Name: ....

ID #: ....

Maxwell's Equations – Faraday's Law

Laboratory P01 Manual
Prof. Aluizio Prata

1 Laboratory Objective

In this laboratory you will learn about Maxwell's Equations under time-varying situations. More specifically, you will learn how to apply Faraday's law to circuits using inductors operating in time-harmonic regimen. You will then measure the circuits' characteristics, compare the results obtained with theoretical predictions, and also measure the permeability (i.e., the μ) of both air and a magnetic material.

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report is actually this manual, which provides a detailed step-by-step description of all tasks, and have fields that you need to complete as you proceed with your experiment. When done the completed laboratory manual will be a detailed document with all your measurement results, calculations, conclusions, drawings, plots, etc. With this in mind, please make sure to properly add your name and identification number to the fields provided on the top right of this page.

Note that, to maximize the learning experience, this laboratory has been designed to be carried out individually, hence each person in class received their own individual laboratory manual. Consequently, and even if you are part of a group, the corresponding report has to be done completely individually.

2 Instruments, Tools, and Parts List

1. Oscilloscope

2. Two 10:1 oscilloscope probes

3. Time-harmonic voltage source (arbitrary waveform generator)

4. Coaxial cable with one BNC connector and two alligator clips

5. Multimeter

6. Two multimeter test leads

Document version: January 17, 2024, at 10:19 o'clock

7. Digital caliper

8. Small long-nose pliers ($\sim$100 mm)

9. Small side-cutting pliers ($\sim$100 mm)

10. Utility knife

11. $\sim$40 W soldering iron

12. 60/40 Sn-Pb solder, 1 mm diameter

13. One powdered-iron toroidal core (Blue colored. Amidon Associates, Inc. part number T-80-1, $\mu_r \approx 20$, 0.5 - 5.0 MHz. Nominal dimensions: 20.2 mm outer diameter, 12.6 mm inner diameter, and 6.4 mm height)

14. One no-powdered-iron toroidal core (Tan colored. Amidon Associates, Inc. part number T-80-0, $\mu_r = 1$, 0.0 - 300.0 MHz. Nominal dimensions: 20.2 mm outer diameter, 12.6 mm inner diameter, and 6.4 mm height)

15. Two 2.7 Ω, ± 5 %, 1/4 W resistors

16. Two pieces of #20 AWG enameled insulated copper wire (each piece 100 cm long)

3 Theory

In integral form and SI units Maxwell's equations are[1]

$$\oint_C \vec{e} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \int_S \vec{b} \cdot d\vec{s}, \qquad \text{Faraday's law}, \qquad (1a)$$

$$\oint_C \vec{h} \cdot d\vec{\ell} = +\frac{\partial}{\partial t} \int_S \vec{d} \cdot d\vec{s} + \int_S \vec{j} \cdot d\vec{s}, \quad \text{Ampere's law}, \qquad (1b)$$

$$\oint_S \vec{d} \cdot d\vec{s} = \int_V \rho \, dv, \qquad \text{Gauss' law}, \qquad (1c)$$

$$\oint_S \vec{b} \cdot d\vec{s} = 0, \qquad \text{Gauss' law}, \qquad (1d)$$

where, following electric circuits nomenclature, lower case variables are used to represent arbitrary time-varying field quantities (i.e., $\vec{e}$ instead of $\vec{E}$, $\vec{b}$ instead of $\vec{B}$, etc.). The arbitrary surfaces S of these equations are shown as an open surface in Fig. 1, and

---

[1]Ling, Samual J.; Sanny, Jeff; Moebs, William; Friedman, Gerald; Druger, Stphen D.; Kolakowska, Alice; Anderson, David; Bowman, Daniel; Demaree, Dedra; Ginsberg, Edw. S.; Gasparov, Lev; LaRue, Lee; Lattery, Mark; Ludlow, Richard; Motl, Patrick; Pan, Tao; Podolak, Kenneth; Sato, Takashi; Smith, David; Trout, Joseph; and Wheelock, Kevin, *University Physics Volume 2* (2016). Open Access Textbooks. 2. https://commons.erau.edu/oer-textbook/2

hence with a closed contour C at its opening. Each equation has its own independent surface though (i.e., the surfaces' S of each equation can be different, even though they are being represented by the same letter S). The surfaces of Eqs. 1a and 1b are open in general and hence the direction of the infinitesimal area element $d\vec{s}$ relates to the contour C integration direction according to the right-hand rule. On the other hand the surfaces of Eqs. 1c and 1d are necessarily closed, as indicated by the small circle on the surface integral signs. Consequently the surfaces of the last two equations have neither an opening nor a corresponding contour C, and enclose a volume V. Hence the direction of the infinitesimal area element $d\vec{s}$ points away from the interior of the volume V.
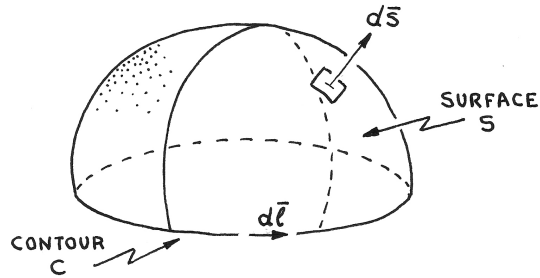

Figure 1: Surface for Faraday's law.

In this laboratory we will deal with Faraday's law (i.e., Eq. 1a). In it $\vec{e}$ is the electric field intensity along the contour C (in Volts per meter, V/m), $\vec{b}$ is the magnetic flux density crossing the surface S (in Volts $\times$ second per meter squared, Vs/m^2, which is Tesla), and t is time (in seconds, s). Observe that the surface S is fictitious (i.e., exists only in our minds) and the field vectors $\vec{e}$ and $\vec{b}$ are all location and time dependent. The term on the right side of Eq. 1a was discovered by Michael Faraday in 1831, through a series of now famous experiments. However, since Faraday had little mathematical training, the convenient mathematical form shown above was only developed many decades later, by James Clerk Maxwell and Oliver Heaviside.

Now let's consider a particular application of Faraday's law to the electric circuit shown in Fig. 2, which is relevant to the current lab. The circuit has a time-harmonic (i.e., sinusoidal time variation) voltage source $v(t)$ (circle on the left side) connected to an inductor (right side) by two wires of negligible electric resistance (wires HF and AC). Except for the rod inside the inductor, which is made of a material with permeability μ, the circuit will be reasonably assumed to be located on air, which has negligible electrical conductivity.

An inductor is a device specifically designed to store a significant amount of energy in its magnetic field, in a highly concentrated and localized manner. This is accomplished by coiling a wire around a core of magnetic material (i.e., a non-conducting material with $\mu > \mu_0$), as shown schematically in Fig. 2, so that the magnetic field produced by each wire loop magnetizes the core and adds constructively to the magnetic field produced by the other loops, to create a region where there is a relatively strong magnetic field. Observe that in this convenient arrangement the current is reused by each loop and hence the magnetic field in the inductor is proportional to the number of

3

loops (or turns) N, provided that the ohmic resistance of the loops is kept small. Since a magnetic mateiral is capable of storing energy in its atomic structure, its presence further increases the total energy stored in the inductor.
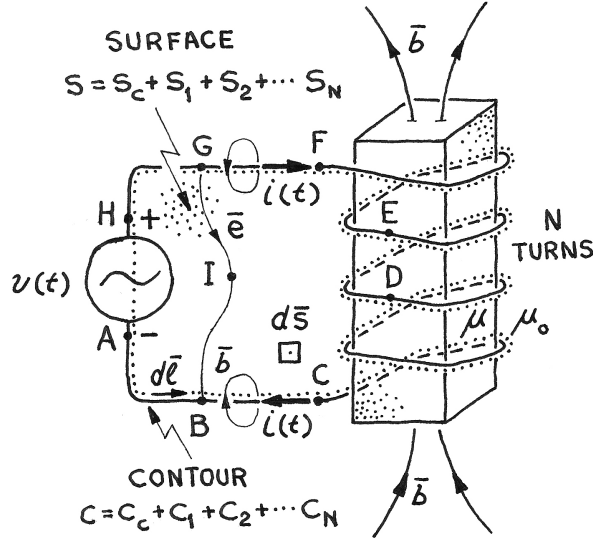

Figure 2: Electric circuit for measurement of Faraday's law

The voltage source of Fig. 2 is used to excite the inductor (i.e., create its magnetic field) by producing a time-dependent current $i(t)$ in its wire. The plus and minus signs on terminals H and A (see Fig. 2), respectively, indicate that, at $t = 0$ the terminal H is at a positive potential relative to the terminal A. Similarly, the wires HF and AC carry a current $i(t)$ with the directions shown in the figure at the time $t = 0$.

In order to apply Faraday's law to the circuit of Fig. 2, the general closed contour C depicted in Fig. 1 is chosen to coincide everywhere with the wire path. Note that $C = C_c + C_1 + C_2 + \ldots C_N$ then follows the path $ABCDEFGHA$, following each turn of the inductor. C_c is the part of the contour just outside the inductor (i.e., $HGF + CBA$) and C_1, C_2, $\ldots C_N$ are the parts of the contour corresponding to the first loop of the inductor, second loop of the inductor, and the Nth (i.e., last) loop of the inductor, respectively. This direction of travel along C then defines the direction of the infinitesimal area element $d\vec{s}$ according to the right-hand rule, as shown in Fig. 2. The closed contour $C = C_c + C_1 + C_2 + \ldots C_N$ then define the corresponding surface $S = S_c + S_1 + S_2 + \ldots S_N$, respectively. Where S_c is the part of S just outside the inductor and S_1, S_2, $\ldots S_N$ are the parts of S associated with the first loop, second loop, and the Nth loop of the inductor, respectively. This particular contour C choice is convenient for analyzing the circuit of Fig. 2, as will become clear below.

Because of the fact that C follows every turn of the the inductor wire, it can be somewhat difficult to visualize all the relevant details of the tri-dimensional area S that it defines. To assist with this visualization the reader is referred to Fig. 3. In this abstraction the inductor core has been removed and the loops of the inductor have been flattened, *without crossing any wires*, making the area S flat. With the assistance of this figure it should be clear what is $S = S_c + S_1 + S_2 + \ldots S_N$. Note that in this

abstraction the magnetic flux density $\vec{b}$ inside the inductor now goes into the page and exists only inside the areas of the loops.
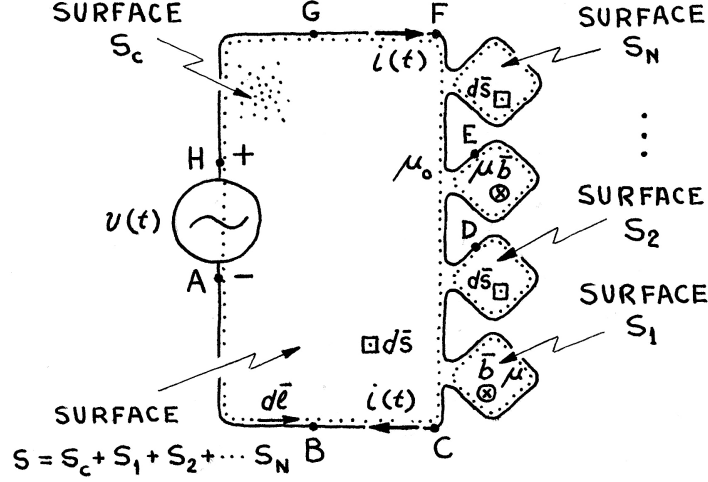

Figure 3: Flattened inductor circuit

With the above contour C choice Eq. 1a then becomes

$$\int_{HA} \vec{e} \cdot d\vec{\ell} + \int_{AB} \vec{e} \cdot d\vec{\ell} + \int_{BC} \vec{e} \cdot d\vec{\ell} + \int_{CD} \vec{e} \cdot d\vec{\ell}$$

$$+ \int_{DE} \vec{e} \cdot d\vec{\ell} + \int_{EF} \vec{e} \cdot d\vec{\ell} + \int_{FG} \vec{e} \cdot d\vec{\ell} + \int_{GH} \vec{e} \cdot d\vec{\ell} = -\int_{S} \frac{\partial \vec{b}}{\partial t} \cdot d\vec{s} , \qquad (2)$$

where the time derivative has been moved inside the integral and operates only on $\vec{b}$ (since the surface S is not varying with time). Now, observing that the AB, BC, CD, DE, EF, FG, and GH integrals are all equal to zero, since the electric field $\vec{e}$ is practically zero inside the wires (the wires are made of an excellent electric conductor), Eq. 2 reduces to

$$\int_{H}^{A} \vec{e} \cdot d\vec{\ell} = -\int_{S} \frac{\partial \vec{b}}{\partial t} \cdot d\vec{s}. \qquad (3)$$

Now recall that the potential difference (i.e., voltage) between two arbitrary spatial points a and b is given by

$$\Phi_{ab} = \Phi_a - \Phi_b = -\int_{b}^{a} \vec{e} \cdot d\vec{\ell}. \qquad (4)$$

Then, since

$$v(t) = \Phi_H - \Phi_A = -\int_{A}^{H} \vec{e} \cdot d\vec{\ell} = +\int_{H}^{A} \vec{e} \cdot d\vec{\ell}, \qquad (5)$$

Eq. 3 becomes

$$v(t) = -\int_S \frac{\partial \vec{b}}{\partial t} \cdot d\vec{s}. \tag{6}$$

Up to this point our derivation is quite general. However, to proceed further we need to carry out the integration on the right side of Eq. 6, and for this we need to determine the magnetic flux density $\vec{b}$ that is going through the area S, which depends on the inductor specific details. Although it is possible to determine the $\vec{b}$ of the solenoid shown in Fig. 2, *a solenoid is by no means a good way to store energy in a magnetic field.* The reason is that the magnetic field of a solenoid is not confined to a closed spatial region (i.e., the solenoid magnetic field spills into the surrounding space. Also, for the same reason, the solenoid can easily pick up interfering magnetic fields). It is then much better to work with an inductor geometry that has a well confined magnetic field; this better geometry is provided by the *toroidal inductor*.
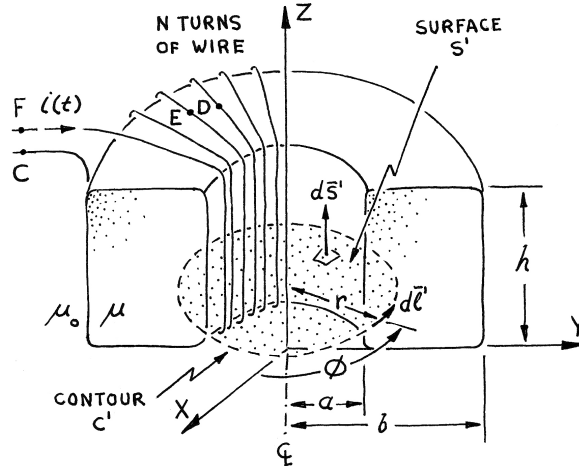

Figure 4: Cut view of a toroidal inductor with a core of rectangular cross section.

Consider then the toroidal inductor, shown in Fig. 4 with its core cut in half along the $x = 0$ plane, for clarity. The core is the toroid with an inner radius a, outer radius b, height h, made with a magnetic material of permeabity μ. The magnetic field of this inductor is created by uniformly winding N turns of tightly-packed high-conductivity wire all around the toroidal core and passing a current i on the wire. Note the points C, D, E, and F on the toroidal inductor wire, which correspond to the same labeled points on the solenoid of Fig. 2. The current i goes into the inductor at F and comes out at C. The toroidal inductor can then be regarded as a curved solenoid with its two ends connected together. As will be seen shortly, this has the very desirable effect of causing the magnetic flux density $\vec{b}$ to be negligible outside the toroidal core (also negligible is then the magnetic field intensity $\vec{h}$ outside the toroidal core, since $\vec{b} = \mu_0 \vec{h}$ there).

In order to assist in determining the magnetic flux density $\vec{b}$ of the toroidal inductor, both Cartesian and cylindrical coordinate systems are shown in Fig. 4. Since we are going to use Ampere's law (i.e., Eq. 1b) to determine $\vec{b}$, also shown in Fig. 4 is a flat

closed circular contour C' of radius r, where $a < r < b$. The closed contour C' encircles a flat open surface S', and the associated $\vec{d\ell}'$ and $\vec{ds}'$ are also shown in Fig. 4. Note that the contour C' is inside the core, resting on a constant z plane with $0 < z < h$.

Let's now apply Ampere's law (i.e., Eq. 1b) to the contour C' of Fig. 4, namely

$$\oint_{C'} \vec{h} \cdot \vec{d\ell} = + \int_{S'} \left(\frac{\partial \vec{d}}{\partial t} + \vec{j} \right) \cdot \vec{ds}' . \tag{7}$$

Note that both the $\partial \vec{d}/\partial t$ and the $\vec{j}$ shown on the right side of this equation are the values on the surface S'. Now, assuming that the voltage source $v(t)$ that is producing the electric flux density $\vec{d}$ on the surface S' is not changing too fast with time, the derivative $\partial \vec{d}/\partial t$ can be assumed negligible when compared to the $\vec{j}$ term, and the above equation then reduces to

$$\oint_{C'} \vec{h} \cdot \vec{d\ell} = + \int_{S'} \vec{j} \cdot \vec{ds}' . \tag{8}$$

Recalling that $\vec{b} = \mu \vec{h}$, one notices that the magnetic field intensity $\vec{h}$ inside the inductor must also obey Gauss' law (i.e., Eq. 1d), namely

$$\oint_{S''} \vec{h} \cdot \vec{ds}'' = 0 , \tag{9}$$

where in this case it is convenient to choose the closed surface S'' as a cylinder of infinitesimal height and base equal to S'. The only way that $\vec{h}$ can simultaneously satisfy both Eqs. 8 and 9, and the symmetry of the toroidal inductor, is if

$$\vec{h}(r, \phi, z, t) = h(r, t)\, \hat{\phi} . \tag{10}$$

Substituting this result into Eq. 8, recalling that $\vec{d\ell}' = \hat{\phi}\, r\, d\phi$ on C' , and that

$$\int_{S'} \vec{j} \cdot \vec{ds} = -N\, i(t) , \tag{11}$$

since $\vec{j} = -j\, \hat{z}$, $\vec{ds}' = \hat{z}\, ds'$, and the current $i(t)$ crosses N times the surface S, one can write Eq. 8 as

$$\oint_{C'} h(r, t)\, \hat{\phi} \cdot \hat{\phi}\, r\, d\phi = -N\, i(t) . \tag{12}$$

Since the radius r is constant on C', the above equation becomes

$$h(r, t)\, r \oint_{C'} d\phi = -N\, i(t) , \tag{13}$$

which yields

$$h(r, t) = \frac{-N\, i(t)}{2\pi r} . \tag{14}$$

7

Substituting this result into Eq. 10 finally yields the desired magnetic flux density inside the toroidal inductor as

$$\vec{b} = \mu\,\vec{h} = \frac{-N\,\mu\,i(t)}{2\pi r}\,\hat{\phi}\,. \tag{15}$$

Observe that in the above $\vec{b}$ derivation it was assumed that the flat contour C' was located inside the inductor core (i.e., C' has $a < r < b$ and is at a plane z such that $0 < z < h$). If for any z we select a contour C' with $r < a$ instead, the derivation yields $\vec{b} = \vec{0}$, since the right side of Eq. 8 will be zero as there will be no current crossing the surface S'. Similarly if for any z we select a contour C' with $r > b$ instead, the derivation yields $\vec{b} = \vec{0}$, since the right side of Eq. 8 will be zero as there will be either no current or no *net* current crossing the surface S'. The bottom line is that $\vec{b} = \vec{0}$ anywhere outside the toroidal inductor; the magnetic field of the toroidal inductor is then very desirably confined to the inductor interior.

Let's now return to the circuit of Fig. 2 with the toroidal inductor replacing the solenoid. Substituting Eq. 15 into Eq. 6 yields

$$v(t) = -\int\limits_{S_1+S_2+S_3+\ldots S_N} \frac{\partial}{\partial t}\left[\frac{-N\,\mu\,i(t)}{2\pi r}\,\hat{\phi}\right] \cdot d\vec{s}\,. \tag{16}$$

Note that the integration over the surface $S = S_c + S_1 + S_2 + S_3 + \ldots S_N$ has been replaced by an integration over just $S_1 + S_2 + S_3 + \ldots S_N$, which are the only parts of S with a significant magnetic flux density (i.e., the sum of the rectangular regions inside each one of the toroidal inductor turns). However, since the integrand is the same over each loop of the toroid we can write

$$v(t) = -N\int\limits_{z=0}^{z=h}\int\limits_{r=a}^{r=b} \frac{\partial}{\partial t}\left[\frac{-N\,\mu\,i(t)}{2\pi r}\,\hat{\phi}\right] \cdot d\vec{s}\,, \tag{17}$$

where $d\vec{s} = \hat{\phi}\,dr\,dz$ is the area element of the inductor core cross section. Using this $d\vec{s}$ Eq. 17 becomes

$$v(t) = \frac{\mu N^2}{2\pi}\frac{di(t)}{dt}\int\limits_{z=0}^{z=h}\int\limits_{r=a}^{r=b}\frac{1}{r}\,dr\,dz = \frac{\mu N^2 h}{2\pi}\,\ln(b/a)\,\frac{di(t)}{dt}\,, \tag{18}$$

which can be conveniently rewritten as

$$\boxed{v(t) = L\,\frac{\partial i(t)}{\partial t}}\,, \tag{19}$$

where L is the inductance of the toroidal inductor with rectangular cross section (in Henries), namely

$$\boxed{L = \frac{\mu N^2 h}{2\pi}\,\ln(b/a)}\,, \tag{20}$$

μ is the permeability of the toroidal core, N is the number of wire turns, h is the height of the toroidal core, and a and b are the inner and outer radii of the toroidal core, respectively. The inductance L depends only on the physical characteristics of the inductor and measures the ability of the inductor to store energy in the magnetic field (the energy stored is proportional to L and the square of the inductor current).

Equation 19 can be considered Maxwell's Faraday law for an inductor. It is a very important equation for circuit theory, as it completely characterizes the inductor electric behavior. Among other things it shows that, as long as the current flowing in the inductor is not constant with time, an inductor does not behave as an open circuit, since the changing magnetic field in its interior creates a voltage $v(t)$ on its terminals.

For this laboratory we will use a time-harmonic (i.e., sinusoidal) voltage source $v(t)$ to excite the inductor (i.e., create a current and hence a magnetic field in its core). This voltage source can be assumed to be described by

$$v(t) = v_0 \cos(\omega t). \tag{21}$$

In this equation v_0 is a constant (in Volts) that determines the amplitude of the applied voltage and ω is the angular frequency of the produced sinusoidal voltage (i.e., $\omega = 2\pi f$, where f is the sinusoid frequency, in Hertz). Substituting Eq. 21 into Eq. 19 yields

$$i(t) = \frac{v_0}{\omega L} \sin(\omega t) = \frac{v_0}{\omega L} \cos(\omega t - \pi/2). \tag{22}$$

Note that Eq. 22 indicates that, when a time harmonic voltage is applied to an inductor, a time-harmonic current will flow through the inductor. The amplitude of this induced time-harmonic current $i(t)$ is proportional to the amplitude of the applied time-harmonic voltage, the frequency, and the inductance. Also note that, even thought the current $i(t)$ is time harmonic, it is not it phase with the applied voltage $v(t)$; the current $i(t)$ *lags* the source voltage by 90°.

Armed with the above equation we will now proceed towards our Faraday's law experiment.

4   Constructing the Inductors

In this section you will construct two small toroidal inductors, one with a non-conducting magnetizable core (i.e., a powdered-iron core, $\mu \gg \mu_0$) and the other with a non-conducting non-magnetizable core (i.e., a no-powdered-iron core, $\mu = \mu_0$), and precisely measure their relevant physical dimensions. The completed inductors will look as shown in Fig. 5. Note the enameled copper wire uniformly tightly wrapped around the two cores. Also note how the two wire ends protrude to create the terminals for connecting the inductors to a circuit (the thin cylinders in the middle of the bottom wires are resistors).
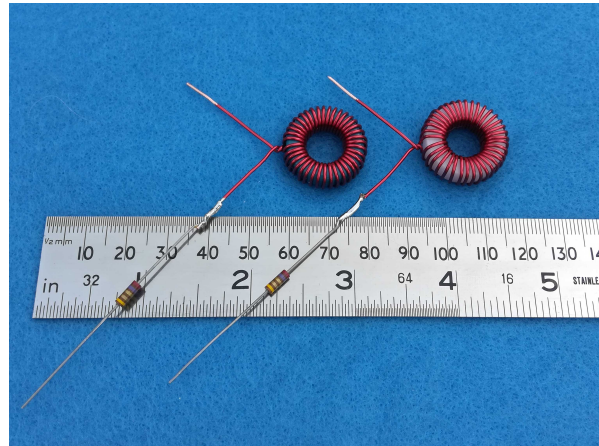
Figure 5: Completed Precision inductors.

1. Obtain one (blue-colored) powdered-iron toroidal core.

   As discussed in the theory section, we are using a toroidal-shaped inductor because of its very desirable ability to keep the magnetic field confined to its interior volume.

2. Use a digital caliper to measure the average height h, outer diameter $2b$, and inner diameter $2a$ of the toroidal core. Take several measurements (at least four, angularly spaced by about 90°) of different heights and diameters and record each result in the table below with two decimal places.

DIMENSIONS OF THE POWDERED-IRON CORE			
Measurement number	Height, h (in mm)	Outer diameter, $2b$ (in mm)	Inner diameter, $2a$ (in mm)
1			
2			
3			
4			

Average all the above measured results to determine the most probable values of h, b, and a, and record them in the fields below, with two decimal places.

POWDERED-IRON CORE	
Average value of h (in mm)	
Average value of b (in mm)	
Average value of a (in mm)	

You will be using these values later.

3. Obtain a 100 cm long piece of #20 AWG enameled copper wire.

4. Coil the enameled wire around the toroidal core to make an inductor. The best way to do this is to hold the coil with your right hand and to start by passing half the length of the wire through the toroid center hole. Then to keep on repeatedly coiling one of the two half lengths until only a few centimeters of the enameled wire remains uncoiled. At this point grab the other length of wire and repeatedly coil it until again only a few centimeters of the enameled wire remains uncoiled.

The above coiling process needs to be done by pulling very strongly on the wire after each turn, with your left hand, to force the wire to conform tightly against the toroid surface. If you decide to use the long-nose pliers to do the pulling, please make sure to always grab the wire only very near its end, because the pliers will damage the enamel insulation (later you will be able to cut off the very end of the wire, to remove any damaged part). To be able to pull strongly on the wire, without causing it to slip from the core, you will also need to hold the coils in place by pressing on them with the thumb of your right hand. This process needs to be repeated until the entire core inner hole is covered all around by closely spaced tightly wound turns.

The diameter of the #20 AWG wire is such that the toroidal core will be able to accommodate precisely $N = 39$ full turns of wire on a *single layer*. Count the turns of your inductor and make sure that you indeed have 39 turns (to do the counting without losing track, it helps to place the tip of a pencil against each wire turns and move it along as you count). If your inductor does not have 39, turns correct the problem before proceeding.

When satisfied with the inductor coils, twist the two inductor terminals together by 1+1/4 turns, to prevent the coils from breaking loose. The end result should look as depicted in Fig. 5.

5. Obtain a 2.7 Ω and $\pm 5\%$ accuracy resistor. Using a multimeter, measure the actual resistance R of the resistor and record it in the field below.

   Note that, because the resistance value being measured is relatively small, it is prudent to use the "relative" feature the multimeter, as it allows you to subtract out the undesirable contribution of the resistance of the two multimeter test leads.

POWDERED-IRON CORE	
Measured value of R (in Ω)	

   You will be using this R value later.

6. Using a small side-cutting pliers, cut each one of the inductor's and resistor's terminals to a length of about 30 mm.

   Using the utility knife, completely scrape out about 10 mm of the enamel insulation from the inductor two terminals. When you do this make sure that the enamel coating is entirely removed and you have only the light yellow copper color showing all around the end of the wire.

   Solder one of the resistor terminals to one of the inductor terminals according to the instructions below (please see Fig. 5 for the end result of this process).

   When soldering keep in mind that the soldering iron is used primarily to provide heat, and not to position parts or move solder around. Also be very careful because a powered soldering iron runs very hot (above 300 °C) and will hurt you and damage anything it comes in contact with as well (it will damage plastics immediately!). *Keep then the hot soldering iron on the table and far away from everything!*

   To properly solder, start by powering up the soldering iron (if your particular soldering iron has a temperature dial, set it to 300° C, as the 60/40 Sn-Pb solder has a 188° C melting temperature). After waiting a few minutes for the soldering iron tip to warm up, melt a small amount of solder in the soldering iron tip and then clean the tip using a wet paper towel folded several times to insulate your hand from the heat. This will clean any undesirable old (i.e., oxidized) solder from the soldering iron tip.

   To solder the resistor and inductor wires together, first place their two wires side-by-side, overlaping by about 10 mm. Then twist the resistor wire around the

inductor wire about two turns so the two stay locked together. Lay the inductor on top several pieces of paper (to protect the table from heat), with the twisted wire about 10 mm above the paper. Then melt a *small* amount of solder in the soldering iron tip and without delay place the soldering iron tip in good contact with the two twisted wires. This will heat the wires and start wetting them with the solder. After the wetting starts, add some *small* amount of fresh solder at the junction of the soldering iron and the wires. This step is needed because the solder (i.e., soldering wire) has rosin on its interior, to assist with the soldering process. The twisted wires should now be fully wetted with solder. The last step is to steadly move the soldering iron tip away from the twisted wires and to wait several seconds for the solder to solidify.

When done, immediately turn off the soldering iron and let it cool before storing it away (this will take at least ten minutes).

The entire soldering process needs to be completed without delays, as parts can get damaged by excessive heat and solder oxidizes undesirably if you keep it heated too long.

Doing a good quality solder requires some skill, but it can be easily mastered if you follow the above instructions and are patient with your learning process. The mark of a well done solder is a small, well wetted, smooth, and shiny pool of solder on the soldered parts.

The completed inductor should look as shown in Fig. 5.

DIMENSIONS OF THE NO-POWDERED-IRON CORE			
Measurement number	Height, h (in mm)	Outer diameter, $2b$ (in mm)	Inner diameter, $2a$ (in mm)
1			
2			
3			
4			

7. Obtain one (tan-colored) no-powdered-iron toroidal core to construct the non-magnetic core inductor. You will do this by repeating all the previous steps with the no-powdered-iron core.

   Start by measuring the dimensions of your core and writing it in the table above.

   Then average all the above measured results to determine the most probable values of h, b, and a, and record them in the table below, with two decimal places.

NO-POWDERED-IRON CORE	
Average value of h (in mm)	
Average value of b (in mm)	
Average value of a (in mm)	

   You will be using these dimensions later.

8. Then obtain another 2.7 Ω and ±5% accuracy resistor. Using a multimeter, measure the actual resistance R of the resistor and record it in the field below.

NO-POWDERED-IRON CORE	
Measured value of R (in Ω)	

   You will be using this R value later.

9. Prepare the resistor and no-powdered-iron-core inductor terminals as previously described and solder the resistor in place. Again, the completed no-powdered-iron-core inductor should look as shown in Fig. 5.

5 Assembling the Faraday's Law Circuit

In this section you will assemble the circuit used to verify Faraday's law. In schematic (left) and physical (right) forms, the circuit will look as depicted in Fig. 6. Since

the circuit is identical for either of the two inductors, this section proceeds under the assumption that the powdered-iron-core inductor will be measured first.

Note again the 2.7 Ω resistor. Its function is to add a small resistance R to the inductor connecting wire. This resistance creates a small voltage $v_R(t)$ that can be used to indirectly measure the inductor current $i(t)$ using Ohm's law, namely

$$i(t) = \frac{v_R(t)}{R} \, .$$

The trade-off here is to use a resistor that is large enough to yield a voltage $v_R(t)$ that can be accurately measured, but not so large that it significantly impacts the inductor operation; the 2.7 Ω value does the job adequately.
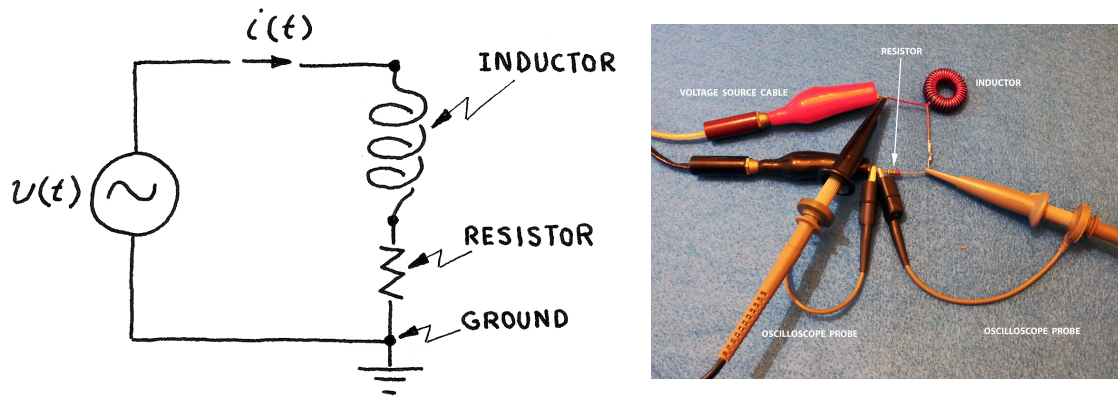

Figure 6: Faraday's law circuit.

1. Obtain an approximately 1 m long coaxial cable (the actual length is not critical) with a BNC connector at one end and two alligator clips at the other end (see Fig. 7 for details). Connect the BNC connector to the Channel 1 connector of the voltage source, and then connect the ground alligator clip (black color) to the inductor terminal that have the 2.7 Ω resistor. This connection point is the *ground* of our circuit, and any other instruments' grounds must be connected to it.
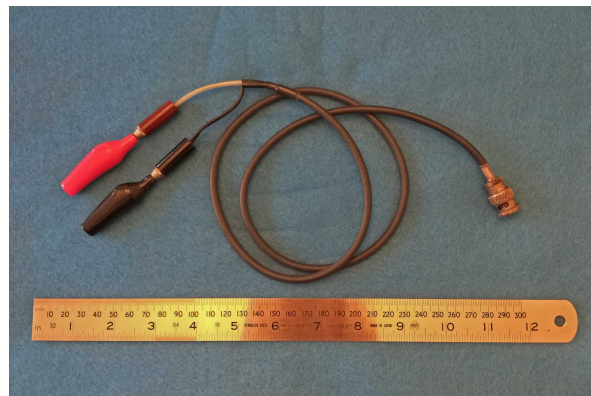

Figure 7: Coaxial Cable.

Note that the voltage source ground is connected to the coaxial cable ground, which is internally connected to the ground plug of the power cord that goes to the wall receptacle.

2. Connect the live alligator clip of the coaxial cable (red color) to the inductor terminal without resistor.

3. Obtain two 10:1 oscilloscope probes for the specific oscilloscope that will be used in the measurements (see Fig. 8 for their appearance). Connect the BNC end of the oscilloscope probe to the Channel 1 connector of the oscilloscope. Then connect the ground end of the oscilloscope probe (end with small alligator clip) to the circuit ground. And then connect the live end of the oscilloscope probe to the inductor terminal without resistor.
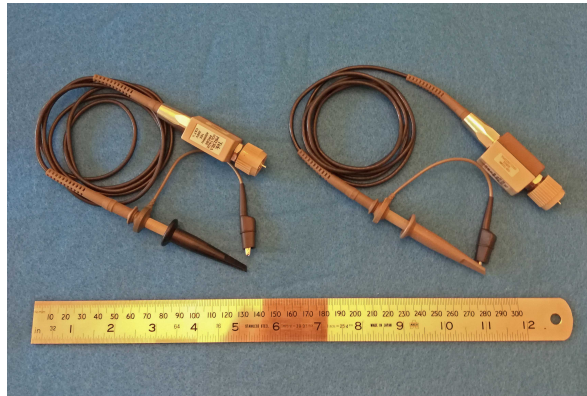

Figure 8: Oscilloscope probes.

The Channel 1 of the oscilloscope will be used to monitor the voltage applied to the inductor.

Note that the oscilloscope ground is internally connected to the ground plug of oscilloscope power cord that goes to the wall receptacle.

4. Connect the BNC end of the other oscilloscope probe to the Channel 2 connector of the oscilloscope. Then connect the ground end of the oscilloscope probe (end with small alligator clip) to the circuit ground. And then connect the live end of the oscilloscope probe to the wire segment that connects the 2.7 Ω resistor to the inductor.

The Channel 2 of the oscilloscope will be used to monitor the resistor voltage, and hence indirectly the current flowing through the inductor.

The Faraday's law circuit is now assembled and should look as depicted in Fig. 6.

6 Laboratory Measurements

Figure 9: Faraday's law circuit connected to instrumentation.

In this section you will measure the characteristics of the circuit assembled in the previous section. The measurement setup is depicted in Fig. 9, with the inductor circuit located near the bottom of the figure, and the voltage source sitting on top of the oscilloscope. Although the specific instruments that you will be using may differ from what is shown, since they are subjected to current availability, their functionality will not.

1. Before proceeding beyond this point, estimate the maximum current that your powdered-iron-core inductor can take (i.e., the inductor current rating) before its core either saturates or heats excessively, whatever occurs first.

   Powdered-irons typically tolerate a maximum magnetic flux density of about 500 mT before saturation effects becomes significant (i.e., the linearity of the relation $\vec{b} = \mu\,\vec{h}$ is compromised). However, excessive heating of the powdered-iron because of its inherent loss mechanisms, in the 0.5 - 5.0 MHz time-harmonic frequency range, can occur if the magnetic flux density exceeds the much lower value of about 6 mT.[2]

   Use the space below to estimate the current rating of your powdered-iron-core inductor, and write the results in the box. The extra space above the box is for

---

[2] *Iron Powder Cores and Materials*, Amidon Associates, Inc. technical note, pag. 9, Dec. 2001.

you to write your equations and comments (i.e., write the rational that lead to the boxed number).

Powdered-iron-core inductor current rating, i_{max} (in mA)	

Note that the above value is only an estimates. For this reason we will use this number only as a guideline, and not as a hard limit, and keep an eye out for any of the effects of heating and non-linearity in our inductor circuit.

Is case you are wondering, the no-powdered-iron core inductor does not suffer from either saturation (μ_0 is always a constant) or significant heating effects (it is not being magnetized). The only heat generation that occurs is the one associated with the enameled wire Joule loss, and this is small in our experiment.

2. Power up the voltage source (i.e., the arbitrary waveform generator), select a time-harmonic signal (i.e., sine wave) and adjust the signal frequency to 1 MHz.

3. Adjust the voltage amplitude of the arbitrary waveform generator to 20 V peak-to-peak (10 V amplitude), provided that this value does not significantly exceed the current rating of your inductor.

4. Turn on the output of Channel 1 of the arbitrary waveform generator.

5. Power up the oscilloscope, find out how to reset it to its factory setup, and perform the reset. This will clean up any undesirable residual settings that might have been left behind by previous users.

6. Modern oscilloscopes have a useful automated setting button, find and press it and you should see the desired time-harmonic signal on the oscilloscope screen (the button is usually labeled Autoset or Autoscale).

7. Make sure that both Channels 1 and 2 are displayed on the oscilloscope screen and adjust the traces' positions and amplitudes to your convenience.

8. Take a quick reading of the voltage waveform across the inductor terminals (i.e., the Channel 1 reading). It should be a clean sinusoid with a peak-to-peak value close to the previously adjusted 20 V peak-to-peak. If this is not the case you will need to troubleshoot your circuit for problems.

Note that modern digital oscilloscopes have many useful features and you may want to use some of them to both expedite and increase the accuracy of your your measurements. In particular, they allow you to automatically average several measurements to reduce measurement noise. It is suggested that you locate and familiarize yourself with this averaging option and use it in your measurements, since the voltage across the resistor terminals (i.e., v_R) can be relatively noisy.

9. Use the oscilloscope to precisely measure the voltage peak-to-peak value across the inductor terminals (i.e., v_{Lpp}), the voltage peak-to-peak value across the resistor terminals (i.e., v_{Rpp}), and the phase difference between the two voltages (i.e., $\angle v_{Rpp} - \angle v_{Lpp}$) for several different frequencies, and write it in the fields below. Make sure to add the phase values with the proper sign.

In case you are wondering, modern digital oscilloscopes are also capable of automatically measuring and numerically displaying peak-to-peak as well as phase values.

Meas. number	Frequency (in MHz)	Inductor peak-to-peak voltage, v_{Lpp} (in V)	Resistor peak-to-peak voltage, v_{Rpp} (in mV)	$\angle v_{Rpp} - \angle v_{Lpp}$ (in deg)
1	0.5			
2	1.0			
3	1.5			
4	2.0			

Observing that the resistor voltage is in phase with its current, confirm that the phase of the inductor current lags the inductor voltage by the expected amount.

10. Calculate the amplitude of the resistor voltage (i.e., v_R), amplitude of the resistor current (i.e., i_R), and the product $\omega \times i_R$ (in Mega radians per second times mili

Amperes), and write them in the table below. The space below is left for you to write down the equation that provides i_R.

Meas. number	Frequency (in MHz)	Resistor amplitude voltage, v_R (in mV)	Resistor amplitude current, i_R (in mA)	$\omega \times i_R$ (in Mrad/s $\times$ mA)
1	0.5			
2	1.0			
3	1.5			
4	2.0			

Observe that, and as predicted by Eq. 22, the product of the frequency and the current flowing on the circuit (i.e., $\omega \times i_R$) should be constant, provided that the applied voltage v_{Lpp} stays constant with frequency.

11. Complete the third and fourth columns of the table below and from them determine the measured inductance of the inductor, L_{PI}, at each frequency. The space below is left for you write down the equation that provides L_{PI}.

Meas. number	Frequency (in MHz)	$\omega \times i_R$ (in Mrad/s × mA)	Inductor amplitude voltage, v_L (in V)	Inductance (in μH)
1	0.5			
2	1.0			
3	1.5			
4	2.0			

12. Average the above inductance values to determine the probable inductance of the powdered-iron-core inductor, L_{PI}, and write it on the field below.

Powdered-iron-core inductor inductance, L_{PI} (in μH)	

13. Using the above inductance value, determine the permeability of the powdered-iron core and write it on the box below. The space below is left for you write down the equation that provides μ_{PI}.

Powdered-iron permeability, μ_{PI} (in H/m)	

14. We will now replace the powdered-iron-core inductor of our circuit by the no-powdered-iron-core inductor, to determine the permeability of air. This is a simple substitution. All you need to do is disconnect the alligator clips and oscilloscope probes, replace the inductor, and reconnect the alligator clips and oscilloscope probes.

    Observe that, because the no-powdered-iron-core inductor has the permeability of vacuum (i.e., it does not get magnetized), its inductance is considerably smaller than the inductance of the powdered-iron-core inductor. For this reason we will be performing measurements at much higher time-harmonic frequencies. To start set then the voltage generator frequency to 10 MHz.

15. Make sure that both Channels 1 and 2 are displayed on the oscilloscope screen and adjust the traces' positions and amplitudes to your convenience.

16. Use the oscilloscope to measure the voltage peak-to-peak value across the inductor plates (i.e., v_{Lpp}), the voltage peak-to-peak value across the resistor terminals (i.e., v_{Rpp}), and the phase difference between the two voltages (i.e., $\angle v_{Rpp} - \angle v_{Lpp}$) for several different frequencies, and write it in the fields below. Make sure to add the phase values with the proper sign.

Meas. number	Frequency (in MHz)	Inductor peak-to-peak voltage, v_{Lpp} (in V)	Resistor peak-to-peak voltage, v_{Rpp} (in mV)	$\angle v_{Rpp} - \angle v_{Lpp}$ (in deg)
1	10			
2	13			
3	16			
4	19			

Observing that the resistor voltage is in phase with its current, confirm that the phase of the inductor current lags the inductor voltage by the expected amount.

17. Calculate the amplitude of the resistor voltage (i.e., v_R), amplitude of the resistor current (i.e., i_R), and the ratio $\omega \times i_R$ (in Mega radians per second times mili

Amperes), and write them in the table below. The space below is left for you write down the equation that provides i_R.

Meas. number	Frequency (in MHz)	Resistor amplitude voltage, v_R (in mV)	Resistor amplitude current, i_R (in mA)	$\omega \times i_R$ (in Mrad/s $\times$ mA)
1	10			
2	13			
3	16			
4	19			

Observe that, and as predicted by Eq. 22, the product of the frequency and the current flowing on the circuit (i.e., $\omega \times i_R$) is constant provided that the generator voltage does not change with frequency.

Meas. number	Frequency (in MHz)	$\omega \times i_R$ (in Mrad/s × mA)	Inductor amplitude voltage, v_L (in V)	Inductance (in μH)
1	10			
2	13			
3	16			
4	19			

18. Complete the third and fourth columns of the table above and from them determine the measured inductance of the no-powdered-iron-core inductor, L_{air}, at each frequency. The space below is left for you to write down the equation that provides L_{air}.

19. Average the above inductance values to determine the probable inductance of the no-powdered-iron-core inductor, L_{air}, and write it on the field below.

No-powdered-iron-core inductor inductance, L_{air} (in μH)	

20. Using the above inductance value, determine the permeability of air and write it on the box below. The space above the box is left for you write down the equation

that provides μ_0.

Air permeability, μ_0 (in H/m)	