you are dealing with waves moving at breakneck speed. Nevertheless, you and the NanoVNA-F should be able to easily handle the above task very precisely.

3 Simulated Pulse Propagation in Transmission Lines

In the previous section you used the time that signals take to propagate along a transmission line to measure distances. The methodology used then hints that if one knows the characteristics of S_{11} as a function of frequency, for example, one should be able to extract information about what is happening on the transmission line of Port 1 as a function of time. The technique that allows this to be done is based on the Fourier

transform pair, which is given by[2]

$$F(\omega) = \mathcal{F}[f(t)] = \int\limits_{-\infty}^{+\infty} f(t)\, e^{-j\omega t}\, dt \,, \tag{11}$$

$$f(t) = \mathcal{F}^i[F(\omega)] = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} F(\omega)\, e^{+j\omega t}\, d\omega \,. \tag{12}$$

The $F(\omega)$ given by Eq. 11 is called the Fourier transform of the time-dependent signal $f(t)$ (represented by $\mathcal{F}[f(t)]$), and the $f(t)$ given by Eq. 12 is called the inverse Fourier transform of the frequency-dependent signal $F(\omega)$ (represented by $\mathcal{F}^i[F(\omega)]$). In the above equations t is time and ω is the angular frequency, given by

$$\omega = 2\pi f \,, \tag{13}$$

with f being the frequency of the signal.

When you measured S_{11} as a function of frequency (i.e., $S_{11}(\omega)$), you measured the $F(\omega) = S_{11}(\omega)$ of Eq. 12. Hence by performing the indicated integral you should be able to calculate the corresponding $S_{11}(t)$.

A very important and useful result is the physical interpretation of this calculated $S_{11}(t)$: it can be shown that $S_{11}(t)$ is the time response of the Port 1 of the two-port device to a wave impulse of unit strength that fired at $t = 0$. In other words, $S_{11}(t)$ *is the time response of the Port 1 of the device when a v_1^+ wave impulse of unit strength, with peak at the calibration plane at the time $t = 0$, is sent towards the device*[3]. This physical interpretation will be proven shortly, but first let's discuss in more detail the unit strength impulse.

An impulse of unit strength, firing at $t = 0$, is usually represented by $\delta(t)$ and has the property that it only has amplitude significantly different than zero near $t = 0$ (e.g., it only lasts for a short time near the firing time), such that

$$\delta(t) = \begin{cases} \infty & \text{if } t = 0 \\ 0 & \text{if } t \neq 0 \end{cases} \,, \tag{14}$$

with the property

$$\int\limits_{-\infty}^{+\infty} \delta(t)\, dt = 1 \,. \tag{15}$$

The impulse is said to have unit strength because of this equal to one integral property[4]. An intuitive way to physically grasp the characteristics of the unit strength impulse can

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[2]D. K. Cheng, *Analysis of Linear Systems*, Addison-Wesley Publishing Co., Inc., 1959, Sec. 5-6.

[3]The situation is similar to the one discussed in the Example 9-11, pag. 478, of D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., 1989, provided that you assume that the pulse duration is very small and its peak voltage is very large.

[4]The impulse is commonly known as the Dirac Delta Function. It is a generalized function which was actually created before Prof. Paul A. M. Dirac's time.

be achieved by considering its Gaussian approximation

$$\delta(t) \approx \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{t^2}{2\sigma^2}}\,,\tag{16}$$

which satisfies Eq. 15 for any chosen standard deviation value (i.e., σ). Note that by reducing σ the impulse can be concentrated as much as desired to the neighborhood of $t = 0$, at the expense of having higher peak values. And in the limit of $\sigma \to 0$, Eq. 16 becomes $\delta(t)$.

Before moving forward let's prove the italicized statement made above, namely: $S_{11}(t)$ is the time response of the Port 1 of the device when a v_1^+ wave impulse of unit strength, with peak at the calibration plane at the time $t = 0$, is sent towards the device. To prove this fact first observe that the spectrum $G_i(\omega)$ associated with an arbitrary signal $g_i(t)$ is given by Eq. 11 as

$$G_i(\omega) = \mathcal{F}[g_i(t)] = \int\limits_{-\infty}^{+\infty} g_i(t)\, e^{-j\omega t}\, dt\,,\tag{17}$$

and since the $S_{11}(\omega)$ provides the S_{11} of the two-port network at each frequency ω, the corresponding frequency-domain response $G_o(\omega)$ of the two-port network will be

$$G_o(\omega) = G_i(\omega) \times S_{11}(\omega)\,,\tag{18}$$

and the associated time-domain response of the two-port network, when it is excited by $g_i(t)$, is then be given by Eq. 12 as

$$g_o(t) = \mathcal{F}^i[G_i(\omega) \times S_{11}(\omega)] = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} G_i(\omega) \times S_{11}(\omega)\, e^{+j\omega t}\, d\omega\,.\tag{19}$$

Now, if $g_i(t)$ is an impulse of unit strength (i.e., $g_i(t) = \delta(t)$), Eqs. 17, 14, and 15 show that

$$G_i(\omega) = \mathcal{F}[\delta(t)] = \int\limits_{-\infty}^{+\infty} \delta(t)\, e^{-j\omega t}\, dt = 1\,,\tag{20}$$

and Eq. 19 yields the impulse response of the two-port network as

$$g_o(t) = \mathcal{F}^i[G_i(\omega) \times S_{11}(\omega)] = \frac{1}{2\pi}\int\limits_{-\infty}^{+\infty} 1 \times S_{11}(\omega)\, e^{+j\omega t}\, d\omega = S_{11}(t)\,,\tag{21}$$

which proves that $S_{11}(t)$ is the time response of the Port 1 of the device when a v_1^+ wave impulse of unit strength, with peak at the calibration plane at the time $t = 0$, is sent towards the device.

Equation 12 carries several difficulties in a practical implementation. The first one is that it requires an integration over negative ω values, and negative frequencies do

not exist in practical measurements of $F(\omega)$. To see how to handle this problem recall that $f(t)$ must always be a real function of time and substitute Euler's identity

$$e^{-j\omega t} = \cos(\omega t) - j\sin(\omega t)\tag{22}$$

in Eq. 11 to obtain

$$F(\omega) = \int\limits_{-\infty}^{+\infty} f(t)\,\cos(\omega t)\,dt + j\int\limits_{-\infty}^{+\infty} f(t)\,\sin(\omega t)\,dt\,.\tag{23}$$

Equation 23 can now be used to mathematically extend $F(\omega)$ for negative frequencies by replacing ω by $-\omega$. This yields

$$F(-\omega) = \int\limits_{-\infty}^{+\infty} f(t)\,\cos(\omega t)\,dt - j\int\limits_{-\infty}^{+\infty} f(t)\,\sin(\omega t)\,dt\,.\tag{24}$$

We then see that $F(-\omega)$ can easily be obtained from any measured $F(\omega)$ using

$$F(-\omega) = F^*(\omega)\,,\tag{25}$$

where the asterisk denotes complex conjugation.

Another difficulty of Eq. 12 is that on frequency-domain measurements $F(\omega)$ is only known at the discrete measurement frequencies ω_n. Assuming that these frequencies are all uniformly spaced by $\Delta\omega$, we then in principle only have available the discrete $F(\omega)$ values

$$F(\omega_n) = F(n\,\Delta\omega),\tag{26}$$

where $n = -\infty, \ldots -3, -2, -1, 0, +1, +2, +3, \ldots +\infty$. We can then approximate the integral of Eq. 12 by a sum to obtain

$$f(t) = \frac{1}{2\pi}\sum_{n=-\infty}^{n=+\infty} F(n\,\Delta\omega)\,e^{+jn\,\Delta\omega\,t}\,\Delta\omega\,.\tag{27}$$

Although Eqs. 25 and 27 solve the two practical difficulties mentioned previously, another difficulty still remains: the required infinite number of $F(n\,\Delta\omega)$ values is never available. However, this can be remedied by assuming that, as $|n|$ increases towards infinite, the $F(n\,\Delta\omega)$ values becomes negligible. We can then truncate our sumation at a sufficiently large $n = N$ value to write

$$f(t) = \frac{\Delta\omega}{2\pi}\sum_{n=-N}^{n=+N} F(n\,\Delta\omega)\,e^{+jn\,\Delta\omega\,t}\,.\tag{28}$$

In practical situations often happens that one is forced to use an N that does not assure that $F(n\,\Delta\omega)$ has reached values that can be neglected (an example of such a case is considered below, in the numerical simulations). However, and this may surprise you, even if $F(n\,\Delta\omega)$ has not reached values that can be neglected at the frequency

band edge, Eq. 28 still works reasonably well. Nevertheless, to handle the situations where the truncation is a problem, a windowing (or apodizing) function $W(n\,\Delta\omega)$ can be introduced into Eq. 28 to yield

$$f(t) = \frac{\Delta\omega}{2\pi} \sum_{n=-N}^{n=+N} F(n\,\Delta\omega)\,W(n\,\Delta\omega)\,e^{+jn\,\Delta\omega\,t}\,. \tag{29}$$

The function $W(n\,\Delta\omega)$ is carefully selected to artificially taper down the values of $F(n\,\Delta\omega) \times W(n\,\Delta\omega)$ to negligible levels at $n = N$, without introducing significant artifacts into the $f(t)$ result. How this can be successfully achieved is discussed in more detail below, but for now let's use $W(n\,\Delta\omega) = 1$ (i.e., no windowing function will be applied).

There is one characteristic left in Eq. 29 that is inconvenient in many practical situations. Since unit strength impulses have very high amplitudes (theoretically infinite), the $f(t)$ results provided by Eq. 29 will be very large for times coinciding with the impulse response peak location. A commonly occurring situation of this is when $F(n\,\Delta\omega) = 1$ and $t = 0$ (this happens, for example, in the S_{11} of very short open-circuited cables). In these situations Eq. 29 becomes

$$f(t) = W_0 = \frac{\Delta\omega}{2\pi} \sum_{n=-N}^{n=+N} W(n\,\Delta\omega)\,. \tag{30}$$

To correct for this undesirable high-amplitude effect, Eq. 29 is normalized using the W_0 provided by Eq. 30 to yield the final result

$$\boxed{f(t) = \frac{1}{W_0}\frac{\Delta\omega}{2\pi} \sum_{n=-N}^{n=+N} F(n\,\Delta\omega)\,W(n\,\Delta\omega)\,e^{+jn\,\Delta\omega\,t}}\,, \tag{31}$$

therefore limiting the peak $f(t)$ of an open- or short-circuited cable to be $+1$ or -1, respectively.

From this point onwards Eq. 31 will be referred to as the *Sampled Inverse Fourier Transform (SIFT)*[5], and it will be used in this laboratory to determine the time-domain (pulse) response of our two-port networks (e.g., transmission lines). More specifically,

> when $F(n\,\Delta\omega) = S_{11}(\omega)$, the $f(t)$ given by Eq. 31 is the time response of the Port 1 of the device when a v_1^+ wave pulse of unit amplitude (not an impulse of unit strength), with peak at the calibration plane at the time $t = 0$, is sent towards the device.

Observe that Eq. 31 is actually a Fourier series where the frequency of the highest harmonic is

$$f_{max} = \frac{N\Delta\omega}{2\pi}\,, \tag{32}$$

[5]J. P. Dunsmore, *Handbook of Microwave Component Measurements*, John Wiley & Sons, Ltd., 2012.

and in practice this f_{max} value is determined by the highest frequency that the VNA can reach in its measurements. Since a Fourier series is unable to accurately represent time details smaller than about half the period associated with its highest harmonic (simply because the highest harmonic of the Fourier series needs to be oscillating fast enough to represent all relevant details of the function being interpolated), the time resolution $\Delta t|_{min}$ of the $SIFT$ algorithm (i.e., the smallest detail that can be resolved in time by Eq. 31) is given by

$$\Delta t|_{min} \approx \frac{1}{2 f_{max}} \,. \tag{33}$$

This is a very important equation, since it provides an estimate of the time resolution yielded by Eq. 31. It shows that the time resolution of a VNA is inversely proportional to its maximum measurement frequency. It also shows that, if two pulses are separated in time by less than the $\Delta t|_{min}$ provided by Eq. 33, the $SIFT$ processed VNA data will be unable to display (or resolve) them as two independent pulses.

One should mention that, since algorithms are already widely available to calculate the inverse Fourier transform of discrete frequency-domain data, apparently there seems to be no need to have developed our own algorithm here. The celebrated Inverse Fast Fourier Transform (IFFT) is one of these algorithms. The main advantage of the IFFT is its unparalleled numerical efficiency (i.e., computational speed), which allows it to process incredibly large amounts of data points very fast. However, this speed comes with the inconvenience of array manipulation acrobatics and limits on the time spanned and the spacing of the sample points in time. Although the numerical implementation of Eq. 31 is much slower by comparison, it is still sufficiently fast for our needs, and it does not suffer from the IFFT inconveniences; the pulse response $f(t)$ can be easily calculated at any desired time value t. For this reason Eq. 31 is then what is used to determine time responses in modern VNA instruments, including the NanoVNA-F (perhaps this may come as a surprise to you, but the Fast Fourier Transform algorithm is not what is used in VNAs).

We will now use Eq. 31 to simulate and investigate how pulses propagate in transmission lines. We will start by developing the required numerical Matlab tools, and after that we will theoretically (in a numerical sense) apply these tools to two well understood and relatively simple practical $S_{11}(f)$ situations: shorted and opened transmission lines. This will allow you to master well most of the the techniques involved in obtaining the time response of two-port networks, before heading out to use them on NanoVNA-F measured data (done in the next section).

1. Start by writing a Matlab code to implement the SIFT according to Eqs. 31 and therefore calculate the $f(t)$ associated with and arbitrary array of $F(n\,\Delta\omega)$ values. Call this Matlab code $SIFT\_simulation.m$. To assist with both the understanding as well as with the debugging of your code, at first $F(n\,\Delta\omega)$ will be a test array generated inside the code itself (by Eqs. 34 - 36), and later the array will be the NanoVNA-F measured data.

    Your code should allow the user to freely specify the number N of positive frequency points and the maximum frequency $fmax$ of the array $F(n\,\Delta\omega)$. Below

is the example of a code that can be used to specify the frequencies in Matlab (in this example $N = 51$ and the frequency band goes from -1.5 to +1.5 GHz):

```
nfreq = 51;
fstrt = 0.0*10^9;
fstop = 1.5*10^9;
fstep = (fstop-fstrt)/(nfreq-1);
freq  = -fstop:fstep:fstop;
omega = 2*pi*freq;
omega_step = 2*pi*fstep;
```

Your code should also allow the user to freely control the times where the pulse response is computed. Below is the example of a code that can be use to specify the times in Matlab (in this example the time is running from -10 to +80 ns with 1001 time steps):

```
ntime = 1001;
tstrt = -10*10^-9;
tstop =  80*10^-9;
tstep = (tstop-tstrt)/(ntime-1);
time  = tstrt:tstep:tstop;
```

2. Derive the reflection coefficient $S_{11}(\omega)$ of both an opened and a shorted transmission line of length ℓ, characteristic impedance Z_0, attenuation factor α, and velocity factor VF, namely

$$S_{11}(\omega)|_{open} = +\frac{1 - \tanh(\gamma\ell)}{1 + \tanh(\gamma\ell)}, \tag{34}$$

$$S_{11}(\omega)|_{short} = -\frac{1 - \tanh(\gamma\ell)}{1 + \tanh(\gamma\ell)}, \tag{35}$$

where

$$\gamma = \alpha + j\,\frac{\omega}{VF\,c_0}. \tag{36}$$

Note that in practice the attenuation factor is frequency dependent (increases with frequency). However, for the transmission lines' lengths and frequencies that we are considering in this laboratory, the attenuation effects are pretty much negligible.

3. Program Eqs. 34 and 35 into your Matlab $SIFT\_simulation.m$ code, so that an array of either $S_{11}(n\,\Delta\omega)|_{open}$ or $S_{11}(n\,\Delta\omega)|_{short}$ is generated as $F(n\,\Delta\omega)$ for your SIFT algorithm, and hence the associated pulse response $f(t)$ of your transmission lines can be calculated.

4. Before heading out to process measured data, let's first carefully experiment with your $SIFT\_simulation.m$ Matlab code to get a solid grasp on how the above

SIFT algorithm works by performing some simulations. To this effect, use your code to determine the frequency-domain reflection coefficient (i.e., $S_{11}(\omega)$) of the $\ell \approx 210$ mm RG316 NanoVNA-F test cable terminated in an open circuit, over -1.5 to +1.5 GHz with $N = 51$ frequency points. Assuming that the RG316 has $Z_0 = 50\,\Omega$, $VF = 0.7$, and $\alpha = 0$, plot the $|S_{11}(\omega)|$ and $\angle S_{11}(\omega)$ obtained. To give you an idea of the plot format expected, Fig. 2 shows a typical result.
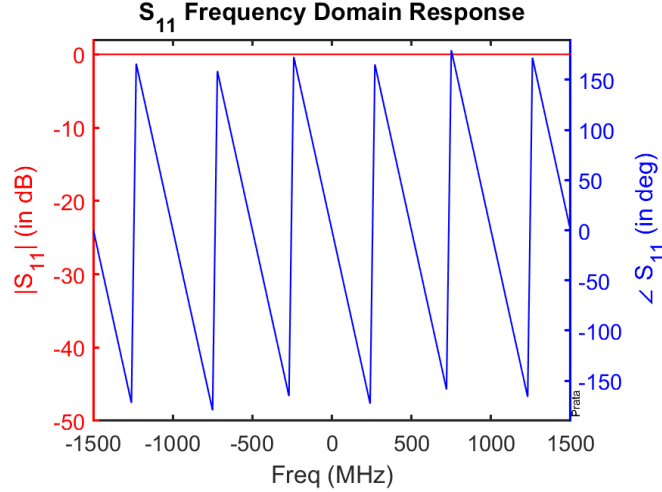

Figure 2: Predicted frequency response of an open-ended 210 mm long RG316 coaxial cable (N=51)

5. Have your code inverse Fourier transform the computed $S_{11}(\omega)$ values of the previous item, with the window $W = 1$ (i.e., no windowing) and $W_0 = 1$ (i.e., no normalization), to obtain the time response of the epen-ended cable excited by a v_1^+ unit amplitude pulse, and plot the obtained result from -10 to 80 ns. Comment on what you are seeing and why. In particular, explain why the impulse peaks are so large and why the values outside the peaks are so low, relatively speaking. In other words, explain what is going on with the Fourier transform mathematics that causes this.

6. Add the normalization W_0 to your code (given by Eq. 30) and again provide a plot of the result, which is the time response of the Port 1 of the transmission line when a v_1^+ wave pulse of unit amplitude, with peak at the calibration plane at the time $t = 0$, is sent in. Figure 3 displays a typical result.

   Note that what Figure 3 depicts is actually v_1^-, which is the reflection of v_1^+ by the end termination of the transmission line. Because the VNA provides a matched termination at its Port 1 (this is implicitly assumed in in the S_{11} of Eqs. 34 and 35), there is no higher order reflection of v_1^- by the VNA (i.e., no v_2^+).

7. Calculate the expected reference plane arrival time t_a, of the pulse reflected by the open end of the cable, and compare with the result provided by your plot.
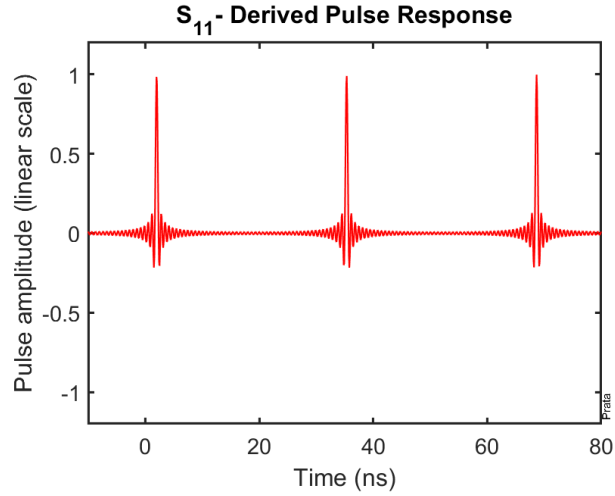
Figure 3: Predicted pulse response of an open-ended 210 mm long RG316 coaxial cable (N=51)

8. By now at least three details should have caught your eyes in the time-domain result (if not, please take a careful look at your figure): there is more that one pulse over the -10 to +80 ns range; there is significant ringing on both sides of the pulse peaks, and the ringing of the first pulse starts even before $t = 0$. Let's consider only the multiple pulses in this item, and the other two effects in the subsequent item.

From the expected t_a result, you probably already suspect that the pulses located around $t = 35$ and $t = 69$ ns may be numerical artifacts. This suspicion is indeed correct. The artifact is caused by the fact that when the Eq. 12 integral was discretized with frequencies spaced by Δf, it caused the $f(t)$ to become a periodic function with period $T = 1/\Delta f$ (the inverse Fourier integral became a Fourier series). In other words, the sampling frequency interval Δf determines the *spurious-free time range* T_{max} (also referred to as the alias-free time range in the available literature), which is given by

$$T_{max} = \frac{1}{\Delta f} \,. \tag{37}$$

If a longer spurious-free time range is desired (or required), you will need to reduce the Δf used. If the frequency range must be kept constant, this can only be achieved by increasing N. To confirm these conclusions, increase N to 101, and then increase it again to 201. And make sure to include plots of these two calculated pulse responses in your laboratory report.

9. Determine the maximum spurious-free time range available when $N = 101$ and provide the pulse response of the cable using this maximum range in the abscissa. Do this for both the openned and shorted cables.

10. Take a closer look at the pulse shown in your plots. For good detail generate plots using an abscissa going only from -1 to +4 ns (Fig. 4 is an example of the

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expected result). Observe that the pulse is symmetric about its peak (if this is not the case in your plot, you still have programming errors to find and correct) and is far from infinitely sharp, but this was already expected from Eq. 33.
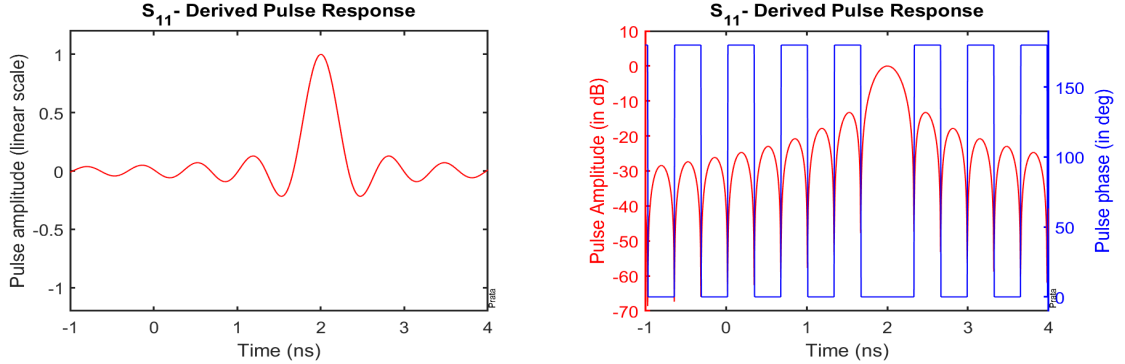

Figure 4: Predicted pulse response of an open-ended 210 mm long RG316 coaxial cable

Recalling that the transmission line currently has its end open-circuited (in other words, the line is terminated by a load producing a reflection coefficient equal to +1), the plot of Fig. 4 is just the excitation pulse delayed by the travel time t_a. This is a useful perspective since it allows you to take a look at the shape of the excitation pulse whenever needed. You may want to do this, for example, to take a look at the ringing present, the available time resolution, etc.

Determine the expected time resolution of the NanoVNA-F and compare the result obtained with the full-width half maximum (i.e., $FWHM$) of the displayed pulse.

11. Let's now tackle the (undesirable) ringing (or sidelobes) present on both sides of the pulse peak shown on our plots. The ringing is caused by truncating the Eq. 31 series too soon. Note that the first sidelobe level is 13.26 dB below the pulse peak value (i.e., $SLL = 13.26$ dB); it can be shown that this is always the case as long as there is no windowing function present (i.e., $W(n\,\Delta\omega) = 1$).

One way to reduce the ringing is by adding attenuation to the cable (we have been assuming it to be negligible). In fact, by having $\alpha = 0$ we have been truncating the Eq. 31 series without allowing the $F(n\,\Delta\omega)$ to decay at all. An equivalent convenient more general way to achieve the same goal as adding attenuation is by using a windowing function $W(n\,\Delta\omega) \neq 1$ in Eq. 31. The technical literature is full of different windowing functions, each with its own particular characteristics and behaviors. For the task at hand we will employ a simple Gaussian window, which is given by

$$W(n\,\Delta\omega) = e^{-\xi p \left(\frac{n}{N}\right)^2},\tag{38}$$

where the constant $\xi = 0.11512925465$ has been selected to make p the desired amplitude taper value at the edge of the frequency band, in dB (e.g., $p = 20$ causes a 20 dB taper at the frequency $N\Delta f$, $p = 30$ causes a 30 dB taper, and so on).

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Implement Eq. 38 in your Matlab $SIFT\_simulation.m$ code and provide both the $S_{11} \times W$ plot as well as the corresponding pulse response of the cable using a 30 dB taper on the Gaussian window. For good detail resolution use again an abscissa going from -1 to +4 ns. Determine the value of the time t_a associated with the pulse response peak, the $FWHM$ value, as well as the SLL value. Make sure to comment on the desirable and undesirable significant effects that the windowing function has on the coaxial cable simulated pulse response.

4   Measured Pulse Propagation on Transmission Lines

Armed with the acquired knowledge and material from the previous section, we will now proceed to time-domain process experimental data acquired using VNAs. For this you will be needing the Matlab $SIFT\_simulation.m$ code developed in the previous section well understood, properly working, and without any bugs. *I strongly suggest that you do not even think about starting to work on the material below before this is the case.*

1. Duplicate the Matlab $SIFT\_simulation.m$ code and name the duplicated version $SIFT\_NanoVNAF.m$. Store the $SIFT\_simulation.m$ code in a safe place (you will very probably still need it) and modify the $SIFT\_NanoVNAF.m$ code to read in the Touchstone s1p data files generated by the NanoVNA-F.

2. Using the NanoVNASaver software, and an appropriate NanoVNA-F calibration, measure and save the S_{11} of one of the RG316 $\ell \approx 210$ mm long test cables with both an open and a short at the end (to eliminate avoidable errors, make sure to use for ℓ the precise measured value of your cable). A total of 101 points, spanning the 10 kHz to 1.5 GHz frequency range, is sufficient. Save the measured data in the files "Opened_cable.s1p" and "Shorted_cable.s1p," respectively.

3. Modify your $SIFT\_NanoVNAF.m$ code to properly handle Touchstone files' data, process the measured values, and provide plots with both the measured $S_{11} \times W$ results as well as the time-domain pulse response of the cable. Determine the value t_a associated with the pulse response peak and compare with both the numerical simulations of the previous section as well as with the expected results.

   Note that the SIFT algorithm requires a measured point at zero frequency, and instead the lowest available point is at 10 kHz (VNAs never operate down to zero frequencies). Observing that the 10 kHz S11 measured value is pretty much the expected value at 0 kHz, provide the required 0 kHz value by simply moving the 10 kHz point to zero frequency (make sure to move only the 10 kHz point).

   When compared to all other points, the above moved frequency point will have a slightly increased spacing to its adjacent points. For the case at hand of 101 points spanning the 10 kHz to 1.5 GHz frequency range, the spacing between points is (1500 - 0.01)/100 = 14,9999 kHz, with exception of the spacing of the 0 kHz

point, which is 14,9999 + 10 = 15,0099 kHz. This very small spacing difference (only 0.07%) is inconsequential for our purposes. In case you are wondering, VNAs routinely numerically extrapolate their lowest measured frequencies to obtain the required zero-frequency value for their time-domain transforms. Our simple shift is just a simplified version of this extrapolation.

4. Let's now tackle a more sophisticated time-domain transmission-line measurement. Start by making a transmission-line cascade by connecting the $\ell_1 \approx 210$ mm long RG316 test cable to the Port 1 of the NanoVNA-F, then connect the other end of the cable to the SMAF-DIP8-SMAF test fixture, then connect the $\ell_2 \approx 200$ mm long 26 AWG parallel-wire line between the two female connectors of the test fixture, then connect the $\ell_3 \approx 2000$ mm long RG316 cable to the free SMA female connector of the test fixture, and finally connect a short circuit to the end of the $\ell_3 \approx 2000$ mm long RG316 cable (see Fig. 5). Make sure to carefully measure and record the lengths ℓ_1, ℓ_2, and ℓ_3 of the three cables used, as they will be needed below.
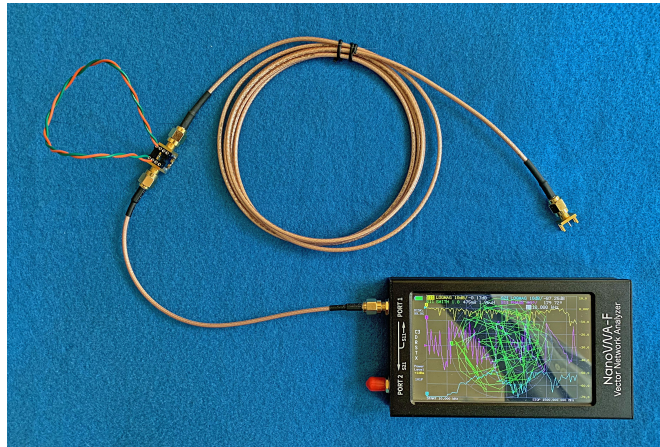

Figure 5: Transmission-line cascade to exercise time-domain measurements

5. Using the NanoVNASaver software, with an appropriate NanoVNA-F calibration, measure and save the S_{11} of the transmission-line cascade over the 10 kHz to 1.5 GHz frequency range. Since you are dealing with a relatively long cable, use a total of 2×101 frequency points to well sample the relatively fast varying S_{11} features with frequency (you should also try the usual 101 frequency points to experience the negative effects of an insufficient number of sample points). Save the measured data in a file named "Cascade_of_cables.s1p."

6. Determine the spurious-free time range of your measurement.

7. Process the measured data to obtain the time-domain pulse response of the transmission-line cascade and plot the result over the entire spurious-free (i.e., alias-free) time range of your measurement, and never beyond that, starting at $t = -5$ ns. Experiment with the window edge taper to get a relatively clean result

(a 30 dB taper is usually a good compromise between maximum resolution and undesired artifacts).

Please make sure to include a computer generated dB plot showing the measured pulse response of the transmission-line cascade, with the scales properly labeled. Your plot should be similar to the one shown on the right side of Fig. 4.

8. Carefully discuss in detail the causes of the three pulses with largest amplitude present in your measured plot. In particular, make sure to provide the timing of each of the three pulses' peaks (i.e., t_1, t_2, and t_3, respectively, in ns) and to explain why some of the pulses are positive and some are negative.

9. Sketch the bounce diagram of the transmission line cascade, when it is excited by a rectangular pulse of unit amplitude and width of 0.4 ns (this is the *FWHM* of the pulse depicted in Fig. 4), over only the 0 to 5 ns time interval, and over only a length equal to the sum of the first two transmission lines of the cascade (i.e., $\ell_1 + \ell_2$).

   Observe that the bounce diagram needs to include reflections at the junction between the $\ell_1 = 210$ mm long RG316 coaxial line and the $\ell_2 = 200$ mm long 26 AWG parallel wire line, and also at the junction between the ℓ_2 long parallel wire line and the $\ell_3 = 2000$ mm long RG316 coaxial line. And in the first junction you will also need to properly determine and account for the associated transmission coefficient.

10. Sketch the expected pulse response versus time diagram of the transmission line cascade, when it is excited by a rectangular pulse of unit amplitude and width of 0.4 ns, over only the 0 to 5 ns time interval. Make sure to clearly indicate the calculated amplitudes as well as the times associated with all the pulses' centers.

11. Use your measured plot to determine the characteristic impedance Z_{0p} and the velocity factor VF of the parallel-wire line.

5 Measured Pulse Propagation on Transmission Lines Using the VNA Internal Algorithm

As a last task for this laboratory, you will become acquainted with the fact that modern VNAs already come with the $SIFT$ capability implemented in their internal software, and it can be used with any one of the measured S-parameters. The NanoVNA-F is no different and the $SIFT$ capability can be used with either the measured $S_{11}(f)$ or $S_{21}(f)$.

The NanoVNA-F $SIFT$ capability can be found in its menu tree, under the "DISPLAY" and "TRANSFORM" buttons. In there you will find a place to turn the $SIFT$ on or off, to input the transmission line velocity factor (in percent), as well as to select between three window tapers (i.e., "MINIMUM," "NORMAL," and "MAXIMUM"). With

the choice of "MINIMUM" no window is applied, while the other two choices progressive impart larger amplitude tapers to the S-parameters being processed[6]. Also in the "TRANSFORM" menu you will find two choices for the low-frequency extrapolation to be used: "LOW PASS IMPULSE" and "BAND PASS." The "LOW PASS IMPULSE" option corresponds to what we have used in our Matlab implemented algorithm, and the "BAND PASS" option applies the selected window to both the near-zero and high $S_{11}(f)$ frequencies.

When the NanoVNA-F $SIFT$ capability is turned on, the horizontal display scale turns to time (in ns), the vertical scale will show the value of the $|f(t)|$ (in dB), and the sign of $f(t)$ is displayed in the associated phase scale (either 0° or 180°, since $f(t)$ is real). The marker information will also change to time and distance, where the distance takes into account the displayed velocity factor (in percent). The range of the time scale selected automatically to display only the spurious-free region, and is controlled by the frequency scan together with the number of frequency points used, according to Eq. 37.

Also in the NanoVNA-F "TRANSFORM" menu you will find a perhaps unfamiliar "LOW PASS STEP" selection. This alternative yields the time response for a unit-step excitation firing at $t = 0$ (i.e., the Heaviside Step Function); a useful capability that has not been previously explored in this laboratory. The time response associated with this type of excitation can in principle be determined with the $\delta(t)$ of Eq. 20 replaced by the Heaviside Step Function $u(t)$, and working out the associated math. This will not be done in this laboratory though. The "LOW PASS STEP" selection is useful, among other things, to get the characteristic impedance of transmission lines, as you will find out below.

When the unit-step excitation is used, the two white values that are provided in the NanoVNA-F screen line that starts with the green words "S11 SMITH" correspond to the measured complex impedance (i.e., ratio of the complex voltage and current) at the VNA calibration plane, at the time associated with the Mark 1.

An important NanoVNA-F characteristic to keep in mind as you move forward is that the SOLT calibration must be done with the NanoVNA-F set to operate in the frequency-domain mode. Calibrating while the NanoVNA-F is operating in the time-domain mode will yield incorrect results. This is also true if the NanoVNA is being calibrated through the NanoVNASaver software.

1. Let's now start by measuring the loss of the $\ell_3 = 2000$ mm long RG316 cable, a task that you already performed in a previous laboratory, but in the frequency domain. For this use the internal NanoVNA-F $SIFT$ capability, in "LOW PASS IMPULSE" mode, to process the $S_{11}(f)$ from an ℓ_3 long RG316 cable connected to

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[6]The NanoVNA-F does not use the Gaussian window of Eq. 38. Instead it implements the so called Kaiser window that is commonly used in almost all VNAs. This window is named after J. F. Kaiser of the paper *Nonrecursive Digital Filter Design Using the I_0-Sinh Window Function,* Proceedings of the 1974 IEEE International Symposium on Circuits and Systems, April 1974, pp. 20–23. The taper provided by the Kaiser window is controlled by a parameter called β. The "MINIMUM," "NORMAL," and "MAXIMUM" settings implement Kaiser windows with parameter $\beta = 0$, 6, and 13, respectively. With $\beta = 0$ no window is applied to the data. The $\beta = 6$ choice applies a taper of about 34 dB, and yields results comparable to the Gaussian window with 30 dB taper that you used previously (i.e., $p = 30$ in Eq. 38). And the $\beta = 13$ of the "MAXIMUM" setting further increases the applied taper.

the Port 1 of the NanoVNA-F. Provide an annotated picture of the NanoVNA-F screen with your result, clearly identifying the reflection coming from the end of the cable, and from it calculate the one-way loss (as opposed to round-trip loss) L_3 of the cable, in dB.

Note that the above L_3 is an averaged result, since all measured frequencies were involved in its determination.

2. Use the technique of the previous item to measure the averaged time-domain one-way loss L_1, in dB, of the $\ell_1 = 210$ mm long RG316 cable.

3. Again use the technique of the previous item to measure the averaged time-domain one-way loss L_2, in dB, of the $\ell_2 = 200$ mm long 26 AWG parallel wire line.

4. Let's now measure the reflection coefficient of the Port 2 of the NanoVNA-F, a task that you already performed in a previous laboratory, but in the frequency domain. For this use again the internal NanoVNA-F $SIFT$ capability, in "LOW PASS IMPULSE" mode, to process the $S_{11}(f)$ from a $\ell_3 = 2000$ mm long RG316 cable connected between Ports 1 and 2 of the NanoVNA-F. Provide an annotated picture of the NanoVNA-F screen with your result, clearly identifying the reflection coming from Port 2, and from it calculate the reflection coefficient Γ_2 of Port 2, in dB.

The use of a long cable in this measurement conveniently allows you to unambiguously identify the reflection coming from Port 2. However, a long cable comes with significant losses, and this has to be properly accounted for in your Γ_2 calculation.

Observe again that your measurement is only able to provide a frequency-averaged value of the Γ_2, since the pulse is made up of all frequencies available in the frequency band of your measurement.

5. Modify the technique that you learned in the previous item to measure the reflection coefficient Γ_1 of Port 1 of the NanoVNA-F, in dB. In VNA parlance this reflection coefficient is referred to as the *Port 1 source match*. Please make sure to provide a picture of your measurement setup.

Note that, for high S-parameter measurement accuracy, the reflection coefficients of Ports 1 and 2 of the VNA need to be made as small as possible. Since, and as we learned in a previous laboratory, S-parameters must to be measured with Ports 1 and 2 providing matched terminations to the device being measured.

In the NanoVNA-F, the small Γ_2 value is implemented by a passive circuit, since Port 2 is passive (no calibration improvement is then possible). However, since Port 1 is active, its Γ_1 value is improved through the SOLT callibration and its associated processing. The advantage of this is that a much smaller Γ_1 value can be obtained. VNAs that have active Ports 1 and 2 have then the advantage that they can provide excellent source matches for both ports.

6. Use the internal NanoVNA-F $SIFT$ capability, in "LOW PASS IMPULSE" mode, to process the $S_{11}(f)$ of the transmission-line cascade that you measured previously. Again carefully identify the three pulses with largest amplitude present in

your measurement. Compare the timing of each of the three pulses with the values you measured before (i.e., t_1, t_2, and t_3, respectively), determine the associated reflection coefficients, and comment on your results.

7. Sketch the expected pulse response versus time diagram of the transmission-line cascade, when it is excited by a Heaviside step function of unit amplitude, over only the 0 to 5 ns time interval. Make sure to clearly indicate the calculated amplitudes as well as the times associated with all the voltage transitions.

8. To conclude our laboratory, use the internal NanoVNA-F $SIFT$ capability, in "LOW PASS STEP" mode, to process the measured $S_{11}(f)$ of the transmission-line cascade. Use the obtained time-domain results, together with the bounce diagram of the previous item, to determine the characteristic impedance Z_0 of the RG316 cable as well as the characteristic impedance Z_{0p} of the 26 AWG parallel-wire line. Feel free to experiment with the available Fourier Transform windows, and the choices for the number of frequency points, in order to best adjust the time plot for accurate readings.

Please make sure to include an annotated picture of the NanoVNA-F screen showing the Heaviside step function response of this item. In particular, the picture should clearly indicate what time markers were used to determine Z_0 and Z_{0p}, and your report should justify these choices. Furthermore, you should clearly indicate how you performed the calculations that yielded the Z_0 and the Z_{0p} values obtained.