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Maxwell's Equations – Ampere's Law

Laboratory P02 Manual

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1 Laboratory Objective

In this laboratory you will continue to learn about Maxwell's Equations under time-varying situations. More specifically, you will learn how to apply Ampere's law to circuits using capacitors operating in time-harmonic regimen, and understand electric displacement currents. You will then measure the circuits' characteristics, compare the results obtained with theoretical predictions, and also measure the permittivity (i.e., the ϵ) of both air and a dielectric material.

As a result of this laboratory you will need to generate and submit a laboratory report for grading. The report is actually this manual, which provides a detailed step-by-step description of all tasks, and have fields that you need to complete as you proceed with your experiment. When done the completed laboratory manual will be a detailed document with all your measurement results, calculations, conclusions, drawings, plots, etc. With this in mind, please make sure to properly add your name and identification number to the fields provided on the top right of this page.

Note that, to maximize the learning experience, this laboratory has been designed to be carried out individually, hence each person in class received their own individual laboratory manual. Consequently, and even if you are part of a group, the corresponding report has to be done completely individually.

2 Instruments, Tools, and Parts List

1. Oscilloscope
2. Two 10:1 oscilloscope probes
3. Time-harmonic voltage source (arbitrary waveform generator)
4. Coaxial cable with one BNC connector and two alligator clips
5. Multimeter
6. Two multimeter test leads

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7. Digital caliper
8. Digital micrometer
9. Small long-nose pliers (∼100 mm)
10. Small side-cutting pliers (∼100 mm)
11. Scissors
12. Jewelers tweezers
13. ∼40 W soldering iron
14. 60/40 Sn-Pb Solder, 1 mm diameter
15. Two brass disks (25 mm diameter and 0.5 mm thickness)
16. One 10 Ω 1/4 W resistor
17. #22 AWG insulated telephone wire (about 100 mm long)
18. 51 μm thick polyimide film (Dupont's Kapton)
19. Two small neodymium magnets (6 mm diameter by 3 mm height)

3 Theory

In integral form and SI units Maxwell's equations are[1]

$$\oint_C \vec{e} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \int_S \vec{b} \cdot d\vec{s}\,, \qquad \text{Faraday's law}\,, \tag{1a}$$

$$\oint_C \vec{h} \cdot d\vec{\ell} = +\frac{\partial}{\partial t} \int_S \vec{d} \cdot d\vec{s} \; + \int_S \vec{j} \cdot d\vec{s}\,, \qquad \text{Ampere's law}\,, \tag{1b}$$

$$\oint_S \vec{d} \cdot d\vec{s} = \int_V \rho \, dv\,, \qquad \text{Gauss' law}\,, \tag{1c}$$

$$\oint_S \vec{b} \cdot d\vec{s} = 0\,, \qquad \text{Gauss' law}\,, \tag{1d}$$

[1] Ling, Samual J.; Sanny, Jeff; Moebs, William; Friedman, Gerald; Druger, Stphen D.; Kolakowska, Alice; Anderson, David; Bowman, Daniel; Demaree, Dedra; Ginsberg, Edw. S.; Gasparov, Lev; LaRue, Lee; Lattery, Mark; Ludlow, Richard; Motl, Patrick; Pan, Tao; Podolak, Kenneth; Sato, Takashi; Smith, David; Trout, Joseph; and Wheelock, Kevin, *University Physics Volume 2* (2016). Open Access Textbooks. 2. https://commons.erau.edu/oer-textbook/2

where, following electric circuits nomenclature, lower case variables are used to represent arbitrary time-varying field quantities (i.e., $\vec{e}$ instead of $\vec{E}$, $\vec{b}$ instead of $\vec{B}$, etc.). The arbitrary surfaces S of these equations are shown as an open surface in Fig. 1, and hence with a closed contour C at its opening. Each equation has its own independent surface though (i.e., the surfaces' S of each equation can be different, even though they are being represented by the same letter S). The surfaces of Eqs. 1a and 1b are open in general and hence the direction of the infinitesimal area element $d\vec{s}$ relates to the contour C integration direction according to the right-hand rule. On the other hand the surfaces of Eqs. 1c and 1d are necessarily closed, as indicated by the small circle on the surface integral signs. Consequently the surfaces of the last two equations have neither an opening nor a corresponding contour C, and enclose a volume V. Hence the direction of the infinitesimal area element $d\vec{s}$ points away from the interior of the volume V.

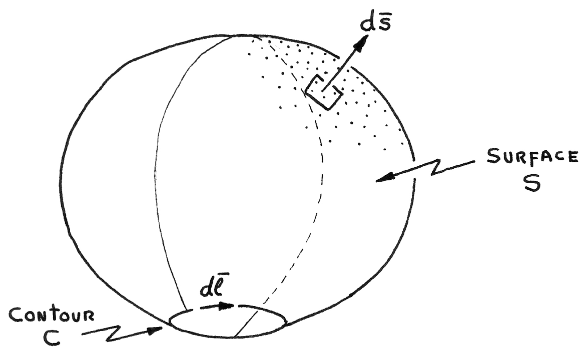


Figure 1: Surface for Ampere's law.

In this laboratory we will deal only with Ampere's law (i.e., Eq. 1b). In it $\vec{h}$ is the magnetic field intensity along the contour C (in Amperes per meter, A/m), $\vec{d}$ is the electric flux density crossing the surface S (in Amperes x second per meter, As/m^2), $\vec{j}$ is the electric current density crossing the surface S (in Amperes per meter squared, A/m^2), and t is time (in seconds, s). Observe that the surface S is fictitious (i.e., exists only in our minds) and the field vectors $\vec{h}$, $\vec{d}$, and $\vec{j}$ are all location and time dependent. Furthermore, since in the situation of interest in this laboratory the surface S is time invariant, Eq. 1b can be rewritten as

$$\oint_C \vec{h} \cdot d\vec{\ell} = \int_S \frac{\partial \vec{d}}{\partial t} \cdot d\vec{s} + \int_S \vec{j} \cdot d\vec{s}. \tag{2}$$

The particular term

$$\vec{j}_d = \frac{\partial \vec{d}}{\partial t}, \tag{3}$$

which behaves as an electric current density, is called *displacement electric current density* (in A/m^2). The need to have it in Eq. 2 was discovered by James Clerk Maxwell himself, in one of the greatest breakthroughs of the nineteenth century physics. Substituting Eq. 3 into Eq. 2 yields a convenient compact way to express Ampere's law when

S is time invariant, namely

$$\oint_C \vec{h} \cdot d\vec{\ell} = \int_S (\vec{j}_d + \vec{j}) \cdot d\vec{s}\,. \tag{4}$$

Now let's consider a particular application of Ampere's law to the electric circuit shown in Fig. 2, which is relevant to the current lab. The circuit has a time-harmonic (i.e., sinusoidal time variation) voltage source $v(t)$ (circle on the left side) connected to a parallel-plate circular capacitor (right side) by two wires of negligible electric resistance (wires HF and AC). The circuit will be reasonably assumed to be located on air, which has negligible electrical conductivity and permittivity ϵ_0.

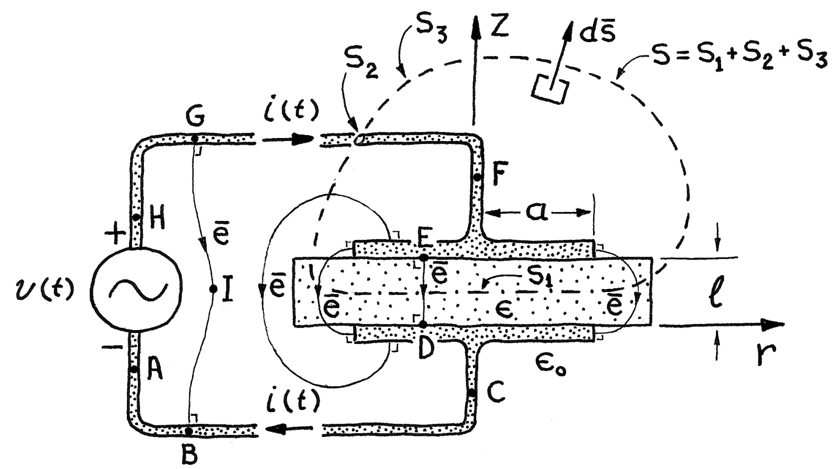


Figure 2: Electric circuit for measurement of Ampere's law

A capacitor is a device specifically designed to store a significant amount of energy in the electric field, in a highly concentrated and localized manner. This is accomplished by creating a region between two closely spaced highly conducting plates where there is a relatively strong electric field. The capacitor of Fig. 2 has two highly conducting circular plates of radius a (can be assumed perfectly conducting for the purposes of this laboratory), separated by a small distance ℓ. By having ℓ/a small a strong localized electric field is produced in the region between the two plates when a voltage is applied to the two capacitor terminals (i.e., F and C). The space between the plates is filled by a dielectric (insulating) material with permittivity ϵ, which will be reasonably assumed to have negligible conductivity. Since a dielectric is capable of storing energy in its molecular dipole moments, its presence further increases the total energy stored in the capacitor. The ammount of energy stored in the capacitor is proportional to the area A of its plates, namely

$$A = \pi a^2\,. \tag{5}$$

To assist in applying Ampere's law, a cylindrical coordinate system is shown in Fig. 2 with its origin located at the center of the lower capacitor plate and z axis aligned with the capacitor axis of symmetry (the lower and upper capacitor plates are then located at $z = 0$ and $z = \ell$, respectively).

The voltage source of Fig. 2 is used to excite the capacitor (i.e., create the electric field between its two plates) by producing a time-dependent voltage between its two terminals (i.e., points H and A). The plus and minus signs on terminals H and A (see Fig. 2), respectively, indicate that, at $t = 0$ the terminal H is at a positive potential relative to the terminal A. Similarly, the wires HF and AC carry a current $i(t)$ with the directions shown in the figure at the time $t = 0$.

In order to apply Ampere's law to the circuit of Fig. 2, the general open surface depicted in Fig. 1 is now the *closed* surface S enclosing just the upper plate of the capacitor. This surface choice is convenient for analyzing the capacitor of Fig. 2, as will become clear below.

Let's now obtain an expression for the current $i(t)$ in terms of the voltage $v(t)$. To accomplish this observe that the surface S is made of three parts: S_1, which is the part inside the capacitor (i.e., region with $0 < z < \ell$ and $r < 0$); S_2, which is the area where S intersects the upper wire; and S_3, which is the remaining part of S (i.e., $S = S_1 + S_2 + S_3$). Since the contour C now has zero length, as S is a surface without opening (a closed surface), Eq. 4 reduces to

$$0 = \int_{S_1} (\vec{j}_d + \vec{j}) \cdot d\vec{s} + \int_{S_2} (\vec{j}_d + \vec{j}) \cdot d\vec{s} + \int_{S_3} (\vec{j}_d + \vec{j}) \cdot d\vec{s}. \tag{6}$$

Assuming that the plates' separation ℓ is much smaller than the capacitor radius (i.e., $\ell/a << 1$), the electric field outside the capacitor plates will be insignificant when compared to the field between the capacitor plates. Consequently the $\vec{j}_d$ of the third integral in Eq. 6, which runs on the surface outside the capacitor and connecting wire, is negligible. Since there is no electric current density $\vec{j}$ crossing the surface S_3 Eq. 6 becomes

$$0 = \int_{S_1} (\vec{j}_d + \vec{j}) \cdot d\vec{s} + \int_{S_2} (\vec{j}_d + \vec{j}) \cdot d\vec{s}. \tag{7}$$

The integration of the second term on the right side of Eq. 7 runs over S_2, with is the cross section of the top connecting wire. Since on S_2 there is no significant electric field, as the wire is made of an excellent electric conductor, $\vec{j}_d$ is negligible. Eq. 7 then reduces to

$$0 = \int_{S_1} (\vec{j}_d + \vec{j}) \cdot d\vec{s} + \int_{S_2} \vec{j} \cdot d\vec{s}. \tag{8}$$

The last integral on the right side of Eq. 8 is simply equal to the negative of the current flowing on top wire, namely $-i(t)$. The negative sign is due to the fact that the direction of the drawn $i(t)$ in Fig. 2 is opposite to the direction of $d\vec{s}$ on S_2 (the result of the dot product $\vec{j} \cdot d\vec{s}$ will then have a negative sign). With this result Eq. 8 then becomes

$$i(t) = \int_{S_1} (\vec{j}_d + \vec{j}) \cdot d\vec{s}. \tag{9}$$

Now let's consider the integration over S_1. Since there is no electric current flowing between the plates of the capacitor (the dielectric has negligible conductivity), Eq. 9

further reduces to

$$i(t) = \int_{S_1} \vec{j}_d \cdot d\vec{s}\,. \tag{10}$$

Or, using Eq. 3 and recalling that $\vec{d} = \epsilon\,\vec{e}$,

$$i(t) = \epsilon \int_{S_1} \frac{\partial \vec{e}}{\partial t} \cdot d\vec{s}\,. \tag{11}$$

Furthermore, if $\ell/a << 1$, the electric field curvature (also called field fringing) that only occurs very near the edge of the capacitor plates (i.e., region with $r \approx a$ in Fig. 2) can be ignored in Eq. 11, and the electric field inside the capacitor can be assumed spatially constant and given by

$$\vec{e}(r, \phi, z, t) = -\,e(t)\,\hat{z}. \tag{12}$$

Substituting Eq. 12 into Eq. 11, and observing that $d\vec{s} = -\hat{z}\,ds$ on the surface S_1 yields,

$$i(t) = \epsilon\,\frac{\partial e(t)}{\partial t} \int_{S_1} ds\,, \tag{13}$$

or, since the area of the surface S_1 is equal to A (given by Eq. 5),

$$i(t) = \epsilon\,\frac{\partial \vec{e}(t)}{\partial t}\,A\,. \tag{14}$$

To determine $e(t)$ in Eq. 14 recall that the potential difference (i.e., voltage) between two arbitrary spatial points a and b is given by

$$\Phi_{ab} = \Phi_a - \Phi_b = -\int_b^a \vec{e} \cdot d\vec{\ell}, \tag{15}$$

where the integral is independent of the integration path taken from b to a. We can then use Eq. 15 on the closed path $HABIGH$ of Fig. 2 to write

$$\Phi_{AA} = \Phi_A - \Phi_A = 0 = \int_{HA} \vec{e} \cdot d\vec{\ell} + \int_{AB} \vec{e} \cdot d\vec{\ell} + \int_{BIG} \vec{e} \cdot d\vec{\ell} + \int_{GH} \vec{e} \cdot d\vec{\ell}. \tag{16}$$

Now, observing that the AB and GH integrals are both equal to zero, since the electric field $\vec{e}$ is practically zero inside the wires (the wires are made of an excellent electric conductor), Eq. 16 reduces to

$$0 = \int_H^A \vec{e} \cdot d\vec{\ell} + \int_{BIG} \vec{e} \cdot d\vec{\ell}. \tag{17}$$

However, since

$$v(t) = \Phi_H - \Phi_A = -\int_A^H \vec{e} \cdot d\vec{\ell} = +\int_H^A \vec{e} \cdot d\vec{\ell}\,, \tag{18}$$

Eq. 17 becomes

$$v(t) = -\int_{BIG} \vec{e} \cdot d\vec{\ell}\,, \tag{19}$$

or, since all the connecting wires, as well as the capacitor plates, are made of excellent electric conductors,

$$v(t) = -\int_{DE} \vec{e} \cdot d\vec{\ell}\,, \tag{20}$$

where the contour integral is now running over an integration path inside the capacitor, running from its lower to its upper plate. In other words, the excellent electric conductors used in the circuit cause the voltage source to apply its voltage directly between the two plates of the capacitor.

We now substitute Eq. 12 into Eq. 20 to obtain, after observing that $d\vec{\ell} = \hat{z}\,dz$ over a straight line connecting D and E,

$$v(t) = \int_{DE} e(t)\,\hat{z} \cdot \hat{z}\,dz = e(t) \int_D^E dz\,, \tag{21}$$

and hence

$$v(t) = e(t)\,\ell\,. \tag{22}$$

Substituting the above $e(t)$ result into Eq. 14 yelds

$$i(t) = \frac{\epsilon A}{\ell}\,\frac{\partial v(t)}{\partial t}\,, \tag{23}$$

which can be conveniently rewritten as

$$\boxed{i(t) = C\,\frac{\partial v(t)}{\partial t}}\,, \tag{24}$$

where C is the capacitance of the capacitor (in Farads), namely

$$\boxed{C = \frac{\epsilon A}{\ell}}\,, \tag{25}$$

A is the capacitor plates' area, given by Eq. 5, and ℓ is the plates' separation. The capacitance C depends only on the physical characteristics of the capacitor and measures the ability of the capacitor to store energy in the electric field (the energy stored is proportional to C and the square of the capacitor voltage).

Equation 24 can be considered Maxwell's Ampere law for a capacitor. It is a very important equation for circuit theory, as it completely characterizes the electric behavior of capacitors. Among other things it shows that, as long as the voltage between the two capacitor plates is not constant with time, a capacitor does not behave as an open circuit, since the displacement current between its plates provides continuity to the current $i(t)$ flowing through the capacitor terminals.

For this laboratory we will use a time-harmonic (i.e., sinusoidal) voltage source $v(t)$ to excite the capacitor (i.e., create the electric field between its two plates). This voltage source can be assumed to be described by

$$v(t) = v_0 \cos(\omega t)\,. \tag{26}$$

In this equation v_0 is a constant (in Volts) that determines the amplitude of the applied voltage and ω is the angular frequency of the produced sinusoidal voltage (i.e., $\omega = 2\pi f$, where f is the sinusoid frequency, in Hertz). Substituting Eq. 26 into Eq. 24 yields

$$i(t) = -\,v_0\,\omega\,C\,\sin(\omega t) = +v_0\,\omega\,C\,\cos(\omega t + \pi/2)\,. \tag{27}$$

Note that Eq. 27 indicates that, when a time harmonic voltage is applied to a capacitor, a time-harmonic current will flow through the capacitor. The amplitude of this induced time-harmonic current $i(t)$ is proportional to the amplitude of the applied time-harmonic voltage, the frequency, and the capacitance. Also note that, even thought the current $i(t)$ is time harmonic, it is not it phase with the applied voltage $v(t)$; the current $i(t)$ *leads* the source voltage by 90°.

Armed with the above equation we will now proceed towards our Ampere's law experiment.

4 Constructing the Capacitor

In this section you will construct a small capacitor and precisely measure its relevant physical dimensions. The completed capacitor will look as shown in Fig. 3. Note the

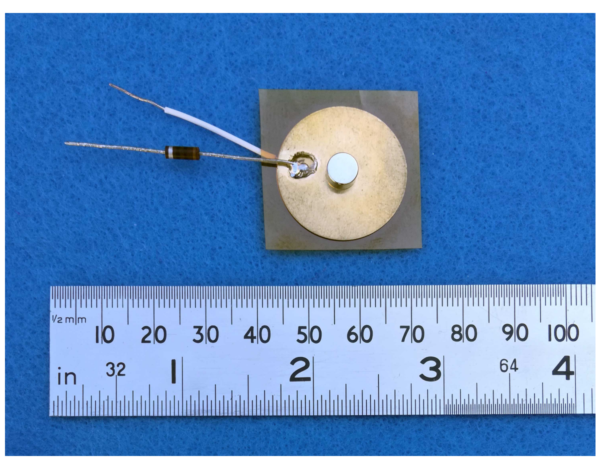


Figure 3: Completed Precision Capacitor.

two yellow plates of the capacitor (brass disks), the dielectric (thin transparent plastic film), and two soldered wires for connecting the capacitor to a circuit (the thin cylinder in the middle of the bottom wire is a resistor). The grey cylinder at the middle of the brass disk is one of two neodymium (Nd) magnets that are used to hold the capacitor plates together; there is another Nd magnet, blocked from view, underneath the capacitor.

1. Obtain two very flat brass disks of 25 mm diameter and 0.5 mm thickness. These two brass disks will constitute the two plates of the capacitor. Because the capacitance depends, among other things, on the plates' separation ℓ (see Eq. 25), to assure good measurement accuracy it is cardinal to start with very flat disks and not bending or nicking their rim by dropping them along the way.

   We are using brass disks for basically two reasons: they can be stamped very flat during the manufacturing process, and it is very easy to solder wires to brass (in soldering parlance, solder wetts brass well).

2. Obtain a 10 Ω and ±5% accuracy resistor. Using a multimeter, measure the actual resistance R of the resistor and record it in the field below.

Measured value of R (in Ω)	

   You will be using this R value later.

3. Using a small side-cutting pliers, cut each one of the resistor's terminals to a length of about 20 mm, and solder one of the terminals' end near the edge of one of the brass disks. Solder near but not at the disk edge (refer to Fig. 3). There must be no solder at the very edge of the disk, otherwise the solder will interfere with the separation of the plates. The resistor can be seen in Fig. 3, the small dark cylinder on the wire shown on the middle left of the figure.

   When soldering keep in mind that the soldering iron is used primarily to provide heat, and not to position parts or move solder around. Also be very careful because a powered soldering iron runs very hot (above 300 °C) and can hurt you and damage things as well.

   To properly solder, start by powering up the soldering iron (if your particular soldering iron has a temperature dial, set it to 300° C, as the 60/40 Sn-Pb solder has a 188° C melting temperature). After waiting a few minutes for the soldering iron tip to warm up, melt a small amount of solder in the soldering iron tip and then clean the tip using a wet paper towel folded several times to insulate your hand from the heat. This will clean any undesirable old (i.e., oxidized) solder from the soldering iron tip.

   To solder the wire to the brass disk, first place the disk flat on top of several *very clean* pieces of paper. This will protect the table surface from the heat, as well as prevent undesirable foreign matter to melt and become part of the disk surface (if this happens the accuracy of your experiment will be compromised). Then melt a *small* amount of solder in the soldering iron tip and without delay place the soldering iron tip on top of the brass disk, at the location where you want to solder the resistor, with the resistor tip held between the disk and the soldering iron. This will heat the disk and melt the solder. When the melting occurs add some *small* amount of fresh solder at the junction of the soldering iron and the disk. This step is needed because the solder (i.e., soldering wire) has rosin on its interior, to assist with the soldering process. The last step is to remove the soldering iron while very steadly holding the resistor in place using a small long-nose pliers, and to wait several seconds for the solder to solidify.

The entire soldering process needs to be completed without delays, as parts can get damaged by excessive heat and solder oxidizes undesirably if you keep it heated too long.

Doing a good quality solder requires some skill, but it can be easily mastered if you follow the above instructions and are patient with your learning process. The mark of a well done solder is a small, well wetted, smooth, and shiny pool of solder on the soldered parts.

4. Using a small side-cutting pliers, cut a ∼40 mm long piece of telephone wire (insulated #22 AWG copper wire) remove about 10 mm of the insulation from each end, and solder one of the wire extremities to the face of the other brass disk. Again, solder near but not at the disk edge; there must be no solder at the very edge of the disk. The insulated #22 wire can be seen in Fig. 3; it is the wire with white insulation.

5. Carefully clean the two brass disks of any dust and foreign matter and assemble the capacitor *without dielectric* by placing the two brass disks in direct contact and holding them firmly together using the attraction force of two cylindrical Nd magnets (6 mm diameter and 3 mm height) placed at the center of the disks. Your capacitor should look as shown in Fig. 3, but without the dielectric film.

   Exercise caution when using the neodimum magnets, as they have very strong magnetic fields and can behave surprisingly. In particular do not bring them anywhere near objects that have magnetic strips with stored information (e.g., credit cards, identity cards, computer magnetic hard disk drives, etc.), as the stored information will be permanently destroyed. In case you are wondering, the magnets' DC magnetic fields have no effect in our experiment.

6. Use a digital caliper to measure the average diameter $2a$ of the plates of the assembled capacitor. Take several measurements (at least four, angularly spaced by about 90°) of different diameters and record each result in the table below with two decimal places.

Measurement number	Disk diameter, $2a$ (in mm)
1	
2	
3	
4	

Average all the above measured results to determine the most probable value of a and record it in the field below, with two decimal places.

Average value of a (in mm)	

Use the above disk radius value to calculate the area of the capacitor plates and record it in the field below, with one decimal place.

Area of the capacitor plates, A (in mm^2)	

7. Use a scissors to cut an ~30 mm square piece of a 50 μm thick polyimide film (i.e., Kapton).

8. Measure the Kapton film thickness at several different locations using a micrometer and record each result in the table below.

Measurement number	Value of ℓ (in μm)
1	
2	
3	
4	

When using the micrometer, make sure to always use the ratchet thimble when closing the jaws to perform a measurement (the ratchet thimble assures a repeatable pressure of the jaws), to close the two jaws very slowly when they are about to touch the part to be measured, and to always start by zeroing out the micrometer readout after bringing the two jaws in direct contact to each other.

Note that our micrometer resolves 1 μm, and it would be desirable to use one that resolves at least 0.1 μm (to have a measurement uncertainty of at least 0.1 μm). Although available, 0.1 μm micrometers are currently still relatively rare, and hence what we have will then have to do.

Average the measured results to determine the Kapton film thickness and record the value in the field below, with zero decimal places (since unfortunately this is all we have).

Average value of ℓ (in μm)	

The above averaged value is the most probable value for the capacitor plates' separation ℓ. You will be using this ℓ value in what follows.

9. Remove the two Nd magnets, separate the two capacitor plates, and lay them on the table with their inner faces up.

   The easiest way to remove the two Nd magnets is to slide one a time off the plates, and safely placing each on the table, away from each other.

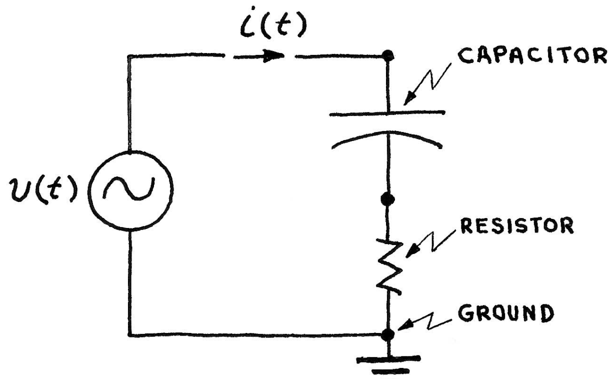
10. Carefully place the square piece of Kapton film on top of one of the capacitor plates. The film is the capacitor dielectric. Without stretching, position the film

so it completely covers the plate and that there are absolutely no folds or crinkles on the film. Assemble the capacitor back together with the Kapton film spilling past the rim of the two brass disks, make sure that the disks' rims precisely align, and put back the two Nd magnets to hold everything in place. The fact that the film edges spill past the disks' rim assures that no air is part of the capacitor dielectric.

The completed capacitor should look as shown in Fig. 3.

5 Assembling the Ampere's Law Circuit

In this section you will assemble the circuit used to verify Ampere's law. In schematic (left) and physical (right) forms, the circuit will look as depicted in Fig. 4.



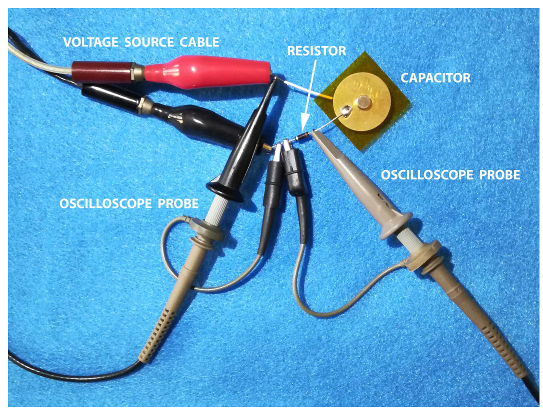


Figure 4: Ampere's law circuit.

Note again the 10 Ω resistor. Its function is to add a small resistance R to the capacitor connecting wire. This resistance creates a small voltage $v_R(t)$ that can be used to indirectly measure the capacitor current $i(t)$ using Ohm's law, namely

$$i(t) = \frac{v_R(t)}{R}\,.$$

The trade-off here is to use a resistor that is large enough to yield a voltage $v_R(t)$ that can be accurately measured, but not so large that it significantly impacts the capacitor operation; the 10 Ω value does the job adequately.

1. Obtain an approximately 1 m long coaxial cable (the actual length is not critical) with a BNC connector at one end and two alligator clips at the other end (see Fig. 5 for details). Connect the BNC connector to the Channel 1 connector of the voltage source, and then connect the ground alligator clip (black color) to the capacitor terminal that have the 10 Ω resistor. This connection point is the *ground* of our circuit, and any other instruments' grounds must be connected to it.

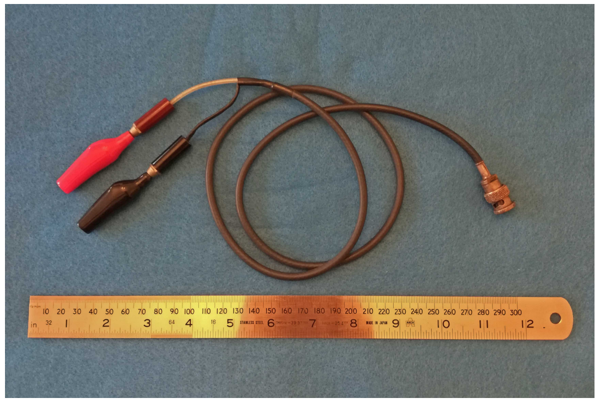


Figure 5: Coaxial Cable.

Note that the voltage source ground is connected to the coaxial cable ground, which is internally connected to the ground plug of the power cord that goes to the wall receptacle.

2. Connect the live alligator clip of the coaxial cable (red color) to the capacitor terminal without resistor.

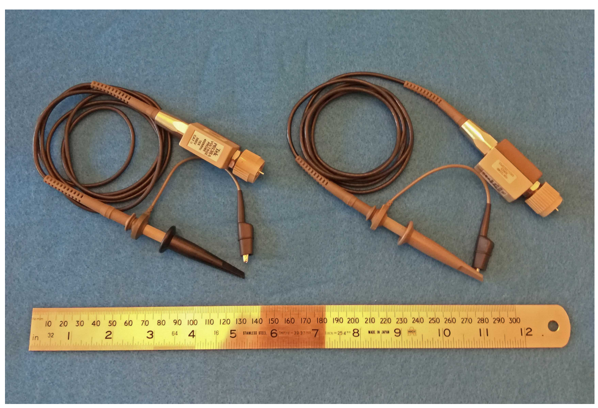


Figure 6: Oscilloscope probes.

3. Obtain two 10:1 oscilloscope probes for the specific oscilloscope that will be used in the measurements (see Fig. 6 for their appearance). Connect the BNC end of the oscilloscope probe to the Channel 1 connector of the oscilloscope. Then connect the ground end of the oscilloscope probe (end with small alligator clip) to the circuit ground. And then connect the live end of the oscilloscope probe to the capacitor terminal without resistor.

   The Channel 1 of the oscilloscope will be used to monitor the voltage applied to the capacitor.

   Note that the oscilloscope ground is internally connected to the ground plug of oscilloscope power cord that goes to the wall receptacle.

4. Connect the BNC end of the other oscilloscope probe to the Channel 2 connector of the oscilloscope. Then connect the ground end of the oscilloscope probe (end with small alligator clip) to the circuit ground. And then connect the live end of the oscilloscope probe to the wire segment that connects the 10 Ω resistor to the capacitor.

   The Channel 2 of the oscilloscope will be used to monitor the resistor voltage, and hence indirectly the current flowing through the capacitor.

   The Ampere law circuit is now assembled and should look as depicted in Fig. 4.

6 Laboratory Measurements

In this section you will measure the characteristics of the circuit assembled in the previous section. The measurement setup is depicted in Fig. 7, with the capacitor

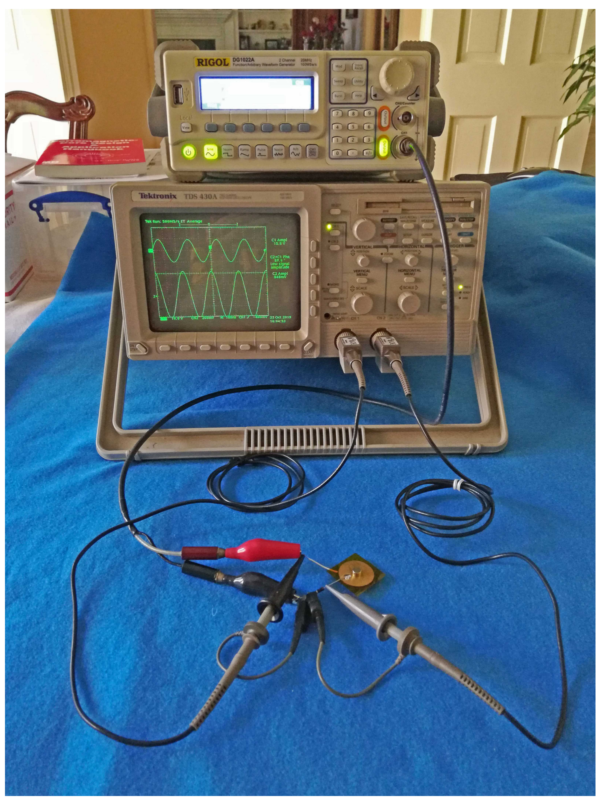


Figure 7: Ampere's law circuit connected to instrumentation.

circuit located near the bottom of the figure, and the voltage source sitting on top of the oscilloscope. Although the specific instruments that you will be using may differ from what is shown, since they are subjected to current availability, their functionality will not.

1. Power up the oscilloscope.

2. Before proceeding beyond this point, estimate the maximum voltage that your capacitor can take before dielectric breakdown occurs (i.e., the capacitor voltage rating).

   To perform this estimate, note that the tabulated dielectric strengths of dry air and Kapton (the dielectric of your capacitor) are about

$$e_{max}|_{air} = 3\,\mathrm{V}/\mu\mathrm{m}\,, \tag{28}$$

$$e_{max}|_{Kpt} = 240\,\mathrm{V}/\mu\mathrm{m}\,, \tag{29}$$

   respectively. These are the values of the electric field beyond which sparks will happen in either air or Kapton, respectively. These two values then determine the maximum voltage that can be applied to the capacitor, since if a spark starts either on air or Kapton it eventually shorts the capacitor, and the capacitor stops behaving as a capacitor. It is then very important to determine the approximate maximum voltage that can be safely applied to the capacitor, and not to significantly exceed it.

   At first you probably would think that Eq. 22 directly provides the capacitor voltage rating. However, the situation is far more complicated since the largest electric field occurs near the edge or the discs (regions with $r = a$ and $z = 0, \ell$ in Fig. 2) and is dictated by the (so far ignored) fringing fields, which depend strongly on the capacitor geometry and dielectric permittivity. Also, since the points with $r = a$ and $z = 0, \ell$ are at the very boundary between metal, air, and dielectric, if dielectric breakdown occurs the spark will start on the material with the smallest dielectric strength (i.e., either dielectric or air) and straddle the two capacitor disks.

   For a capacitor with a Kapton dielectric sheet extending significantly beyond the discs' edge, the edge field (i.e., field at $r = a$ and $z = 0, \ell$) is about 10 times the field at $r = 0$ (i.e., a factor $\gamma_{Kpt} = 10$). And for a capacitor with a air dielectric, the edge field (i.e., field at $r = a$ and $z = 0, \ell$) is about 5 times the field at $r = 0$ (i.e., a factor $\gamma_{air} = 5$).[2]

   Use the space below to estimate the voltage rating of your capacitor with both Kapton and air dielectrics, and write the results in the corresponding boxes. The extra space above the boxes is for you to write your equations and comments (i.e.,

[2] R. Blecic, Q. Diduck, and A. Baric, *Minimization of maximum Electric Field in High-Voltage Parallel-Plate Capacitor*, Proceedings of the 2016 39th International Convention on Information and Communication Technology, Electronics and Microelectronics, MIPRO 2016, May 30 – June 3, 2016, Opatija, Croatia, pp. 99 – 103.

write your rational for the boxed numbers).

Kapton capacitor voltage rating, v_{max} (in V)	

Air capacitor voltage rating, v_{max} (in V)	

Note that the above values are only estimates, since dielectric breakdown is notoriously difficult to determine accurately. The reason is that it depends on usually hard to control factors, such us the actual shape and roughness of the disks edges (both at macroscopic and microscopic levels), the purity of the dielectric material, the air and dielectric humidities, etc. For this reason we will use the above numbers only as guidelines, and not as hard limits, and keep an eye out for the effects of dielectric breakdown in our capacitor circuit.

3. Power up the voltage source (i.e., the arbitrary waveform generator), select a time-harmonic signal (i.e., sine wave) and adjust the signal frequency to 1 MHz.

4. Adjust the voltage amplitude of the arbitrary waveform generator to 20 V peak-to-peak (10 V amplitude), provided that this value does not significantly exceed the voltage rating of your capacitor.

5. Turn on the output of Channel 1 of the arbitrary waveform generator.

6. Power up the oscilloscope, find out how to reset it to its factory setup, and perform the reset. This will clean up any undesirable residual settings that might have been left behind by previous users.

7. Modern oscilloscopes have a useful automated setting button, find and press it and you should see the desired time-harmonic signal on the oscilloscope screen (the button is usually labeled Autoset or Autoscale).

8. Make sure that both Channels 1 and 2 are displayed on the oscilloscope screen and adjust the traces' positions and amplitudes to your convenience.

9. Take a quick reading of the voltage waveform across the capacitor plates. It should be a clean sinusoid with a peak-to-peak value close to the previously adjusted 20 V. If this is not the case you will need to troubleshot your circuit.

   If, after trouble shooting, the voltage across the capacitor plates is still significantly smaller than 20 V peak-to-peak, a dielectric breakdown may have occurred and the capacitor plates may now be partially shorted. In this case the capacitor dielectric has been compromised. You should then turn off the output of the arbitrary waveform generator, open the capacitor, and replace the Kapton dielectric film.[3]

   Note that modern digital oscilloscopes have many useful features and you may want to use some of them to both expedite and increase the accuracy of your your measurements. In particular, they allow you to automatically average several measurements to reduce measurement noise. It is suggested that you locate and familiarize yourself with this averaging option and use it in your measurements, since the voltage across the resistor terminals (i.e., v_R) can be relatively noisy.

10. Use the oscilloscope to precisely measure the voltage peak-to-peak value across the capacitor plates (i.e., v_{Cpp}), the voltage peak-to-peak value across the resistor terminals (i.e., v_{Rpp}), and the phase difference between the two voltages (i.e., $\angle v_{Rpp} - \angle v_{Cpp}$) for several different frequencies, and write it in the fields of the table below. Make sure to add the phase values with the proper sign.

    In case you are wondering, modern digital oscilloscopes are also capable of automatically measuring and numerically displaying peak-to-peak as well as phase values.

[3] The author experienced dielectric breakdown a few times while developing this laboratory experiment, but only with a film of about 12 μm thick that was not Kapton.

Meas. number	Frequency (in MHz)	Capacitor peak-to-peak voltage, v_{Cpp} (in V)	Resistor peak-to-peak voltage, v_{Rpp} (in mV)	$\angle v_{Rpp} - \angle v_{Cpp}$ (in deg)
1	1.0			
2	2.0			
3	3.0			
4	4.0			

Observing that the resistor voltage is in phase with its current, confirm that the phase of the capacitor current leads the capacitor voltage by the expected amount.

11. Calculate the amplitude of the resistor voltage (i.e., v_R), amplitude of the resistor current (i.e., i_R), and the ratio i_R/ω (in mili Amperes per Mega radians per second), and write them in the table below. The space before the table is left for you to write down the equation that provides i_R.

Meas. number	Frequency (in MHz)	Resistor amplitude voltage, v_R (in mV)	Resistor amplitude current, i_R (in mA)	i_R/ω (in mA/(Mrad/s))
1	1.0			
2	2.0			
3	3.0			
4	4.0			

Observe that, and as predicted by Eq. 27, the ratio of the current flowing on the circuit and the frequency (i.e., i_R/ω) is constant within measurement accuracy.

12. Complete the third and fourth columns of the table below and from them determine the measured capacitance C_{Kpt} of the capacitor, at each frequency. The space below is left for you write down the equation that provides C_{Kpt}.

Meas. number	Frequency (in MHz)	i_R/ω (in mA/(Mrad/s))	Capacitor amplitude voltage, v_C (in V)	Capacitance C_{Kpt} (in pF)
1	1.0			
2	2.0			
3	3.0			
4	4.0			

13. Average the above capacitance values to determine the probable capacitance of the Kapton capacitor and write it on the field below.

Kapton capacitor capacitance, C_{Kpt} (in pF)	

14. Using the above capacitance value, determine the permittivity ϵ_{Kpt} of the Kapton dielectric and write it on the box below. The space below is left for you write down both the equation that provides ϵ_{Kpt} as well as your calculation.

Kapton permittivity, ϵ_{Kpt} (in pF/m)	

15. We will now modify the capacitor to determine the permittivity of air. To do this we need to remove the Kapton dielectric while keeping the discs' separation. This can be done by cutting four pieces of Kapton film measuring approximately 4 x 2 mm (or less), removing the capacitor from the measuring circuit, removing the two Nd magnets, opening the capacitor, removing the dielectric sheet, replacing the dielectric sheet by the four small Kapton film pieces, assembling back the capacitor, and connecting it to the measuring circuit. You may want to use the jewelers tweezers to assist in performing these tasks.

    Measure and write in the box below the approximate combined area of your four dielectric pieces.

Combined area of the four dielectric pieces (in mm^2)	

    When reassembling the capacitor, put one of the Kapton pieces at the center of the disks (to hold the pressure applied by the Nd magnets and hence assure the plate separation ℓ) and the other three pieces near the disks' rims (to prevent the disks from touching and shorting the capacitor). The principle here is that, since the area of the four PE pieces is much smaller than the area of the disks, for practical purposes the capacitor dielectric is now air.

16. Make sure that both Channels 1 and 2 are displayed on the oscilloscope screen and adjust the traces' positions and amplitudes to your convenience.

17. Use the oscilloscope to measure the voltage peak-to-peak value across the capacitor plates (i.e., v_{Cpp}), the voltage peak-to-peak value across the resistor terminals (i.e., v_{Rpp}), and the phase difference between the two voltages (i.e., $\angle v_{Rpp} - \angle v_{Cpp}$) for several different frequencies, and write it in the fields below. Make sure to add the phase values with the proper sign.

Meas. number	Frequency (in MHz)	Capacitor peak-to-peak voltage, v_{Cpp} (in V)	Resistor peak-to=peak voltage, v_{Rpp} (in mV)	$\angle v_{Rpp} - \angle v_{Cpp}$ (in deg)
1	1.0			
2	2.0			
3	3.0			
4	4.0			

Observing that the resistor voltage is in phase with its current, confirm that the phase of the capacitor current leads the capacitor voltage by the expected amount.

18. Calculate the amplitude of the resistor voltage (i.e., v_R), amplitude of the resistor current (i.e., i_R), and the ratio i_R/ω (in mili Amperes per Mega radians per second), and write them in the table below. The space below is left for you write down the equation that provides i_R.

Observe that, and as predicted by Eq. 27, the ratio of the current flowing on the circuit and the frequency (i.e., i_R/ω) is constant within measurement accuracy.

Meas. number	Frequency (in MHz)	Resistor amplitude voltage, v_R (in mV)	Resistor amplitude current, i_R (in mA)	i_R/ω (in mA/(Mrad/s))
1	1.0			
2	2.0			
3	3.0			
4	4.0			

19. Complete the third and fourth columns of the table below and from them determine the measured capacitance C_{air} of the capacitor, at each frequency. The space below the table is left for you to write down the equation that provides C_{air}.

Meas. number	Frequency (in MHz)	i_R/ω (in mA/(Mrad/s))	Capacitor amplitude voltage, v_C (in V)	Capacitance C_{air} (in pF)
1	1.0			
2	2.0			
3	3.0			
4	4.0			

20. Average the above capacitance values to determine the probable capacitance of the air capacitor and write it on the field below.

Air capacitor capacitance, C_{air} (in pF)	

21. Using the above capacitance value, determine the permittivity of air and write it on the box below. The space above the box is left for you write down both the equation that provides ϵ_{air} as well as your calculation.

Air permittivity, ϵ_{air} (in pF/m)	