Antennas in Transmission

Laboratory 06 Manual

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1 Laboratory Objective

So far in our laboratories we have only dealt with devices that use heavily confined electromagnetic fields (e.g., circuits and transmission lines). In many applications this is precisely what is desired; the electromagnetic fields should pretty much stay inside well defined spatial regions, and therefore minimize interference to and from surrounding devices.

However, in many applications it is desired to create electromagnetic fields in space, use these fields to carry information without employing any electric conductors (this is done in mobile communications and radar applications, for example), and then retrieve part of these fields at another location. In order to produce fields in space, starting from electric currents, a transducer is required. For electromagnetic fields this type of transducer is called an antenna. In many aspects a transmitting antenna is similar to a loudspeaker, as both take electric currents and produce fields in space. The primary conceptual difference between the two is that a loudspeaker produces acoustic fields, as opposed to electromagnetic fields.

There are many different types of antennas. As an example, if the end of an optical fiber is left unconnected it will radiate light, and hence can be considered an antenna (of a general type called an aperture antenna). In this laboratory we will only consider another type of antenna, which is by far the most common: a wire antenna. Wire antennas have the convenience of providing a two-terminal circuit port (like a loudspeaker), and hence can be easily connected to a transmission line (this is the general type of antenna that is widely used in mobile phones, for example).

We will study below linear wire antennas (called linear because they are made using straight wire segments) when they radiate (i.e., transmit energy). Their behavior when they receive energy, which is equally important, will be considered in a subsequent laboratory. We will start by understanding the radiation mechanism of linear antennas and then consider in detail one of their most important characteristic: the impedance that they present at their circuit terminals. A wire antenna needs to be connected to a transmission line, and hence if their impedance is substantially different than the transmission line characteristic impedance they will operate with an excessively high reflection coefficient, and hence will not be able to efficiently transfer power to the surrounding space.

To be successful in this laboratory you will need to have studied in detail the basic material covered in the initial sections of Chap. 11 of our textbook[1], and also study

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[1]D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., 1989.

the theory presented ahead. As always, we will approach the material at hand from both the theoretical and the experimental viewpoints. First we will look into the theory involved, then we will implement some Matlab simulations, and subsequently we will confirm the predictions through accurate experiments.

As I am sure you have already observed, each one of our laboratories progressively rely on the material learned in preceding laboratories. This will be even truer now, since the laboratory that will follow this one will deal with antennas receiving energy, and hence will need to rely heavily on the material learned, codes developed, and hardware constructed in the current laboratory. It can't then be over stressed that the successful completion of the current laboratory, in its totality, is essential for what lies in the future.

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report should have each of its sections and subsections numbered according to this laboratory manual, and be a detailed document with all your derivations, calculations, design efforts, associated Smith Charts, measurement results, conclusions, drawings, plots, relevant photos of all constructed components (to showcase your very important high-frequency craftsmanship), and printouts of any developed software.

Note that, to maximize the learning experience, the laboratory has been designed to be carried out individually, hence each person in the class received their own individual lab kits. Consequently, the experiment and the corresponding report has to be done completely individually.

2 Radiation of a Linear Wire Antenna

Whenever a small segment of wire of length $d \ll \lambda_0$ and negligibly small radius a, located in free space, carries a spatially-constant time-harmonic electric current $I = \hat{z}\, I$, it radiates an electromagnetic field given by[2,3]

$$\vec{E} = \frac{j\eta_0}{4\pi}\,\beta_0 I d\,\frac{e^{-j\beta_0 R}}{R}\left\{\hat{R}\,\frac{2}{j\beta_0 R}\left(1+\frac{1}{j\beta_0 R}\right)\cos\theta+\hat{\theta}\left[1+\frac{1}{j\beta_0 R}+\frac{1}{(j\beta_0 R)^2}\right]\sin\theta\right\}, \quad (1)$$

$$\vec{H} = \frac{j}{4\pi}\,\beta_0 I d\,\frac{e^{-j\beta_0 R}}{R}\,\hat{\phi}\left(1+\frac{1}{j\beta_0 R}\right)\sin\theta\,, \quad (2)$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields radiated at the point with arbitrary spherical coordinates (R, θ, ϕ), and β_0 and η_0 are the intrinsic free-space propagation factor and characteristic impedance, respectively.

[2]D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., 1989, Sec. 11-2.

[3]This fundamental result was first derived and published by Prof. Heinrich R. Hertz more than 130 years ago (in 1888), and is now referred to as the field produced by the *Hertz dipole*. Concurrently with the derivation, Prof. Hertz also invented the first transmitter, antenna, and receiver, and used them to produce and detect electromagnetic waves in space. Had Prof. Hertz lived long enough he most certainly would have been awarded the Nobel prize in physics for this work, but unfortunately he died in 1894, when he was only 36 years old. Sadly, he did not even see his inventions give birth to the field of wireless telecommunications.

These equations are somewhat complicated, and hence we will wait to grasp some of their details when we use them a bit later. But for now it is very important to note that these equations clearly show that the radiation produced by an electric current is directly proportional to the product $\beta_0 Id$. In other words, and this is a very important rule of antenna engineering:

To radiate a substantial field a significant current must be made to flow on a wire of sufficiently long electrical length (i.e., Id/λ_0 can't be small).

Considering the above rule, one should then naturally wonder why a coaxial cable does not radiate (yes, coaxial cables do not radiate, and it is precisely because they don't radiate that they are used to carry electromagnetic energy). The answer lies on the fact that at any cross section the currents carried by the inner and outer conductors of a coaxial cable are equal and in phase opposition (i.e., $I_{inner} = -I_{outer}$), and the outer current symmetrically encircles the inner current. Anywhere outside the coaxial cable the radiation of the inner and outer currents then cancel out perfectly.

Note that, since parallel-wire transmission lines do not benefit from coaxial symmetry, they radiate. However, since again here at any cross section the currents carried by the two conductors are equal and in phase opposition, the radiation tends to cancel out. The radiation does not cancel perfectly though, since the two wires are not collocated. Parallel-wire transmission lines then radiate a little bit, and in direct proportion to the separation between the two conductors.

Small overall size (and hence small product Id) together with the radiation cancellation effect caused by equal currents in phase opposition is also what fortunately keeps radiation by electronic circuits at a negligible level.

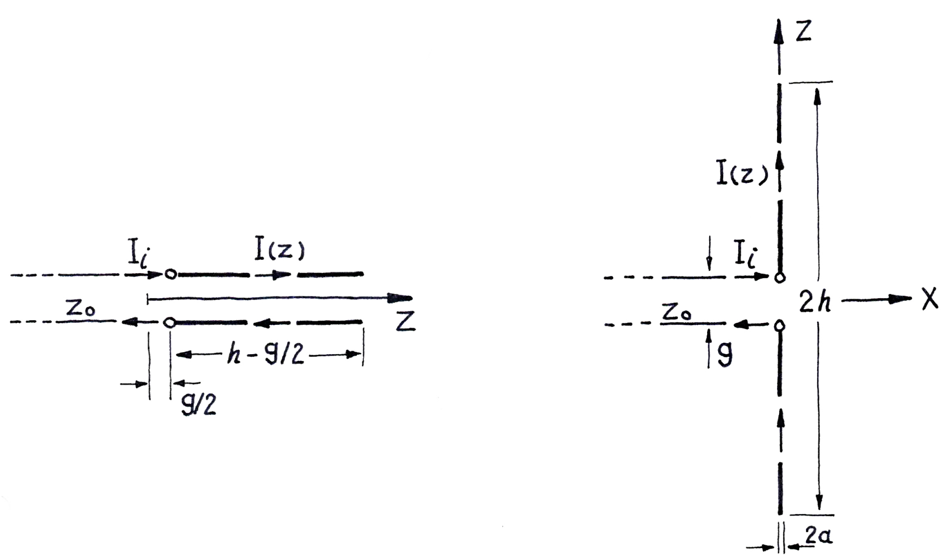

Figure 1: Creation of a dipole antenna (right) starting from a parallel-wire transmission line (left)

To intentionally cause radiation (i.e., to create an efficient antenna) one then must implement a relatively large Id product (e.g., use a wire length d comparable to the operation wavelength) while simultaneously avoiding paired currents in phase opposition.

This is the basic reasoning that guided Prof. Hertz in inventing the very first linear wire antenna, the dipole. To see how he achieved this result, consider Fig. 1: he cleverly bent a significant length of the extremity of a parallel-wire transmission line by 90°. This causes the currents on the two wires to now flow in the same direction, and hence their radiation no longer cancel each other. The dipole antenna obtained is shown aligned with the z axis, has a total length $2h$, diameter $2a$, and is excited by an air-spaced parallel-wire transmission line of characteristic impedance Z_0 that produces a current I_i at the antenna two input terminals (the two small circles depicted in Fig. 1).

Recalling that in general the time-harmonic current that is flowing on the conductors of an air-spaced transmission line (shown on Fig. 1, on the left) is composed of two waves traveling in opposite directions with propagation factor $\gamma = \alpha + j\beta$, we can write for the current flowing on the top conductor

$$I(z) = A^+ e^{-\gamma z} + A^- e^{+\gamma z}\,, \tag{3}$$

where A^+ and A^- are the complex amplitudes of the $+\hat{z}$ and the $-\hat{z}$ traveling waves, respectively. Neglecting the stray capacitance from the open at the end of the transmission line we can also say that $I(z = h) = 0$. Hence, further assuming that $g \ll h$ we can say that $I(z = 0) = I_i$, and consequently one can determine A^+ and A^- as

$$A^+ = +\frac{I_i}{2\sinh(\gamma h)}\, e^{\gamma h}\,, \tag{4}$$

$$A^- = -\frac{I_i}{2\sinh(\gamma h)}\, e^{\gamma h}\,, \tag{5}$$

and hence Eq. 3 becomes

$$I(z) = \frac{I_i}{\sinh(\gamma h)}\, \sinh[\gamma(h - z)]\,. \tag{6}$$

A difficulty with this equation is that, even though α comes primarily from the energy "lost" to radiation, at this point $\gamma = \alpha + j\beta$ is an unknown quantity. In order to proceed we will then assume that $\alpha \approx 0$ and $\beta \approx \beta_0$ and hence approximate the current flowing on the dipole antenna by

$$I(z) \approx I_m\, \sin[\beta_0(h - |z|)]\,, \tag{7}$$

where

$$I_m = \frac{I_i}{\sin(\beta_0 h)}\,, \tag{8}$$

and the $|z|$ has been introduced to make Eq. 6 valid for both halves of the dipole. Note that I_m is the maximum amplitude of the current standing wave on the dipole. A value that in general is different than I_i, since I_i is the current at the input terminals of the dipole. Also, since Eq. 8 has singularities whenever $\beta_0 h = n\pi$, $n = 0, 1, 2, \ldots$, Eq. 7 becomes invalid whenever $\beta_0 h \approx n\pi$.

In the antenna literature Eq. 7 is referred to as the *sinusoidal current approximation.* If you consider it carefully you will find reasons to doubt that it is correct. After all, and although it correctly describes the current on a parallel-wire transmission line, a dipole antenna is geometrically significantly different from a parallel-wire transmission line. Furthermore, a dipole antenna is losing energy through radiation, which should cause Eq. 3 to have a complex propagation factor γ, instead of just a β_0. However, it turns out that it can be shown that Eq. 7 provides a very good approximation for the current that flows on any smooth wire (i.e., a wire without any kinks), even if the wire is curved. The only requirement is that the wire be thin and not excessively long compared with the operation wavelength (i.e., $2a \ll \lambda_0$ and $h \lesssim \lambda_0/2$), as long wires make impossible to approximate the γ in Eq. 6 by just $j\beta_0$. This sinusoidal current approximation was first obtained experimentally more than 100 years ago, by actually measuring currents flowing on antenna wires, and it has been successfully and widely used since then. We will then use it in all that follows without further questioning, and leave to our experiments to confirm its validity range.

We are now in the position to determine the electromagnetic field radiated by a dipole antenna of significant length $2h$ (i.e., $2h \not\ll \lambda_0$), and negligibly small diameter $2a$, by using Eqs. 1 and 2, together with Eq. 7. Note however that Eqs. 1 and 2 can't be used directly though, since they are valid only when $d \ll \lambda_0$. This difficulty can be overcome by observing that the dipole of Fig. 1 can be regarded as a large number of very small Hertz dipoles staked end-to-end and radiating together, each carrying its own individual z-dependent current $I(z)$ (given by Eq. 7) and radiating according to Eqs. 1 and 2. This leads to a sum (or an integration) over all the individual Hertz dipole's contributions, and the details of how to do this, for an observation point in the far-zone (i.e., when the distance from the point where we want to know the field to the antenna is large, $R \gg \lambda_0$) can be found in our textbook[4] and hence will not be repeated in here.

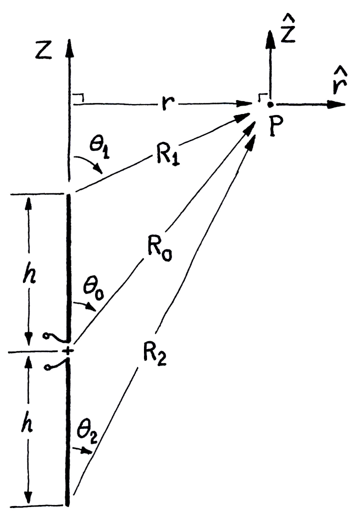

Figure 2: Parameters associated with the near-zone field of a straight dipole

For reasons that will become clear below, for the purposes of this laboratory we are interested on the electromagnetic field radiated at any distance from the dipole wires (i.e., observation points where R is arbitrary, and not just $R \gg \lambda_0$). In such case the integration of all the concatenated Hertz dipoles is more involved, but nevertheless can still be done in closed form using

[4]D. K. Cheng, *Field and Wave Electromagnetics*, Second edition, Addison-Wesley Pub. Co., 1989, Sec. 11-4.

cylindrical coordinates, and it can be shown to yield[5]

$$\vec{E}(P) = \frac{j\,\eta_0\,I_m}{4\pi}\left\{-\;\hat{r}\;\frac{1}{r}\;\left[2\,e^{-j\beta_0 R_0}\cos\theta_0\;\cos(\beta_0 h) - e^{-j\beta_0 R_1}\cos\theta_1 - e^{-j\beta_0 R_2}\cos\theta_2\right]\right.$$
$$\left. +\;\hat{z}\left[2\,\frac{e^{-j\beta_0 R_0}}{R_0}\;\cos(\beta_0 h) - \frac{e^{-j\beta_0 R_1}}{R_1} - \frac{e^{-j\beta_0 R_2}}{R_2}\right]\right\}\,, \tag{9}$$

$$\vec{H}(P) = \frac{-j\,I_m}{4\pi}\,\hat{\phi}\;\frac{1}{r}\left[2\,e^{-j\beta_0 R_0}\cos(\beta_0 h) - e^{-j\beta_0 R_1} - e^{-j\beta_0 R_2}\right]\,. \tag{10}$$

As shown in Fig. 2, the observation point P is located by the cylindrical coordinates (r, ϕ, z), the fields are described using the corresponding cylindrical unit vectors (i.e., $\hat{r}$, $\hat{\phi}$, and $\hat{z}$), the dipole has length $2h$, negligibly small radius (the radius a is not even present in the above equations), and the variables R_0, R_1, R_2, θ_0, θ_1, and θ_2 are given by

$$R_0 = \sqrt{r^2 + z^2}\,, \tag{11}$$
$$R_1 = \sqrt{r^2 + (z-h)^2}\,, \tag{12}$$
$$R_2 = \sqrt{r^2 + (z+h)^2}\,, \tag{13}$$
$$\cos\theta_0 = z/R_0\,, \tag{14}$$
$$\cos\theta_1 = (z-h)/R_1\,, \tag{15}$$
$$\cos\theta_2 = (z+h)/R_2\,. \tag{16}$$

To get you well familiarized with the usage of the above equations, and the corresponding geometry, let's now derive some associated results.

1. A Hertz dipole assumes a spatially-constant time-harmonic current radiating. However, this type of current can't be achieved in practice when a small wire dipole (i.e., a dipole constructed using a wire with $d << \lambda_0$) is excited at its central terminals. Use the sinusoidal current approximation to derive the current $I(z)$ that flows on the arms of a small dipole.

   How should Eqs. 1 and 2 be modified to yield the field radiated by the small dipole?

2. Assuming that the observation point is far away from the antenna (i.e., $R \gg 2h$), use Eqs. 9 – 16 to obtain the far-zone electromagnetic fields $\vec{E}$ and $\vec{H}$ of the dipole, in spherical coordinates (i.e., in terms of R, θ, and ϕ, and the associated unit vectors $\hat{R}$, $\hat{\theta}$, and $\hat{\phi}$).

   Far-zone fields require approximations that take advantage of the fact for amplitude terms one can say that, for example, $R_1 \approx R$. However, this type of approximation is way too crude for the complex exponents, since they are associated with periodic functions. For the complex exponents one needs instead, for example, $R_1 \approx R - h\cos\theta$.

[5] This result can be found in many undergraduate books that cover antennas a bit more extensively than our textbook. See for example E. C. Jordan *Electromagnetic Waves and Radiating Systems*, Prentice-Hall, Inc., April 1960, pag. 320–324.

3 Input Impedance of a Linear Dipole Antenna

Armed with the results of the previous section, we can now proceed to determine the impedance Z_i that is present at the input terminals of a linear dipole antenna (i.e., the impedance presented by the antenna terminals to the transmission line that is connected to the antenna). For this consider the linear dipole geometry shown in Fig. 3, snugly enclosed by a mathematical cylindrical surface S_a (depicted in dashed lines); since the dipole has height $2h$ and diameter $2a$, the surface S_a is a cylinder with the same dimensions.

We know from circuit theory that the time-averaged complex power P_t being fed to the antenna terminals is given by

$$P_t = \frac{1}{2} V_i I_i^* , \tag{17}$$

where V_i and I_i are the voltage and current at the antenna terminals, respectively. Recalling that V_i and I_i are related to each other through the antenna impedance Z_i, according to the equation

$$Z_i = V_i / I_i , \tag{18}$$

we can write

$$P_t = \frac{1}{2} Z_i I_i I_i^* . \tag{19}$$

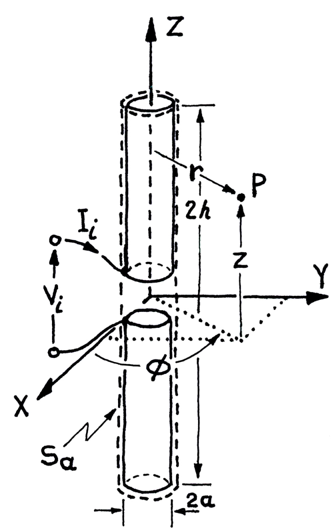

Figure 3: Geometry for determining the dipole antenna impedance

Now, reasonably assuming that the dipole antenna losses can be neglected, all the time-averaged complex power P_t provided by the transmission line must be crossing the surface S_a and going into the space surrounding the antenna, therefore we must have

$$P_t = \oint_{S_a} \vec{S} \cdot \vec{ds} , \tag{20}$$

where $\vec{ds}$ points away from the volume occupied by the dipole and $\vec{S}$ is the time-averaged complex Poynting vector, namely

$$\vec{S} = \frac{1}{2}\,\vec{E} \times \vec{H}^* \,. \tag{21}$$

Substituting Eqs. 19 and 21 into Eq. 20 then finally yields a convenient equation for calculating the input impedance Z_i of the dipole antenna, namely

$$Z_i = \frac{1}{I_i I_i^*} \oint\limits_{S_a} \vec{E} \times \vec{H}^* \cdot \vec{ds}\,, \tag{22}$$

where $\vec{E}$ and $\vec{H}$ are given by Eqs. 9 and 10, respectively. Note that the $\vec{E}$ and $\vec{H}$ to be used in Eq. 22 is *just the field radiated by the induced current that is flowing on the dipole*, which is not equal to the total electromagnetic field that exists over the surface S_a. This is a subtle detail that can be quite confusing at first. To help appreciate the difference between the two fields observe that the total electromagnetic field that exists over the surface S_a has $E_z = 0$ (not then what is given by Eq. 9), since the electric field tangential to the surface of any perfect electric conductor is equal to zero.

The cylindrical surface S_a has top and and bottom caps located at $z = \pm h$, respectively. Hence the surface S_a is constituted of three separate surfaces. However, if the antenna diameter $2a$ is small, which is usually the case when $2a \ll \lambda_0$, the integration over these top and bottom caps will yield a relatively small contribution to the total integral value, and hence can be safely neglected when compared with the integration over the side surface of the cylinder S_a. In this case $\vec{ds} = \hat{r}dz\,rd\phi$ and Eq. 22 becomes

$$Z_i = \frac{1}{I_i I_i^*} \int\limits_0^{2\pi} \int\limits_{-h}^{+h} (\vec{E} \times \vec{H}^*)\Big|_{r=a} \cdot \hat{r}dz\,ad\phi\,, \tag{23}$$

and since from Eqs. 9 and 10 we see the amplitude of the vector $(\vec{E} \times \vec{H}^*)\Big|_{r=a}$ is independent of ϕ,

$$Z_i = \frac{2\pi a}{I_i I_i^*} \int\limits_{-h}^{+h} (\vec{E} \times \vec{H}^*)\Big|_{r=a} \cdot \hat{r}dz\,. \tag{24}$$

Equation 24 provides the desired result for the input impedance of a dipole antenna. To use it one needs to know the electromagnetic field $\vec{E}$ and $\vec{H}$ radiated by the dipole on its surface though (i.e., surface with $r = a$). Whenever $2a \ll \lambda_0$, a very good approximation for this field is provided by Eqs. 9 and 10, with $r = a$.

There is an alternative way to write Eq. 24 that you may find a bit more convenient. To derive it observe from Eqs. 9 and 10 that $\vec{E} = \hat{r}E_r + \hat{z}E_z$ and $\vec{H} = \hat{\phi}H_\phi$, and hence Eq. 24 can be rewritten as

$$Z_i = \frac{-2\pi a}{I_i I_i^*} \int\limits_{-h}^{+h} (E_z H_\phi^*)\Big|_{r=a} dz\,. \tag{25}$$

There is yet another alternative way to rewrite this last equation, which yields the form most commonly found in the literature. To obtain it first recall Ampere's law, namely

$$\oint_C \vec{H} \cdot \vec{d\ell} = j\omega\epsilon_0 \int_{S_c} \vec{E} \cdot \vec{ds} + \int_{S_c} \vec{J} \cdot \vec{ds}\,, \tag{26}$$

where for the application at hand the contour C will be assumed to be a circle of radius a enclosing the antenna, with center at the coordinate z, and the surface S_c is then the surface of this circle. Recalling again from Eq. 10 that $\vec{H} = \hat{\phi}\, H_\phi$ we can write

$$2\pi a H_\phi \Big|_{r=a} = j\omega\epsilon_0 \int_{S_c} \vec{E} \cdot \vec{ds} + \int_{S_c} \vec{J} \cdot \vec{ds}\,. \tag{27}$$

Now, observing that the first term on the right side of Eq. 27 is negligibly small and

$$\int_{S_c} \vec{J} \cdot \vec{ds} = I(z)\,, \tag{28}$$

where $I(z)$ is given by Eq. 7, we obtain

$$2\pi a H_\phi \Big|_{r=a} = I(z)\,, \tag{29}$$

With this last result Eq. 25 becomes

$$Z_i = \frac{-I_m^*}{I_i I_i^*} \int_{-h}^{+h} E_z \Big|_{r=a} \sin[\beta_0(h - |z|)]\, dz\,. \tag{30}$$

Observe that the cylindrical integration surface S_a was intentionally made snug with the two dipole antenna arms (see Fig. 3). This is required to capture all the reactive power present in the space surrounding the antenna (this reactive power is responsible for the reactive part of the antenna impedance). There is a bit of energy stored in both the electric and magnetic fields that exist in the antenna feed point region (in the region with $z \approx 0$ where the two antenna arms come together and are connected to the transmission line). In other words, there is some reactive power present on the approximately parallel-plate capacitor and wire inductances that exist at the antenna feed point. This reactive power is not being captured by either Eq. 23, 25, or 30, since any fringe field effects present in the $z \approx 0$ region is being ignored and also the integrations only capture the reactive power outside S_a. If for any reason this parasitic reactive power is deemed relevant to the antenna operation (perhaps because the antenna terminals are excessively close to each other), it then needs to be accounted for separately, by adding the corresponding stray inductance L_f and capacitance C_f in series and parallel with Z_i, respectively.

1. Generate a Matlab code to implement the above Eq. 25, numerically carry out the associated integral, and therefore compute the input impedance Z_i of linear

dipoles. Your code should generate a plot of both the real and the imaginary parts of Z_i in Ω, displayed in the same figure (the real part on the left scale and the imaginary part on the right scale), as a function of frequency.

Matlab has available several different intrinsic functions for performing numerical integration, and a good simple and robust one to use in this laboratory is the function "trapz," which implements the trapezoidal rule (the function being integrated is approximated by straight line segments, causing the area under a real integration function to be approximated by trapezoids). The trapezoidal rule has the advantage of being capable of handling integration points non-uniformly spaced, but I personally prefer to use Simpson's rule for my integrations. Although Simpson's rule can't handle non-uniform point spacing, the function being integrated is instead better approximated by parabola segments. Since unfortunately Matlab does not seem to have an intrinsic function that implements Simpson's rule, the trapezoidal-rule alternative will have to suffice.

When using numerical integration algorithms keep in mind that you usually have the choice of the number of integration points to be used. In the intrinsic Matlab function "trapz" this choice is implicitly made through the number of provided integrand values. Although more points yields higher accuracy, it requires longer execution times. On this light, please make sure to experiment a bit and operate with the minimum number of points that accurately handles the task at hand. A good strategy is to start with a reasonably large number of integration points and keep on doubling it until the answer stabilizes to some very amall acceptable error.

2. Calculate the input impedance of a dipole of length $2h = 250$ mm with the frequency running from 0 to 2.0 GHz, for three values of the antenna diameter $2a$, namely $2a = 0.001$, 4, and 20 mm and provide the corresponding three plots in your laboratory report. Figure 4 gives you some idea of two useful plot formats. Both formats complement each other, with the logarithm format providing visualization of the full range of input impedances while facilitating the location of the all-important resonant frequencies.

3. Add to your code the option of computing Z_i using instead Eq. 30, use this option to calculate again the same cases of the previous item, provide the corresponding plots, and discuss the results.

4. Observing that dipoles have resonant frequencies, provide the lowest series and parallel resonant frequency values (i.e., f_{0s} and f_{0p}, respectively) of the three dipoles of the previous item. Also provide the values of the input impedances Z_i at the above resonance frequencies.

   Make sure to comment about your results. In particular, provide a precise numerical value for the $2h/\lambda_0$ associated with the very important first series resonance (i.e., the electrical length of the dipole), and explain why thinner dipoles have faster reactance variation with frequency near the resonance frequencies (i.e.,

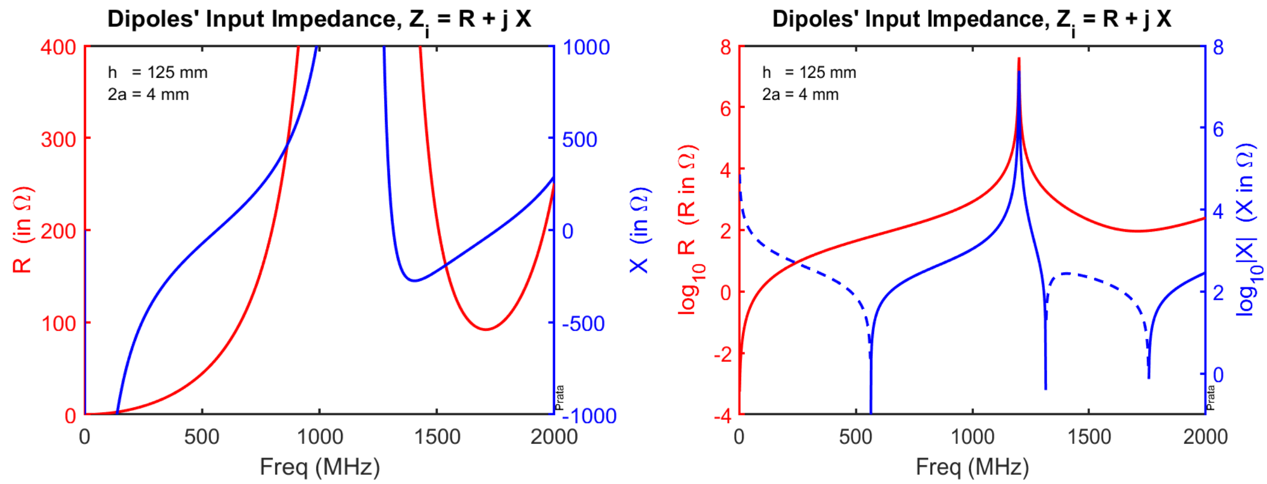

Figure 4: Dipole antenna impedance ($2h = 250$ mm and $2a = 4$ mm). Dashed lines represent negative reactance

larger diameters $2a$ provide broader bands of operation). Where are the resistive and reactive parts of the input impedance coming from?

5. If you were to use the $2h = 250$ mm and $2a = 0.4$ mm dipole to efficiently radiate electromagnetic energy by connecting it to a $Z_0 = 50\,\Omega$ transmission line, what would would be its optimum frequency of operation, and why?

6. Use what you learned in the previous items to design a dipole to precisely operate at 600 MHz with $2a = 0.4$ mm (the diameter of a 26 AWG wire). Provide the dipole length $2h$ and the associated input impedance plot (for good resolution I suggest that you use a horizontal frequency scale going from 550 to 650 MHz, and vertical scales going from -100 to +100 Ω).

4 Input Impedance of a Monopole Antenna

We are now going to make the antenna that you designed in the previous item, and then measure its input impedance Z_i. To determine Z_i we need to perform a differential measurement of the impedance present between the two dipole terminals, when it is radiating energy. The word differential is very important here, as it stresses the fact that, in order to produce the required equal currents in the two dipole arms, the dipole must be excited by a symmetric generator. Unfortunately the NanoVNA-F is unable to do this since the outside of its coaxial connectors are attached to its metal case (in other words, by construction the ports of the NanoVNA-F are not differential, or symmetric).

To circumvent the above measurement difficulty, instead of constructing a dipole we will instead make and measure its very close cousin, the monopole antenna. Monopole and dipole antennas differ by the fact that, since in the monopole antenna half of the dipole is implemented by a ground plane, it does not require a symmetric excitation[6].

[6]The monopole antenna is sometimes referred to as the Marconi antenna, since it was invented

1. Using all that you learned above, construct a monopole antenna to operate at 600 MHz. Make your monopole antenna using a piece of the 26 AWG insulated hookup wire of the proper length h, carefully insert it in the SMA-DIP8-SMA test fixture, and then connect the test fixture to the properly calibrated Port 1 of the NanoVNA-F.

   If you are wondering, the insulation of the 26 AWG wire has little effect on the monopole operation since the insulation is very thin and hence most of the electric field that runs between the two dipole arms is located in free space.

2. Measure the reflection coefficient S_{11} of your antenna over the 50 kHz to 1000 MHz frequency range and provide the corresponding picture of the NanoVNA-F screen. Do you see the expected resonance? Approach your hand to the antenna and observe the effect on the monopole $|S_{11}|$. Also hold the NanoVNA-F box and again observe the effect on the monopole impedance.

3. Provide a sketch clearly showing what are the two halfs of your dipole antenna and where are the electric currents flowing, and discuss your conclusions.

   When addressing this item keep in mind that, unless they are DC, electric currents always flow in a region very near the surface of conductors (the thickness of this region is only a few skin depths thick). Since Ohms law indicates that an induced electric current density $\vec{J}_c$ is related to the electric field $\vec{E}$ through $\vec{J}_c = \sigma_c \vec{E}$, where σ_c is the electric conductivity of the material, and $\vec{E} = \vec{0}$ inside any excellent conductor, then $\vec{J}_c = \vec{0}$ inside any excellent conductor. Hence under no circumstances the electric currents are able to burrow through the conductor interior.

   Note that *wire antennas always have two terminals!* It is then of paramount importance to understand were are all the currents flowing and where the other half of your antenna is located.

If you can't tell where all the electric currents are flowing on your antenna then you have not understood its operation, and bad things will probably happen.

   As a corollary of the above, do not "design" or use an antenna in which the currents are not well understood and controlled. Doing so is a recipe for disaster.

4. Carefully explain why the antenna you just made is not at all an acceptable engineering device.

5. To correct the undesirable features of your previous antenna, let's now make and use a proper ground plane. For this we will be using aluminum foils that are

by Guglielmo Marconi, the wireless radio entrepreneur, around 1895. Nineteen century land telegraph transmission lines often employed just a single wire and used the earth as the additional required conductor. Marconi then hypothesized that perhaps a similar approach could work with wireless radio. He then tried it out and was successful.

currently widely available in pretty much any household kitchen[7].

There are different ways to make a ground plane. A very simple way is cut an approximately square piece of the aluminum foil and tape its four corners to a table (or to the floor), making sure that it is flat and at least one wavelength away form any surrounding objects (since we are dealing with radiation, to curb interference by surrounding environment the farther away the better, but there is no need to get carried away).

Another way to make a ground plane is to cut a square piece of cardboard and to cover it with aluminum foil. This way is better because the resulting antenna can be moved around. I suggest that you go with this other alternative.

6. Connect one end of the 2000 mm long RG316 coaxial cable to the NanoVNA-F and calibrate out its effect.

7. Connect your monopole to the SMA-DIP8-SMA test fixture, connect the calibrated open end of the 2000 mm long RG316 cable to the test fixture, and then place the test fixture flat at the center of the ground plane, with the monopole perpendicular to the ground plane.

   Make sure that the ground plane and the test fixture make good contact with each other. A couple of carefully placed pieces of adhesive tape may be of value in keeping the test fixture at the desired orientation and location, and making good contact with the ground plane. If a cardboard is used for the ground plane, the SMA-DIP8-SMA test fixture can be fastened to the ground plane by making two holes in the cardboard and then passing a piece of wire through them. This is the arrangement that was used in the monopole shown in Fig. 5.

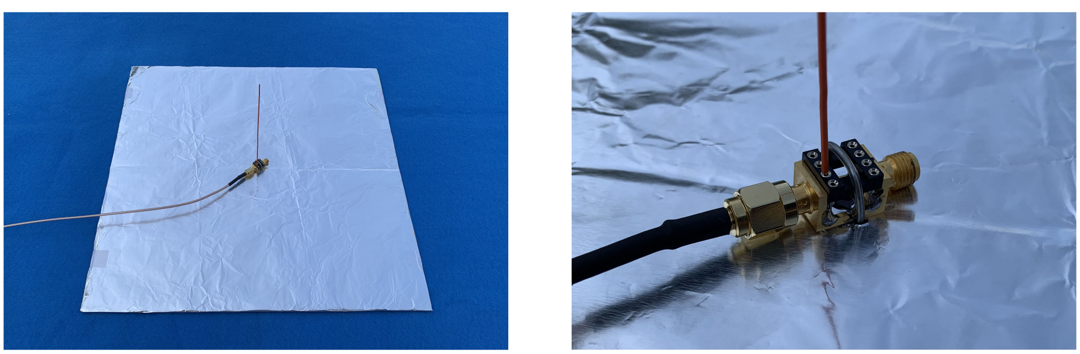

Figure 5: Monopole antenna and its feeding detail

Note that the length h of your monopole is the distance from the tip of the wire to the ground plane. Since the ground plane is a much wider conductor,

[7] I used the *Raynolds Wrap* brand for my antenna—it is 304 mm wide and 16 μm thick. However, pretty much any aluminum foil brand that is used for cooking will work well for the ground plane, as long as its smaller dimension is larger than $\sim 2 \times \lambda_0/4$ (the cooking application assures that the aluminum foil is not dielectric coated).

its terminating distance is not critical, and the height h of the monopole is the dominant factor establishing the antenna resonance frequency.

8. Provide a sketch clearly showing what are the two halfs of your monopole over a ground plane antenna and where are the electric currents flowing. Do the ground plane currents flow on top or underneath its surface?

9. How does the impedance Z_{id} of a dipole antenna and the impedance Z_{im} of a monopole plus infinite flat ground plane antenna relate to each other? Justify your answer using Eq. 22.

10. Augment your previously developed Matlab code to calculate and plot the reflection coefficient S_{11} of your antenna over the 50 kHz to 1000 MHz frequency range and provide the corresponding $|S_{11}|$ (in dB) and $\angle S_{11}$ (in deg) plots.

11. Use the NanoVNA-F, controlled by the the NanoVNASaver software, to measure the reflection coefficient S_{11} of your antenna over the 50 kHz to 1000 MHz frequency range and provide the corresponding $|S_{11}|$ (in dB) and $\angle S_{11}$ (in deg) plots.

12. What is the measured resonance frequency f_0 of your antenna and what is the corresponding value of Z_i? Compare the calculated and measured results and discuss all relevant points.

5 Improving the Monopole Antenna

If you implemented the calculated monopole length correctly, the measured operation frequency is coming significantly under the predicted 600 MHz design value. Observe that you can always tweak the length of your monopole to adjust it to precisely resonate at the 600 MHz design frequency. We will not be doing any length adjustments in this laboratory though. Instead we will be handling this discrepancy in a better way, so please read on.

A possible reason for the resonance frequency discrepancy is that in the calculations you ignored the parasitic reactances of your SMA-DIP8-SMA test fixture and, although they are small, the impact of their associated reactances perhaps can't be safely neglected at our 600 MHz high operation frequency. In other words, instead of measuring the desired Z_i, what you are measuring is the value of Z_i adulterated (or *embedded*) by the test fixture parasitic reactances, which is the Z_e value depicted in Fig. 6. Note that the test fixture basically has two parasitic reactances (you already measured and used both of them in past laboratories): a series stray inductance L_s and a parallel stray capacitance C_s.

1. To test the above hypothesis first show that the desired *de-embedded* Z_i can be

calculated using[8]

$$Z_i = \frac{1}{1/Z_e - j\omega C_s} - j\omega L_s\,. \tag{31}$$

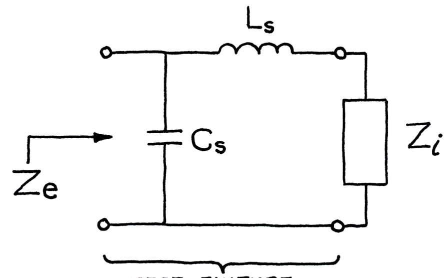

Figure 6: Effect of the SMA-DIP8-SMA test fixture parasitic reactances on the monopole input impedance

Note that the effect of the test fixture stray capacitance is conveniently represented in Eq. 31 using an admitance (i.e., $Y_c = j\omega C_s$), instead of an impedance (i.e., $Z_c = 1/j\omega C_s$), as this eliminates numerical overflow problems (and the corresponding accuracy loss) associated with small ωC_s values, and also allows you to conveniently make C_s equal to zero if desired. On the other hand, and as done in Eq. 31, the effect of the stray inductance is better handled using an impedance, as this allows you to make L_s equal to zero if desired.

2. Use Eq. 31 to augment your Matlab code to properly de-embed the measured Z_e of the monopole and hence obtain Z_i. Then plot the measured reflection coefficient S_{11} of just your antenna over the 50 kHz to 1000 MHz frequency range. As before, provide the corresponding $|S_{11}|$ (in dB) and $\angle S_{11}$ (in deg) plots, compare the calculated and measured results, and discuss all relevant points. Also provide the measured Z_i and the corresponding resonance frequency f_0.

   Figure 7 indicates the type of results that you should expect, and also include in your laboratory report. In particular note the excellent agreement levels between theory and experiment that can be achieved.

[8] *De embedding* is a prevalent VNA terminology, since unfortunately the various unavoidable parasitic impedances present in practical devices make the need to de embed a common feature of VNA measurements.

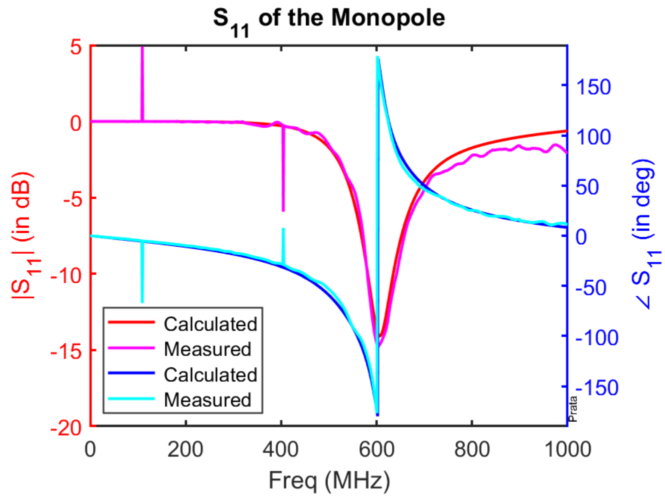

Figure 7: Typical de-embeded monopole reflection coefficient results

It is important to observe that de-embedding could be avoided by calibrating the VNA right at the antenna input terminals (plane established by the two Z_i circles shown in Fig. 6). However, since unfortunately this is not easy to do in the case of the SMA-DIP8-SMA test fixture, we had to resort to de-embedding instead.

A main difficulty with de-embedding is that the equations involved can get out of hand quite fast as the complexity of the embedding circuit increases. You should then keep in mind that it is always easier to calibrate out undesirable effects instead of de-embedding them, and hence plan your work accordingly.

3. What is the calculated and measured percent operation bandwith BW of your monopole antenna (i.e., $BW = (\Delta f/f_0) \times 100$), assuming that a reflection coefficient of -10 dB is acceptable?

4. As you verified in the previous items, and as long as you properly handle the radiating currents, monopole antennas work very well. Unfortunately the aluminum foil ground plane implementation that we have been using is inconvenient though, as it is impractical for situations that require long term durability under weather exposure.

   Fortunately there is a very convenient alternative to the continuous ground plane that you implemented previously, one based on the observation that the ground plane currents flow radially from the base of the monopole, and electromagnetic waves average the effect of obstacles (and conductors) over their wavelength scale. These two facts enable a simplified yet very effective version of the ground plane; an implementation using a few radial wires about $\sim \lambda_0/4$ long.

   Replace the ground plane of your monopole antenna by four 125 mm long 90° radials made of 26 AWG hookup wire (see Fig. 8 for guidance on what to implement), measure the reflection coefficient S_{11} of your antenna over the 50 kHz to 1000 MHz frequency range, and provide the corresponding $|S_{11}|$ (in dB) and $\angle S_{11}$ (in deg) plots.

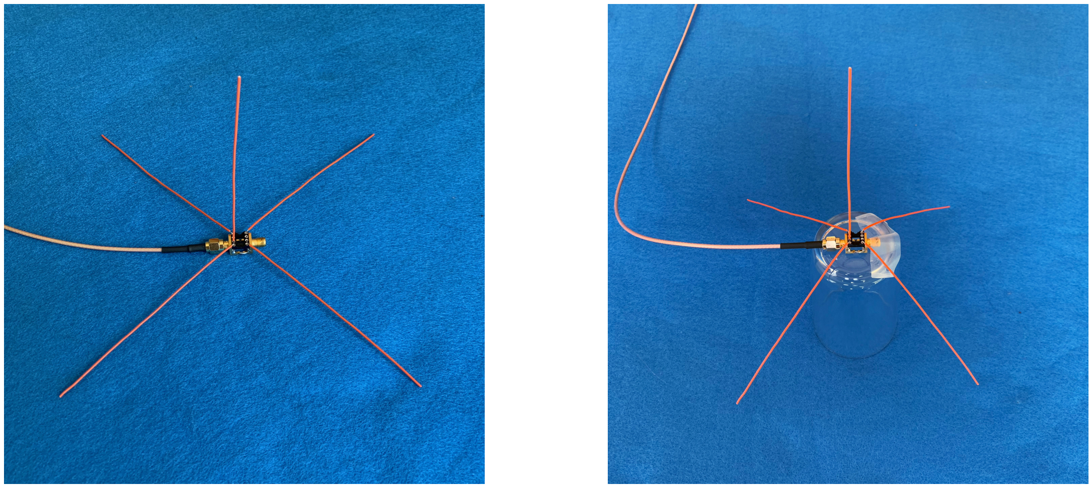

Figure 8: Monopole antennas using 90° and 135° radials
(left and right, respectively)

Although not critical, the length of the radials is measured from the attachment point of the monopole to the SMA-DIP8-SMA test fixture. Also, there is nothing particularly special with using four radials, three radials also work well, as well as more than four radials (as mentioned previously, the electromagnetic field averages the effect of the radials and sees them as a ground plane).

5. What is the measured resonance frequency f_0 of your 90° radials monopole antenna and what is the corresponding value of Z_i?

6. Observing that the radials do not really need to make a 90° angle with the monopole, implement another version of the monopole by placing the SMA-DIP8-SMA test fixture on top of a small non-metallic object and bending the radials downward away from the monopole (a suitable angle relative to the monopole is about 135°). The bent-down radials antenna shown on Fig. 8 uses an upside-down small glass of water to provide the needed support.

   Again measure the reflection coefficient S_{11} of your antenna over the 50 kHz to 1000 MHz frequency range, and provide the corresponding $|S_{11}|$ (in dB) and $\angle S_{11}$ (in deg) plots.

7. What is the measured resonance frequency f_0 of your bent-down radials monopole antenna and what is the corresponding value of Z_i? Make sure to comment on what happened to the Z_i value when you bent down the radials, and to explain what principle caused the beneficial effect obtained.

6 Measurement of the Input Impedance of a Linear Dipole Antenna

In the previous sections we overcame the difficulty associated with measuring the (symmetric or balanced) dipole impedance using an asymmetric (or unbalanced) instrument by taking advantage of the operational equivalences between dipole and monopole antennas. Although this approach worked very well, in many situations one has to measure a symmetric device for which there is no asymmetric equivalent. In this section we will then consider how to properly perform measurements on a balanced device using an unbalanced instrument, such as the NanoVNA-F.

To work with a concrete example, let's consider the voltage source excited dipole antenna shown on the left of Fig. 9. The dimensions of the voltage source V_i and its connecting wires are assumed to be very small compared to the operation wavelength, so circuit theory concepts can be applied to them. Observe that, due to the electrical and mechanical symmetry of the shown voltage source, the currents produced on the two dipole arms are guaranteed to be identical. Furthermore, since the physical dimensions

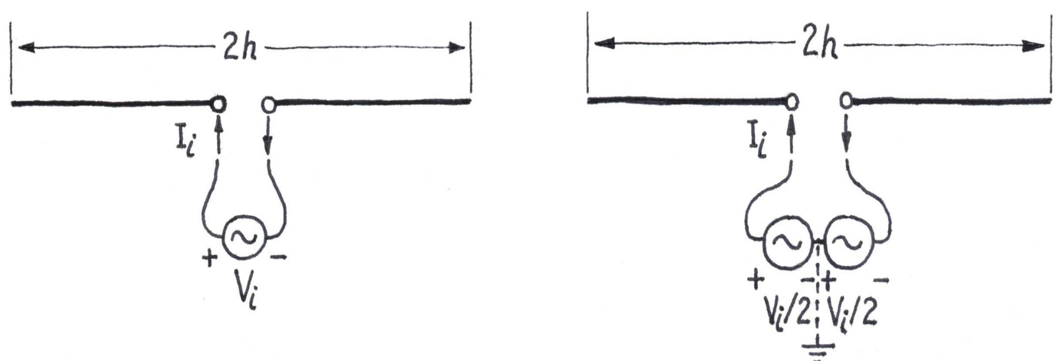

Figure 9: Two variations of a properly excited dipole antenna

of the voltage source are small compared to the dipole dimensions, stray impedances between the voltage source and the antenna arms can be neglected, and hence this is precisely the geometry analyzed earlier on in this manual (i.e., the geometry shown in Fig. 3).

As we proceed it will be more convenient to consider instead the equivalent dipole geometry shown on the right side of Fig. 9, since it more clearly depicts the symmetry present when a proper excitation is employed. In this geometry the voltage source has been split into two identical voltage sources of half value each, which still produce the same excitation voltage and current as before (i.e., V_i and I_i, respectively). However, now it is particularly apparent the symmetry induced *virtual ground* that exists between the two arms of the antenna (i.e., a zero voltage point, and hence effectively a ground). Observe that in both geometries the input impedance of the dipole is given by $Z_i = V_i/I_i$.

On light of the above it should be apparent that simply connecting the terminals of the dipole antenna directly to a coaxial cable, as shown on Fig. 10, will not generate the

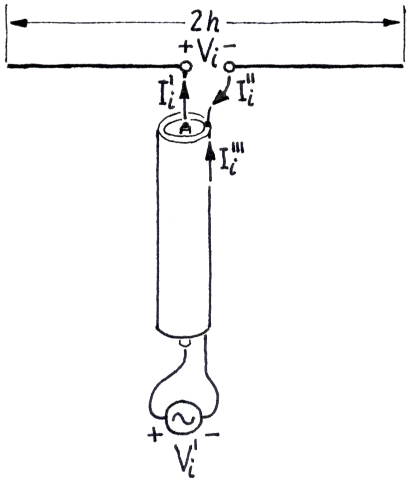

Figure 10: Incorrect coaxial cable excitation of a dipole antenna

desired equal currents on the two dipole arms, since the right dipole arm is now effectively the arm plus the outer surface of the outer conductor of the coaxial cable (observe that $I_i' = I_i'' + I_i'''$). In other words, since both mechanical and electrical symmetries have been destroyed, a symmetry induced ground no longer exists, and hence the dipole antenna will not operate properly, as the currents flowing on its arms will be unequal. Conversely, connecting the Port 1 of the NanoVNA-F to the dipole this way will not measure the input impedance of a properly excited dipole antenna, since V_i/I_i' is not the desired dipole impedance.

A useful alternative way to understand the operation of the antenna of Fig. 10 is to regard it as a monopole with two asymmetric radials; one radial is the dipole right arm and the other radial is the outside surface of the feeding coaxial cable outer conductor. When the antenna is regarded this way you can use what you learned previously to conclude that the dominant factor establishing the antenna resonance frequency is the length h of its left arm, and you can also conclude that the antenna input impedance at resonance will be acceptable from an operations viewpoint. In fact, the antenna would still work if you remove its right arm completely, since one radial is still left in place (i.e., the outside surface of the feeding coaxial cable outer conductor). However, the uncontrolled currents flowing on the outside surface of the feeding coaxial cable outer conductor are undesirable for several reasons (e.g., transmission line will radiate, the antenna radiation pattern will be a distorted version of the dipole antenna radiation pattern, etc.).

To properly operate and measure a dipole antenna a symmetric connection must be used. An example of such connection is shown on the left side of Fig. 11[9]. Since this arrangement has mechanical and electrical symmetry, it assures identical currents on the two dipole arms (a differential, or balanced, excitation is then being used). Observe that, because of the use of two coaxial cables, this connection corresponds to the two-port network shown on the right side of Fig. 11. Each arm of the dipole, together with the virtual ground, constitutes a single two-terminal port of the two-port network; for clarity corresponding points labeled A, B, C, and D are shown on both figures. The short physical connection between points C and D carries a current I_i and is essential for proper operation (without it the outer conductor of the top end of the coaxial cables would not be connected to anything, and hence the two $V_i'/2$ voltage sources would not even be connected to the antenna). Although this specific symmetric connection is almost always impractical for operating dipole antennas (since it is invariably desirable to use just one generator and coaxial cable, as opposed to two of each), it is nevertheless practical for measurements using a VNA, since two VNA

[9]This geometry has been suggested in R. Meys and F. Janssens, *Measuring the Impedance of Balanced Antennas by an S-parameter Method*, IEEE Antennas Propag. Mag., vol. 40, no. 6, pp. 62–65, Dec. 1998.

coaxial ports are already readily available.

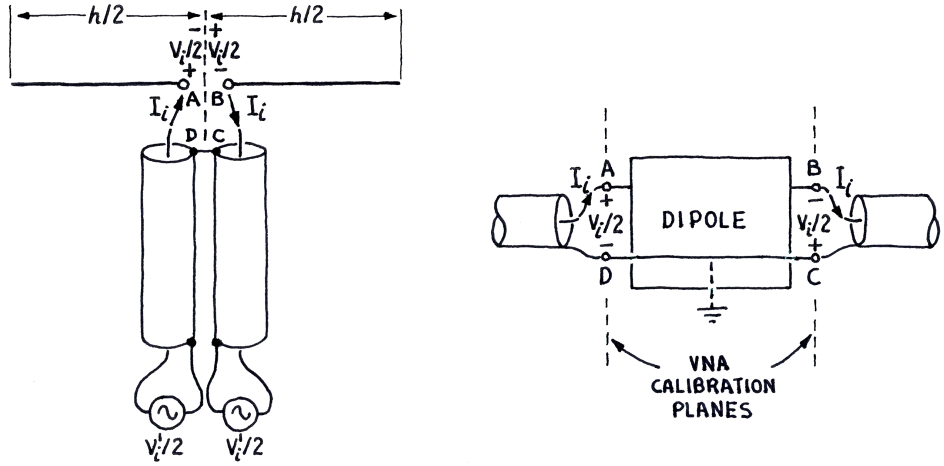

Figure 11: Correct coaxial cable excitation of a dipole antenna (left) and its corresponding two-port network (right)

Let's now implement what we just learned above to measure the impedance of our dipole antenna, without resorting to an equivalent monopole.

1. Relate the two-port network of the dipole antenna depicted on Fig. 11 to the impedance matrix of a general two-port network, and therefore prove that the input impedance of a dipole connected to the two NanoVNA-F coaxial ports is given by
$$Z_i = V_i/I_i = 2\,(Z_{11} - Z_{21})\,. \tag{32}$$

2. Use the SMA-DIP8-SMA test fixture to construct the previously designed 600 MHz dipole antenna (not a monopole), connect it to the NanoVNA-F according to Fig. 10, measure its reflection coefficient, and save the corresponding Touchstone file under the name "unbalanced.s1p."

   Don't forget that the dipole length $2h$ is always the distance measured between the two antenna extremities (in other words, $2h$ includes the space occupied by the test fixture).

   Observe that the purpose of this item is to measure the S_{11} of the incorrectly excited 600 MHz dipole and save the result for posterior analysis. Because somehow you need to hold the dipole away from interfering obstacles (including your hand), and you do not have the benefit of the shielding effect provided by a ground plane (currents are flowing on the outside of the coaxial cable), this is a difficult measurement to perform. Try then to be very careful and precise in order to maximize your measurement accuracy.

3. Modify your previous antenna to implement the correctly excited 600 MHz dipole antenna shown in Fig. 11, measure its scattering parameters, and save the corresponding Touchstone file under the name "balanced.s2p." Observe that the purpose of this item is to measure the S_{11} and S_{21} of the correctly excited 600 MHz

dipole and save the result for posterior processing to obtain the input impedance. Again this is a difficult measurement to perform and hence it is hard to do it accurately.

For your information, Fig. 12 depicts the incorrectly and correctly excited dipole antenna implementations. In particular observe the care taken on preserving the mechanical, and hence electrical, symmetry, on the implementation shown in the right side. Since radiation effects are involved, during measurements the antennas can't be resting on any surface (such as the blue cloth shown on the figure); they have to be held away form any potentially interfering surfaces, otherwise inaccurate results will be produced.

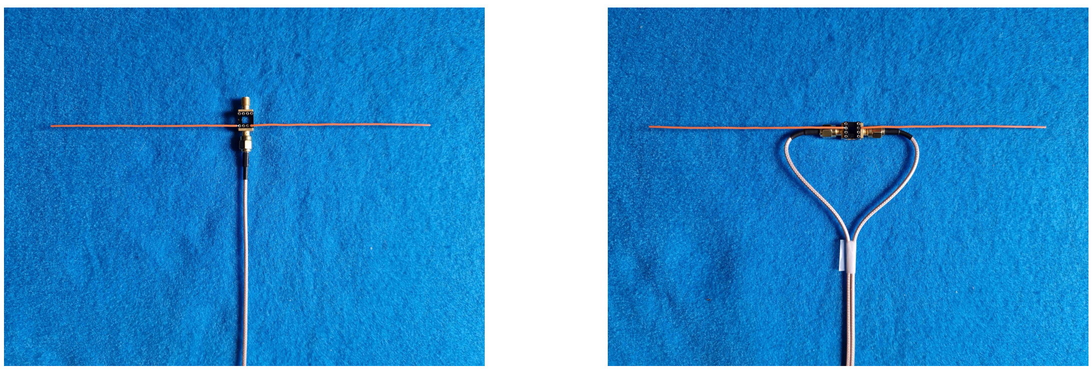

Figure 12: Incorrectly and correctly excited dipole antennas for VNA measurements (left and right, respectively)

4. Duplicate the Matlab code that you formerly generated and modify it to read the previously measured "unbalanced.s1p" and "balanced.s2p" files, process the results, and provide plots showing the dipole antenna predicted Z_i together with the measured values obtained for the incorrectly and correctly excited dipoles. I suggest that the horizontal scale of your plots cover only the 200 MHz region centered at 600 MHz. And please don't forget to comment on all your results.

   Observe that, in spite of not having equal currents flowing on the two dipole arms, having undesirable radiating currents flowing on the outer surface of the outer conductor of the feeding coaxial cable, and suffering from measurement difficulties, as previously mentioned the incorrectly excited dipole still provides a usable input impedance and hence radiates significantly. However, its radiation pattern will be quite different than what is expected for a dipole.

   As previously learned, for good accuracy one would need to properly de-embed the effect of the stray impedances associated with the SMA-DIP8-SMA test fixture used in the Fig. 12 implementations (the impact of the stray impedances are even larger in the dipole configuration that uses both coaxial terminals of the test fixture). In the interest of brevity this refinement will not be pursued at this time though, and we will then live with its undesirable consequences. However, we will again return to this matter in the subsequent laboratory.

5. After all the difficulties that you just experienced operating the properly excited dipole of Fig. 12, it is only natural to wonder how to implement a practical dipole antenna that is correctly fed by a single coaxial cable. The technical literature has available a few alternatives to do this and Fig. 13 depicts one that is particularly convenient.

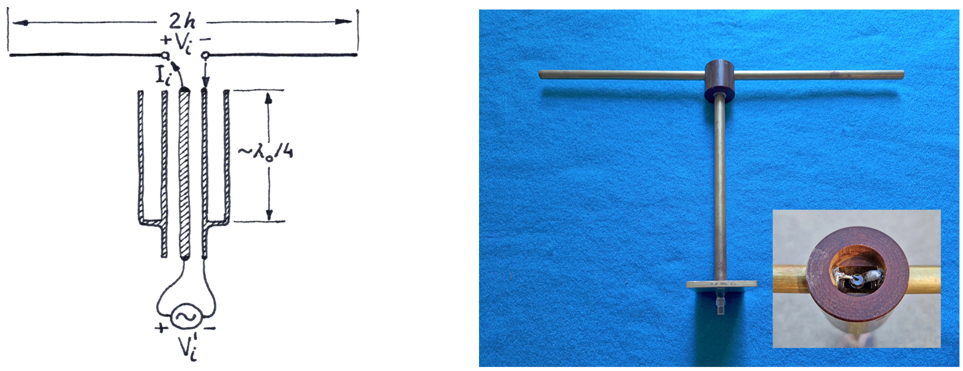

Figure 13: Dipole fed by a bazooka balun. Schematic and implementation (left and right, respectively)

The arrangement of Fig. 13 is made with brass tubes (for rigidity) and employs what is called a choke, or bazooka balun[10], to provide a feed with coaxial symmetry and prevent the excitation current from flowing on the outside surface of the outer conductor of the feeding coaxial cable. The dipole is fed through the SMA-F connector located at the outer sleeve shorting plate (visible at the bottom of the right figure); the plate is also used to support the antenna. The brown cylinder at the center of the dipole arms is a dielectric material that supports the two dipole arms in place. The small insert (at the lower right of the right figure) depicts the details of the connection of the feeding coaxial cable to the two dipole arms; the end of the outer sleeve (i.e., the bazooka) is visible under the solders that connect the coaxial cable conductors to the dipole arms.

As a concluding task for this laboratory, please explain the operation of the bazooka balun, and how it provides the desired balanced excitation to the dipole antenna.

[10]Balun is the name of a general class of devices that provide a balanced excitation starting from an unbalanced transmission line. The word balun is an abbreviation of the words *bal*anced-to-*un*balanced. And bazooka is a somewhat similarly looking musical instrument.